ARITHMETIC

IN

THEORY AND PRACTICE

MACMILLAN AND CO., Limited
LONDON · BOMBAY · CALCUTTA
MELBOURNE

THE MACMILLAN COMPANY
NEW YORK · BOSTON · CHICAGO
ATLANTA · SAN FRANCISCO

THE MACMILLAN CO. OF CANADA, Ltd.
TORONTO

ARITHMETIC

IN

THEORY AND PRACTICE.

BY THE LATE

J. BROOK-SMITH, M.A. LL.B.

ST JOHN'S COLLEGE, CAMBRIDGE; BARRISTER AT LAW;
ONE OF THE MASTERS OF CHELTENHAM COLLEGE.

MACMILLAN AND CO., LIMITED
ST MARTIN'S STREET, LONDON
1910

First Edition 1860.
New Editions 1872, 1873, 1877, 1879, 1880, 1881 (*electrotyped*).
Reprinted 1882, 1883, 1884, 1886, 1887, 1889, 1891,
1893, 1895, 1898, 1901, 1903,
1910.

PREFACE TO THE SIXTH EDITION.

In the following pages I have endeavoured to reason out in a clear and accurate manner the leading propositions of the science of Arithmetic, and to illustrate and apply those propositions in practice.

Every writer on Arithmetic at the present day feels the necessity of explaining the principles upon which the rules of the subject are based, but every writer does not as yet feel the necessity of making these explanations strict and complete; or, failing that, of distinctly pointing out their defective character. Difficulties are still avoided or slurred over, and incomplete proofs without one word of remark or warning are used as though they were full and satisfactory. This surely ought not to be. If the science of Arithmetic is to be made an effective instrument in developing and strengthening the mental powers, it ought to be worked out rationally and conclusively.

In the practical part of the subject, I have advanced somewhat beyond the majority of preceding writers; particularly in Division, in Greatest Common Measure, in Cube Root, in the Chapters on Decimal Money and the Metric System, and more especially in the application of Decimals to Per-Centages and cognate subjects. So long as the mania for *neat* answers continues to exist, so long will Decimals fail to take their legitimate place in the class-room, and be relegated to the office and the counting-house.

The Chapter on Weights and Measures and the Metric System is longer than usual, but not I hope uninteresting.

The tendency of the present day is to make use of the Metric System in international transactions and scientific pursuits; but to the retail dealer and his customers our present system presents so many advantages, that I almost doubt the possibility of uprooting it. They first divide into halves, into quarters, and into eighths, then into thirds and into sixths, but very rarely into fifths or tenths. The probability therefore is that, in this country, the two systems will exist side by side, the scientific and the practical; just as we have two systems of logarithms, and two ways of measuring angles.

In the earlier part of the work I have used the Method of Reduction to the Unit, but I am far from advising an exclusive adherence to that Method: when the student has gained a clear and firm grasp of ratio, it would be unwise of him to neglect the powerful instrument that has come into his possession.

I have to express my many and great obligations to the College Lectures on Arithmetic of Professor Kelland, and of the late Professor De Morgan. I have also consulted with advantage the works of Loomis, Lionnet, Serret, and Bertrand. Any student wishing for a more *detailed* explanation of the earlier portion of Arithmetic, will meet with it in the admirable treatise of Messrs Sonnenschein and Nesbitt. Of the examples about one half are original; the other half are selected from University and College Examination papers, and from papers given in the various competitive Examinations.

I shall be thankful to receive and glad to acknowledge any suggestions or corrections that may be offered to me on any part of the work.

CHELTENHAM COLLEGE,
June, 1881.

CONTENTS.

ARITHMETIC.

CHAPTER I.

DEFINITIONS. NAMES OF NUMBERS. NOTATION AND NUMERATION.

1. WHATEVER is susceptible of increase or diminution is called a *magnitude*.

2. A magnitude may be *continuous*, that is, whole and undivided, as the length of a field, the water of a reservoir; or it may be made up of *separate and distinct* parts, as a heap of pebbles, a flock of sheep.

3. When a magnitude is continuous, we take some well-defined magnitude of that kind which we call its *unit*, and by repeating this unit a sufficient number of times, we make up the given magnitude. If the magnitude be made up of distinct objects, we take an object of that kind as our *unit*, and observe how many such units must be taken to make up the given magnitude.

4. When a magnitude is represented as made up of repetitions of its unit, it is called a *quantity*, and the result of the comparison of the given magnitude with its unit is called a *number:* thus the length of a field, a heap of pebbles are *magnitudes;* twenty yards, a hundred pebbles are *quantities;* twenty and a hundred are *numbers*.

In Mathematics we are only concerned with those magnitudes that can be referred to a unit and expressed as quantities.

5. Numbers are often spoken of as *concrete* or *abstract* according as the nature of the unit in any particular case is or is not mentioned; thus twenty yards, a hundred pebbles, are called *concrete* numbers, but twenty, a hundred, are called *abstract*.

An abstract number is therefore a number in its proper sense, conveying the idea of times or repetitions; and a concrete number is simply a quantity.

6. ARITHMETIC is the science of numbers:—it explains their nomenclature and notation, it investigates their properties, and points out methods of making calculations by means of them.

NAMES OR NUMBERS.

7. To a unit standing by itself we give the name *one* unit: to one unit and one unit taken together, we give the name *two* units; to *two* units and one unit *three* units: to *three* units and one unit *four* units: and so we may proceed to an endless extent. Or, speaking only of the numbers employed, without reference to any particular unit, we may say—one and one taken together is called *two:* two and one, *three:* three and one, *four;* and so on. But if to each of the successive numbers thus formed we were to give an independent name, our range of numbers would of necessity be very limited. We will therefore shew how by a few independent names, and by a systematic combination of them, we can express all the numbers we require.

8. The names of the first numbers in order are—*one, two, three, four, five, six, seven, eight, nine.* These nine numbers are called *simple numbers*, and *units* of the *first* order. Their names are perfectly arbitrary.

9. To nine and one we give the name *ten.* Ten forms a single unit of the *second* order, and we repeat or count by ten, as we before counted by one; thus

one-ten, two-ten, three-ten, four-ten,......nine-ten;

or treating *ten* as a simple number (9), and remembering that "ty" or tig is equivalent to ten, we say

ten, twenty, thirty, forty,.........ninety.

The names of the numbers between ten and twenty are formed irregularly; the first is eleven, supposed to come from the Gothic *ainlif* (*ain* one, and *lif* ten), the next is twelve from the Gothic *tvalif* (*tva* two, and *lif* ten), and the others explain themselves;

thus we have

eleven, twelve, thirteen, fourteen, fifteen,......nineteen,

The names of the nine numbers between twenty and thirty, thirty and forty,......are formed by placing the names of the first nine numbers (8) in order after twenty, thirty,......

10. Ten units of the second order form a single unit of the *third* order, which is called a *hundred:* and we count by hundreds as we counted by simple units, thus

one hundred, two hundred, three hundred,......nine hundred.

The names of the numbers between one hundred and two hundred, two hundred and three hundred,......are formed by placing the names of the first ninety-nine numbers in order after one hundred, two hundred,......

11. Again ten units of the third order form a single unit of the *fourth* order, which is called a *thousand:* also ten thousands form a single unit of the *fifth* order, and ten ten-thousands or a hundred thousand (10) form a single unit of the *sixth* order: but instead of calling these numbers by independent names, we consider a *thousand* as a second principal unit, and count by units, tens, and hundreds of thousands.

The names of the numbers between one thousand and two thousand, two thousand and three thousand,......are formed by placing the first nine hundred and ninety-nine numbers in order after one thousand, two thousand,......

12. Again ten hundreds thousands or a thousand thousands (11) form a single unit of the *seventh* order, which is called a *million;* and we consider a million as a third principal unit, and count by units, tens, hundreds, thousands, ten-thousands, and hundred-thousands of millions.

The names of the numbers between one million and two millions, two millions and three millions,......are formed by placing in order all the numbers from one to nine hundred and ninety-nine thousand nine hundred and ninety-nine after one million, two millions,......

13. Lastly ten hundred thousand millions, or a million millions (12) is called a *billion*, a million billions a *trillion*, a million *trillions*

a *quadrillion*, and so on; and we count by units, tens, hundreds, thousands, ten-thousands, and hundred-thousands of billions, trillions, quadrillions,...precisely as we do in millions.

The names billions, trillions,...... were proposed by Locke (*Essay concerning Human Understanding, Book ii*, 16) in place of millions of millions, millions of millions of millions,......and are very convenient for scientific purposes, but the wants of ordinary life seldom require that we should proceed beyond millions.

In France and some parts of the United States of America, a *thousand* millions is called a billion, a *thousand* billions a trillion, a *thousand* trillions a quadrillion, and so on; hence our billion is their trillion, our trillion their quintillion, &c.

14. It appears then that

(1) We employ practically no more than thirteen independent words: *one, two, three, four, five, six, seven, eight, nine, ten, hundred, thousand, million.*

(2) *Ten* units of any order always make one unit of the next higher order.

(3) Every number is made up of units of successive orders, the number of units in any order never exceeding *nine*.

Ex. 1. Five hundred and sixty-nine, may be expressed as

Five hundred *six* tens and *nine* units.

Ex. 2. Seven millions six hundred and four thousand and thirty-two, may be expressed as

Seven millions *six* hundred-thousands *no* ten-thousand *four* thousand *no* hundred *three* tens and *two* units.

EXERCISE 1.

Express the following numbers as made up of units of each successive order from the highest to the lowest:

1. Sixty-seven thousand and forty-eight; forty thousand and forty.

2. Nine hundred and six thousand five hundred and four.

3. Forty-five millions seven hundred and thirty-four thousand six hundred and ninety-one.

4. Twenty-seven thousand and five millions eighty-seven thousand three hundred and three.

5. What is the number next less than a thousand? a million?

6. Find the regular expression equivalent to *Eighteen* hundred and seventy-seven; *Eleven* hundred and one; *Twenty-three* hundred thousands.

NOTATION AND NUMERATION.

15. NOTATION is the art of representing the names of numbers by means of a few written characters. The system we use is called the Arabic, because it was introduced into Europe by the Arabs in the twelfth century, but it is undoubtedly of Hindu origin.

The converse process of reading the names of numbers when expressed in written characters is called NUMERATION.

16. *To express the name of any number in figures.*

(1) Represent the first nine numbers by the following nine characters, called *figures* or *digits:*

1, 2, 3, 4, 5, 6, 7, 8, 9,

and represent the absence of units by a tenth figure 0, called *nought* or *cipher.*

(2) Express the given number as made up of successive orders, from the highest to the lowest, taking care that no order shall be missing (14, 3); and as the number of units in each order is either one of the first nine numbers, or is nought, replace the number of units in each case by its corresponding figure.

(3) Write now the figures of each order in succession in horizontal line beginning with the highest; and adopt the convention that

Figures occupying the first, second, third...place from the right shall represent units of the first, second, third...order;

and the given number will be completely represented in figures.

Ex. 1. Write in figures, Five-hundred and sixty-nine.

This number is—5 hundred 6 tens and 9 units (14, 3); and is therefore written in figures thus 569

Ex. 2. Write in figures, Seven millions six hundred and four thousand and thirty-two.

This number is—7 millions 6 hundred thousand 0 ten-thousand 4 thousand 0 hundred 3 tens and 2 units (14, 3); and is therefore written in figures thus 7604032.

17. If the given number contains thousands, we shall find it convenient—*First to write down in figures the number expressing the* thousands, *and to its right the rest of the number:—and if the*

rest does not contain three figures, we must prefix the requisite number of ciphers.

Ex. 3. Write in figures Twenty-five thousand three hundred and forty-six.

The number of *thousands* is expressed by 25, and the rest by 346; therefore the given number is expressed by 25,346.

Ex. 4. Write in figures Eighty-thousand and forty-six.

The number of *thousands* is 80, and the rest is 46 or 046; therefore the given number is 80,046.

And if the given number contains higher orders than hundred thousands:—*Write down in figures the number of units of the highest principal order, to its right the number of the next principal order,...then the number of millions, and lastly the rest of the number:—and if any order except the first does not contain six figures, prefix the requisite number of ciphers.*

Ex. 5. Express in figures Forty-six billions three hundred thousand and sixty-nine millions four thousand and forty.

The number of *billions* is expressed by 46, of *millions* by 300069, and the rest by 4040 or by 004040; therefore the number is expressed by 46,300069,004040.

18. To the general rule in Notation there corresponds the following rule in Numeration:—

Proceeding from left to right, write down the name of each figure, annexing the order of its units.

Ex. 1. 736 represents Seven hundreds three tens and six units: or more simply, Seven hundred and thirty-six.

Ex. 2. 910 represents Nine hundreds one ten and no units, or simply, Nine hundred and ten.

But if the number does not contain more than six figures we shall find it convenient:—*To mark off the last three figures to the right, then to write down the period to the left as a number by itself annexing* thousands, *and then the other period.*

Ex. 3. 763054 is marked off thus:—763,054, and represents Seven hundred and sixty-three *thousands* and fifty-four.

And if the number contains more than six figures:—*Mark off the last six figures to the right, then the next six, and so on until not more than six remain: proceeding now from left to right, write down each period as a number by itself, annexing the order of units it represents.*

Ex. 4. 34567008093402 is marked off thus:—34,567008,093402, and is read Thirty-four *billions* five hundred and sixty-seven thousand and eight *millions*, ninety-three thousand four hundred and two.

19. In the system of Notation we have explained, a figure has two distinct values—one *absolute* or *intrinsic*, depending on its shape, the other *local*, depending on the place it occupies in a given number: thus in 65345251 the figure 5 in one place represents *five tens*, in another *five thousands* and in another *five millions*.

20. The number of units in any order which is taken to form a unit of the next higher order is called the *base* of the system. Hence in our system the base is *ten*, and it is therefore called the *decimal* system. Had the base been *twelve*, it would have been called the *duodecimal*. We may also remark that when the base is ten we require ten figures, nine significant and the cipher; and in like manner if the base were twelve, we should require twelve figures.

EXERCISE 2.

Express in figures:—

1. Seven hundred and seven thousand and seventy.
2. Twelve millions twelve thousand and twelve.
3. Six hundred and forty millions sixty-four thousand six hundred.
4. Eight hundred and seven billions eighty thousand and eight millions six hundred thousand and fifty.

Express in words the following numbers:

5. 90960; 70047; 600304; 780983; 6008029; 50706004.
6. 407017170; 370094586061; 304007360020095347.

7. Express in words the following number, both after the English and the French method,—696536483318640035073641037.

8. Write down the greatest and least numbers of *three* figures. How many numbers are represented by three figures?

CHAPTER II.

THE FOUR FUNDAMENTAL OPERATIONS.

ADDITION.

21. ADDITION is the operation by which we find a single number that is equal to two or more given numbers put together.

This single number is called the *sum* of the given numbers.

22. Case I. *To find the sum of two simple numbers.*

For example find the sum of 7 and 3. 3 is the sum of 2 and 1 (8), and therefore the sum of 1, 1 and 1: and adding each of these ones in succession to 7, we have 7 and 1 is 8, 8 and 1 is 9, 9 and 1 is 10; that is, the sum of 7 and 3 is 10. Hence, to add 3 to 7 we have simply to count forwards 3 steps from 7 thus—8, 9, 10: and the last number gives the sum. And in the same way may be found the sum of every two simple numbers.

23. We shall afterwards see that finding the sum of *any* two or more numbers depends on finding the sum of every two simple numbers: it is therefore necessary that all such sums should be thoroughly learnt by heart.

24. Case II. *To find the sum of* any *two or more numbers.*

(1) Suppose each of the numbers decomposed into its simple units, tens, hundreds,...; and find the sum of all the simple units, the sum of all the tens, the sum of all the hundreds,...; the sum of these partial sums will give the sum required, for it will contain all the parts which make up all the given numbers and no more. For convenience in effecting these partial sums write the numbers under one another, so that units of the same order may be in the same vertical column, and draw a line below. In this way the sum of 231, 423 and 135 is found to be 9 units, 8 tens and 7 hundreds, or 789.

$$\begin{array}{r} 231 \\ 423 \\ \underline{135} \\ 789 \end{array}$$

(2) If the sum of the units in any order exceeds 9, we avail ourselves of the following principle usually termed *carrying:*

The tens of any order in a partial sum may be carried *as units to the next higher order;*

for ten units of any order are equivalent to one unit of the next higher order (14).

(3) Lastly instead of finding all the partial sums, and then carrying the tens of any order as units to the next higher order, we may add the number carried from any column to the *first number* in the next column, instead of to the *sum* of that column; and as the carryings are made from any column to the next one on the left, we ought to begin the work at the first column on the right.

We have then the following general Rule:

(1) *Write the numbers under one another, so that units of the same order may be in the same vertical column, and draw a line underneath.* (2) *Begin at the first column on the right, and find the sum of the numbers in that column: set down below the column the units' figure of the sum and carry the tens to the first figure of the next column.* (3) *Having carried thus, find the sum of the second column: set down and carry as before.* (4) *Proceed in this way through all the columns, and below the last write down its sum at full length.*

25. A Proof is a second operation which serves as a test of the correctness of the first.

The Proofs of Addition depend on this principle The *sum* of several numbers is not affected by the *order* in which they are added together. For the sum of the units in the three following groups,

IIIIIII, IIIII, III,

will be the same in whatever order we add them together: thus the sums of 7, 5 and 3; of 5, 3 and 7; of 3, 5 and 7 are the same.

26. Proof. *As we usually begin at the bottom of each column and add upwards, in the proof begin at the top of each column and add downwards;—if at each step the results coincide, we may presume that the work is correct.*

Ex. Find the sum of 82093, 9386, 51764, 475 and 123897.

```
 82093
  9386
 51764
   475
123897
------
267615
```

Beginning at the units' column and repeating the result only of each step, say 12, 16, 22, 25: set down 5, and carry 2 to the first figure of the second column;

then say 11, 18, 24, 32, 41: set down 1 and carry 4;
12, 16, 23, 26, set down 6 and carry 2:

and so proceed through all the columns.

For the Proof, begin at the top of each column and add downwards: thus for the units' column say, 9, 13, 18, 25; for the second column 11, 19, 25, 32, 41, &c. And as at each step these results coincide with the former ones we presume that the work is correct.

Exercise 3.

1. Find the sum of 4578, 34796, 2385, 97, 50405 and 56789.

2. Add together 125, 309486, 7098, 67, 11000, 2106 and 10785.

3. Find the sum of six numbers each equal to 7903856.

4. Add together the sum of five numbers each equal to 4597, and the sum of four numbers each equal to 89796.

5. Add together, Eighty millions sixty-seven thousand and eighteen; nine millions seven hundred and six thousand five hundred and nine; eight hundred and one millions nine hundred and seventy thousand seven hundred and sixty; seven millions and seventy-seven; sixty millions six hundred and six thousand and sixty six; and five hundred and fifty-five thousand and fifty.

6. Find the sum of 69798856 and the six following numbers.

7. In 1871 the population of England and Wales was 22704108, of Scotland 3358613, of Ireland 5402759, of Islands in the British Seas 144430, and of the Army and Navy, &c. 207198: find the total population of the United Kingdom at that date.

8. Find the net revenue of the United Kingdom for the year 1871-2, as derived from the following sources:—Customs £20143044, Excise £23386064, Stamps £9739548, Land Tax and House Duty £2352181, Income Tax £9328103, Post Office £4941510, Telegraph Service £751611, Crown Lands £446801, and Miscellaneous £4060315.

SUBTRACTION.

27. SUBTRACTION is the operation by which we find what number is left when a smaller number is taken from a greater.

The greater number is called the *Minuend*, the smaller one the *Subtrahend*, and the number left the *Remainder*.

28. The number left is the *difference* between the two given numbers; it is also the *excess* of the greater number over the less; it is also the number which must be *added* to the less number to make it equal to the greater.

29. Subtraction is the inverse of Addition:—in Addition two numbers are given to find their sum; in Subtraction the sum of two numbers is given and one of the numbers to find the other.

30. Case I. *When the Subtrahend is a simple number, and the Minuend less than that number increased by* 10.

For example subtract 3 from 7. Now 3 is the sum of 1, 1 and 1 (22): and subtracting each of these ones in succession from 7 we have—1 from 7 leaves 6 (8), 1 from 6 leaves 5, and 1 from 5 leaves 4; therefore 3 from 7 leaves 4. Hence, to subtract 3 from 7 we have simply to count backwards 3 steps from 7, thus—6, 5, 4; and the last number gives the Remainder.

Or, we may ask to what number must we add 3 to get 7: the answer is 4; hence if from 7 we take away 3 the remainder is 4.

31. Case II. *When Minuend and Subtrahend are* any *numbers.*

(1) Suppose the numbers to be decomposed into their simple units, tens, hundreds..., and subtract the units, tens, hundreds... of the Subtrahend, from the corresponding orders of the Minuend; the sum of these partial remainders will give the remainder required, for it gives what is left when all the parts of the Subtrahend have been taken away from the Minuend. For convenience place the Subtrahend under the Minuend so that the units of the same order may be in the same vertical column, and draw a line below. In this way the remainder in subtracting 235 from 978 is found to be 3 units 4 tens 7 hundreds, or 743.

$$\begin{array}{r} 978 \\ \underline{235} \\ 743 \end{array}$$

(2) If the units of any order in the Subtrahend exceed those of the Minuend we avail ourselves of the following principle, usually termed *borrowing*;

The Minuend and Subtrahend may be increased by the same number without altering their difference:

hence we may increase the number of units in any order of the Minuend by 10, if we increase that of the next higher order in the Subtrahend by 1.

(3) Lastly if we proceed from right to left, the partial remainders at each step will be *finally* obtained, for the borrowings are always made from one order to the next higher order; and having found all the partial remainders we have found the final remainder.

We have then the following Rule:

(1) *Write the Subtrahend under the Minuend, so that units of the same order may be under one another, and draw a line below.*
(2) *Beginning at the units' figure, subtract each figure of the Subtrahend from the one above it in the Minuend, and place the remainder immediately below; and if in any case the figure of the Subtrahend be greater than the one above it, add* 10 *to the latter figure and then subtract, taking care to add* 1 *to the next figure of the Subtrahend.*

32. We may use either of the following proofs in Subtraction :—

(1) *Add the Remainder to the Subtrahend*:—we ought to obtain the Minuend (28).

(2) *Subtract the Remainder from the Minuend*:—we ought to obtain the Subtrahend.

Example. Subtract 28549 from 54627.

54627 Minuend	Beginning at the units' figures, proceed thus:—
28549 Subtrahend	9 from 17 leaves 8: set down 8;
26078 Remainder	4 and 1 is 5, 5 from 12 leaves 7: 7;
	5 and 1 is 6, 6 from 6 leaves 0: 0;
	8 from 14 leaves 6: 6;
	2 and 1 is 3, 3 from 5 leaves 2: .... 2.

But the student should accustom himself to repeat the following numbers only:—

9, 8: 5, 7: 6, 0: 8, 6: 3, 2.

where the first number is the figure or increased figure of the Subtrahend, and the second the corresponding figure of the Remainder.

Proofs. Adding the Remainder to the Subtrahend each step gives the corresponding figure in the Minuend. Or, subtracting the Remainder from the Minuend we get at each step the figure in the Subtrahend.

Remark. In France it is usual to *borrow* from the next order in the Minuend; and the preceding example would be gone through thus:—

9 from 17 is 8; 4 from 11 is 7; 5 from 5 is 0; 8 from 14 is 6; 2 from 4 is 2.

In this way the borrowing is delayed one step later than in our method; and this is a disadvantage.

32*. Since the Remainder *added* to the Subtrahend gives the Minuend, we may at each step find what figure added to the figure of Subtrahend will give the figure of the Minuend.

The preceding example would be gone through thus, where the dark figure is the one to be set down on the Remainder:—

9 and **8** is 17; 5 and **7** is 12; 6 and **0** is 6; 8 and **6** is 14; 3 and **2** is 5.

This method is especially convenient when we are required to take from a number the sum of several numbers. In the following example we proceed thus:—

13, 15 and **3** is 18; 7, 10, 17 and **7** is 24;
10, 19, 25 and **8** is 33; 3 and **2** is 5.

From	5348
Take	672
	934
	869
Remd.	2873

33. The *complement* of a number is its defect from 10 units of the number's highest order: thus the complement of 57364 is its defect from 10,0000.

But 100000 is 99999 and 1: hence we may subtract each of the figures of 57364 from 9, except the units' figure and that we must subtract from 10: therefore its complement is written off *from left to right* at sight thus, 42636.

EXERCISE 4.

1. Subtract 38967 from 123456.
2. Find the difference between 78954 and 836523.
3. What number must be added to 7965499 to give 541850036?
4. From Twenty-three millions thirty thousand two hundred and thirty subtract Eight millions eight hundred and seven thousand and sixty-five.
5. From the sum of 75301 and 6456 subtract the sum of 3087 and 56299.

6. From the difference between 3285 and 456 subtract the difference between 19011 and 17455.

7. What number must be added to the sum of 750 and 3287 to make the result equal to the sum of 505, 650, 19 and 9003?

8. Write off the complement of 43; 574; 9999; 542056; 859064.

MULTIPLICATION.

34. MULTIPLICATION is the operation by which we find the sum of a given number repeated as many times as there are ones in another given number.

The number to be repeated is called the *Multiplicand*, the other the *Multiplier*, and the sum found the *Product*.

The Multiplicand and Multiplier are called the *Factors* of the Product.

35. *If we interchange Multiplicand and Multiplier, the Product remains the same.*

For write down 1 in a horizontal line 7 times, and repeat this line 5 times. The sum of each horizontal line is 7, and there are 5 such lines, therefore the sum of all the ones is 7 multiplied by 5. Again the sum of each vertical line is 5 and there are 7 such lines, therefore the sum of all the ones is 5 multiplied by 7: that is 7 multiplied by 5 is the same as 5 multiplied by 7.

1 1 1 1 1 1 1
1 1 1 1 1 1 1
1 1 1 1 1 1 1
1 1 1 1 1 1 1
1 1 1 1 1 1 1

36. *If the Multiplier be a product of two numbers, we may multiply by each of those numbers in succession.*

Let the Multiplier be 15, the product of 5 and 3: then to multiply 67 by 15, we may multiply 67 by 5 and the result by 3.

For write 67 in a horizontal line 5 times, and repeat this line 3 times, then 67 will be repeated 5 times 3 or 15 times, and therefore the sum of all the numbers written down is 67 multiplied by 15. But the sum of each horizontal line is 67 multiplied by 5, and there are 3 such lines, therefore the sum of all the numbers written down is this product multiplied by 3;—that is to multiply 67 by 15 we may multiply 67 by 5 and the result by 3.

67 67 67 67 67
67 67 67 67 67
67 67 67 67 67

37. Case I. *When Multiplicand and Multiplier are simple numbers.*

For example multiply 7 by 3. Here we have to find the sum of 7 repeated 3 times, or the sum of 7, 7 and 7; but this sum is 21; that is 7 multiplied by 3 is 21.

And in the same way must be found the product of every two simple numbers.

38. We shall afterwards see that the product of *any* two numbers depends on the product of two simple numbers: it is therefore necessary that the product of every two such numbers should be thoroughly learnt by heart. Every such product will be found in the following Table, called the Multiplication Table.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

The first horizontal line gives the numbers 1, 2, 3,...12, and each line is derived from the preceding by adding 1, 2, 3,...12 in order; hence the *second* line gives the respective products of these numbers and 2; the *third* line the respective products of these numbers and 3; and so on. Hence to find the product of 7 and 3 we look for the vertical column with 7 at the top and in the third line we

find 21, or (35) we look for the vertical column with 3 at the top and in the seventh line we find 21, the product required.

For the products of simple numbers the Table is carried to 9 times 9 only; but as the products of 10 and 11 are easily learnt and the products of 12 are much used in business transactions, the Table is usually carried to 12 times 12; and the pupil should gradually extend it to 12 times 20.

39. Case II. *When the Multiplicand is any number and the Multiplier a number not greater than 12.*

(1) We have to find the sum of the Multiplicand repeated as many times as there are ones in the Multiplier (34): that is we have to find the sum of the simple units, tens, hundreds,... of the Multiplicand repeated as many times as there are ones in the Multiplier (24); or in other words we have to multiply the simple units, tens, hundreds,... of the Multiplicand by the Multiplier. For convenience write the Multiplier under the units' figure of the Multiplicand and draw a line underneath. In this way, the product of 3122 by 3 is found to be 9366.

$$\begin{array}{r} 3122 \\ 3 \\ \hline 9366 \end{array}$$

(2) If any partial product exceeds 9, we carry as in Addition; that is we set down the units' figure of such product and carry the tens' figure to the next partial product (24, 2).

(3) We begin at the units' figure of the Multiplicand and proceed from right to left for the reason given in Addition (24, 3).

We have then the following Rule:—

(1) *Write the Multiplier under the units' figure of the Multiplicand and draw a line underneath.* (2) *Begin at the units' figure of the Multiplicand, and multiply each figure in succession by the Multiplier, setting down and carrying precisely as in Addition.*

Ex. 1. Multiply 3468 by 7.

$$\begin{array}{r} 3468 \\ 7 \\ \hline 24276 \end{array} \qquad \begin{array}{l} \underline{3468 \times 7} \\ 24276 \end{array}$$

Here say 7 times 8 is 56: set down 6 and carry 5;

7 times 6 is 42 and 5 carried is 47: set down 7 and carry 4;

7 times 4 is 28 and 4 is 32: set down 2 and carry 3;

7 times 3 is 21 and 3 is 24: set down 24.

But in effecting this operation we ought only to say

56; 42, 47; 28, 32; 21, 24.

40. Case III. *When the Multiplier is a simple number followed by one or more noughts.*

(1) Take any number 3468 and multiply it by 10 in the ordinary way: we see that the product is formed by simply placing a nought to the right of the given number. In like manner, we multiply a number by 100, or by 10 times 10, by placing 2 noughts to its right; and by 1000 by placing 3 noughts to its right; and so on.

$$\begin{array}{r} 3468 \\ 10 \\ \hline 34680 \end{array}$$

(2) Suppose now that the Multiplier is 400,—then since 400 is the product of 4 and 100, we may multiply the Multiplicand by 4 and the result by 100 (36); that is we may multiply by 4 and then place 2 noughts to the right of the result.

We have then this Rule:—

Multiply the Multiplicand by the simple number, and to the right of the result place as many noughts as there are noughts to the right of the Multiplier.

Ex. 2. Multiply 5867 by 70; and by 400.

$$\begin{array}{r} 5867 \\ 70 \\ \hline 410690 \end{array} \qquad \begin{array}{r} 5867 \\ 400 \\ \hline 2346800 \end{array}$$

41. Case IV. *When Multiplicand and Multiplier are any numbers.*

Multiply 5867 by 2479. The product is the sum of 5867 repeated 2479 times; and 2479 is the sum of 9, 70, 400 and 2000; if therefore we repeat 5867 9 times, then 70 times, then 400 times, and then 2000 times, we shall have repeated it in all 2479 times; that is the product of 5867 by 2479 is the sum of the products

of 5867 multiplied by 9, by 70, by 400, and by 2000. The first of these partial products is found by Case II., and the others by Case III.

For convenience, write the Multiplier under the Multiplicand and draw a line below; and under this line place as they arise the products by 9, by 70, by 400, and by 2000; and arrange that in Multiplicand, Multiplier, and the partial products, units of the same order may be under one another, and therefore the arrangement will hold for the final product: thus—

5867		5867	(B)
2479		2479	
52803	– product by 9	52803	
410690	= 70	41069	
2346800	= 400	23468	
11734000	= 2000	11734	
14544293	= 2479	14544293	

And if instead of multiplying by 70, 400 and 2000 we were to multiply by 7, 4 and 2, and to place the units' figure of the respective products under the figure we were multiplying by, we should obtain the same result, and our work would stand as at (B).

We have then the following general Rule:—

(1) *Write the Multiplier under the Multiplicand, so that units of the same order may be under one another, and draw a line underneath.* (2) *Begin at the units' figure of the Multiplier and multiply by each of its figures in order, writing down each partial product so that its first figure shall be under the figure of the Multiplier that produces it.* (3) *Add together these partial products; their sum is the product required.*

42. Case V. *When Multiplicand and Multiplier are both terminated by noughts.*

Multiply 347000 by 2100. We multiply by 2100 by multiplying by 21 and placing 2 noughts to the right of the result (40), and we multiply 347000 by 21—which we will suppose to be done in one operation by Case II.—by first setting down 3 noughts and then multiplying 347 by 21;—that is we multiply 347000 by

2100 by multiplying 347 by 21 and then placing 3 noughts and then 2 noughts to the right of the result, that is, as many noughts as there are in Multiplicand and Multiplier together. Hence we have this Rule :—

Suppose the noughts at the right of Multiplicand and Multiplier suppressed, find the product of the resulting numbers, and to the right of this product place as many noughts as were supposed to be suppressed in Multiplicand and Multiplier together.

Ex. 3. Multiply 365700 by 70; and by 67000.

365700	365700
70	67000
25599000	25599
	21942
	24501900000

43. We may adopt the following Proofs in Multiplication:

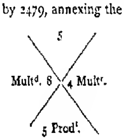
Proof I. *Interchange Multiplicand and Multiplier: the product ought to be unaltered* (35).

Proof II. *By casting out the nines.* We cast the nines out of a number thus: add together all its figures, omitting every 9, and if the sum be greater than 9, replace it by the sum of its figures, and if the new sum be greater, replace it by the sum of its figures, and so proceed till we have a sum less than 9.

Cast the nines out of Multiplicand and Multiplier. Multiply the results, and cast the nines out of their product, noting the new result; now cast the nines out of the Product, and if the result coincide with the one previously noted we presume that the work is correct. This process is explained at Art. 91.

Ex. 4. Multiply 5867 by 2479, annexing the proofs.

5867...8	2479
2479...4	5867
52803...5	17353
41069	14874
23468	19832
11734	12395
14544293...5	14544293



Beginning at the left hand we cast the nines out of the Multiplicand thus —13, 19, 26; replace 26 by the sum of 2 and 6, or 8;—out of the Multiplier thus—6, 13; replace 13 by the sum of 1 and 3, or 4:—now multiply 8 by 4 giving 32, which replace by the sum of 3 and 2, or 5; and note this result. Again we cast the nines out of the Product thus—5, 10, 14, 18, 20, 23; replace 23 by the sum of 2 and 3, or 5; and as this result coincides with the previous one, we presume the work is correct.

Ex. 5. Multiply 43896 by 357, and by 735: making in each case only two partial multiplications.

43896	43896
357	735
307272	307272
1536360	1536360
15670872	32263560

Here we first multiply by 7 and then the result by 5: that is, we first multiply by 7 and then by 35 (36). In multiplying by 7 we set down the first figure of its product under 7 (41, 2); and in multiplying by 35 we set down the first figure under the 5 of 35.

Exercise 5.

1. Multiply 6508794 by 8, by 7, by 9, by 11 and by 12.

2. Multiply 56380477 by 18, by 35, by 48, by 72 and by 132—in each case by two successive multiplications, and also by the usual method.

3. Multiply 67836479 by 356, by 4378 and by 78539.

4. Multiply 70870096 by 404, by 3009 and by 900807.

5. Multiply 57483000 by 40, by 900, by 430 and by 4670000.

6. What is the difference between 23456 multiplied by 996, and the remainder in subtracting 4 times 23456 from 13456000?

7. If the area of England and Wales is 58325 square miles, and there is an average population of 317 persons to every mile, find the total population.

8. If a locomotive travelled from London to Rugby at a uniform speed of 1223 yards a minute, it could perform the distance in 121 minutes and have 380 yards to spare—find the distance between the two places in yards.

9. Multiply 324567 by 486, and by 936, and by 13212, making in each case only two partial multiplications. Multiply 765389 by 64164, and by 189279, and by 83256, making in each case only three partial multiplications.

10. There are 5 piles of shot in an arsenal: the first consists of 8436 68-pounders, the second of 11440 18-pounders, the third of 24395 9-pounders, the fourth of 2567 95-pounders, and the fifth of 7580 36-pounders: find the weight in pounds of the five piles.

DIVISION.

44. DIVISION is the operation by which we find how many times one given number contains another given number.

The first of these numbers is called the *Dividend*, the second the *Divisor*, and the number telling *how many times* the *Quotient.*

45. Since the Quotient tells how many times the Dividend contains the Divisor, it follows that the Dividend is the product of Divisor and Quotient. But since the terms of this product are interchangeable (35), the Divisor may be taken to represent a number of times, and then the Quotient will represent the number to be taken each time. Hence we may further say:—

DIVISION is the operation by which we break up a given number into as many equal parts as there are ones in another given number, and thus find one of these parts.

It is under this view of Division that we have adopted the terms Dividend and Divisor.

46. The Quotient can always be found by successive subtractions of the Divisor; for to divide 26 by 8 means that we are to find how many times 26 contains 8, and the operation at the side shews that 26 contains 8, 3 times with a remainder 2. It also shews that 26 divided into 8 equal parts gives 3 in each part, leaving 2 unoperated upon, that is with remainder 2.

$$\begin{array}{r} 26(1,\ 1,\ 1 \\ \underline{8} \\ 18 \\ \underline{8} \\ 10 \\ \underline{8} \\ 2 \end{array}$$

Here 26 is called the Dividend, 8 the Divisor, and 3 the Quotient.

47. When there is no remainder the division is said to be *exact*, and the Dividend is the product of Divisor and Quotient; but when there is a remainder the Dividend is the product of Divisor and Quotient increased by this remainder.

48. Case I. *When the Divisor is a simple number and the Dividend less than* 10 *times that number.*

Divide 35 by 7. By successive subtractions of 7, putting 1 for each subtraction in the Quotient (46), we find that 35 contains 7 5 times exactly; that is, the Quotient is 5. Or, from the Table we *know that 7 taken 5 times is 35; that is, the Quotient is 5.*

Divide 38 by 7. Now 35 contains 7 5 times, and 38 exceeds 35 by 3, where 3 is less than 7: hence 38 contains 7 5 times with a remainder 3: that is the Quotient is 5 with remainder 3.

49. Case II. *When the Dividend is any number and*

(1) *the Divisor is not greater than* 12. Short Division.

(2) *the Divisor is any number.* Long Division.

For example divide 368549 by 678. Instead of successive subtractions, putting 1 for each subtraction in the Quotient (46), we will assume that at any step we may subtract any number of times the Divisor, if we put this number of times in the Quotient.

For convenience draw curved lines on either side of the Dividend, placing the Divisor on the left, and reserving the right for the Quotient; thus:—

```
678 ) 3685,49 ( 500              678 ) 3685,49 ( 543
      339000     40                    3390
       2954,9     3                     2954
       27120                            2712          (B)
         2429                            2429
         2034                            2034
          395                             395
```

From the left of the Dividend cut off a number not less than 678 but less than 678 multiplied by 10, or than 6780, that is cut off 3685: this is our *first partial dividend* and represents hundreds. It will be found that 3685 contains 678 5 times but not 6 times: hence the Dividend contains 678 500 times but not 600 times (40): put 500 in the Quotient and subtract 678 multiplied by 500, that is 339000, leaving a remainder 29549.

Form a *second* partial dividend as the first was formed: it will be 2954, representing tens. It will be found that 2954 contains 678 4 times but not 5 times: hence this new dividend contains 678 40 times but not 50 times: put 40 in the Quotient and subtract 678 multiplied by 40, that is 27120, leaving a remainder 2429.

Lastly our *third* partial dividend is 2429 itself, and it contains 678 3 times, but not 4 times: put 3 in the Quotient and subtract 678 multiplied by 3, that is 2034, leaving a final remainder 395.

Hence the Quotient is 543 with a remainder 395.

Now with regard to this operation we may observe:—

(1) The partial dividends represent succeeding *orders* of units: and each partial dividend is formed from the partial remainder by placing to its right, or as it is usually expressed *bringing down*, the next figure of the Dividend.

(2) In determining the figures of the Quotient we are only concerned with the *partial* dividends, and therefore we need only write down at each step the figures necessary to their formation.

(3) Since the partial dividends represent every order of units from the highest down to simple units, the figures of the Quotient will represent the same, and no figure of the Quotient can exceed 9; we may therefore lose sight altogether of the *order* of units in each partial dividend, and take them for what they are intrinsically, writing down the figures of the Quotient as they arise one after another. Our work will then stand as at (B).

From these considerations we have the following general Rule:—

(1) *On either side of the Dividend draw curved lines: place the Divisor on the left, and the figures of the Quotient as they arise on the right.*

(2) *From the left of the Dividend cut off a number not less than the Divisor but less than* 10 *times the Divisor, giving the first partial Dividend. Find how often the Divisor is contained in this dividend; put the figure in the Quotient, multiply the Divisor by it, and subtract the product from the partial dividend.*

(3) *To the remainder bring down the next figure of the Dividend forming the second partial dividend: proceed just as before, and continue the process till all the figures of the Dividend have been brought down, and the final remainder obtained.*

Short Division. When the Divisor is not greater than 12, we shall find no difficulty in forming the partial dividends without writing down the preceding products and remainders: and in this case, it will be convenient to write each figure of the Quotient under the units' figure of the partial dividend that gives rise to it. In every other respect we proceed as in Long Division.

50. The Proof usually adopted in Division is this:—*To the product of the Divisor and Quotient add the Remainder; if the result coincides with the Dividend (47) we presume that the work is correct.*

Ex. 1. Divide 612469 by 7.

```
7 ) 612469          7 ) 612469
    87495...4           87495...4
                            7
                        612469
```

From the left of the Dividend cut off a number not less than 7 but less than 70: that is cut off 61, our first partial dividend. Now 7 is contained in 61 8 times and 5 over; put the 8 under the 1 in 61, and to the remainder 5 bring down the next figure of the Dividend 2, giving 52, the second partial dividend. But 7 is contained in 52 7 times and 3 over: put 7 in the Quotient, and to the remainder 3 bring down the next figure 4 giving 34, the third partial dividend: and so proceed.

The above operation is usually performed in saying;

7 in 61, 8 and 5 over; in 52, 7 and 3 over; in 34, 4 and 6 over; in 66, 9 and 3 over; in 39, 5 and 4 over;

but the pupil should accustom himself to bring down the figures of the Dividend to each remainder without previously mentioning the remainder, saying only:

61, 8; 52, 7; 34, 4; 66, 9; 39, 5 and 4 over.

For the Proof we multiply the Quotient 87495 by 7 and add the remainder 4; and as the result coincides with the Dividend we may suppose that the work is correct.

Ex. 2. Divide 104051456 by 12.

```
12 ) 104051456          Here we say:—
      8670954...8       104, 8; 80, 6; 85, 7; 11, 0;
             12         114, 9; 65, 5; 56, 4 and 8 over.
     104051456
```

Ex. 3. Divide 39875365 by 8654.

```
8654 ) 39875365 ( 4607          8654
       34616                    4607
        52593                  60578    Proof.
        51924                 519240
          66965              34616
          60578                 6387
           6387             39875365
```

Cut off from the left of the Dividend a number not less than 8654 but less than 86540; that is cut off 39875, the first partial dividend. It contains the Divisor 4 times; put 4 in the Quotient, multiply 8654 by 4, placing the product under 39875, and subtract, leaving 5259.

To the remainder 5259 bring down the next figure of the Dividend 3, giving 52593, the second partial dividend. It contains the Divisor 6 times, put 6 in the Quotient, multiply 8654 by 6, placing the product under 52593, and subtract, leaving 669.

To 669 bring down the next figure 6, giving 6696, the third partial dividend. It contains the Divisor 0 times; put 0 in the Quotient, and the remainder is now 6696.

To 6696 bring down the last figure 5, and proceed as before.

PROOF. Multiplying the Divisor 8654 by the Quotient 4607 and adding the remainder 6387, we obtain the Dividend.

51. We may dispense with writing down the products under each partial dividend, by carrying on the several steps of the multiplication and subtraction at the same time. To render each step of the subtraction possible, we may increase any order in the dividend by the requisite number of *tens*, if we increase the next order in the product by the same number of units (31, 2). And instead of subtracting the less from the greater at each step, we shall find it much more convenient to add to the less the number which gives the greater: thus instead of saying 28 from 35 leaves 7 we shall say 28 and **7** is 35, where the dark figure **7** is to be set down as soon as it has been pronounced. Therefore in the following example we say—

```
8654)68015,637(7859
       74376
        51443
         81737
          3851
```

28 and **7** is 35, where we have borrowed 30:
35 and 3 carried is 38; 38 and **3** is 41:
42 and 4 .... 46; 46 and **4** is 50:
56 and 5 .... 61; 61 and **7** is 68.

For the next step we say—32 and **4** is 36; 40, 43, and **4** is 47; 48, 52 and **1** is 53; 64, 69 and **5** is 74.

52. Case III. *When the Divisor is the product of two or more factors.*

If we multiply a number by 3 and its product by 5 we multiply the number by 15; hence, conversely, if we divide a number by 5 and its quotient by 3 we divide the number by 15.

Divide 4586 by 105; the factors of 105 being 3, 5 and 7. Dividing in succession by 3, 5 and 7 the final quotient is 43, and the partial remainders are 2, 3 and 4: we must now find the final remainder.

```
3)4586
5)1528...2
 7)305...3...9...11
    43...4...60...71
```

For the first remainder to disappear 4586 must be diminished by 2; for the second remainder also to disappear 1528 must be diminished by 3, and therefore 4586 by 3 times 3 or 9; hence for both remainders to disappear 4586 must be diminished by 9 and by 2, or by 11. For the last remainder to disappear 305 must be diminished by 4, and therefore 1528 by 5 times 4 or 20, and therefore 4586 by 3 times 20 or 60; hence for all the remainders to disappear and the quotient 43 to be exact, 4586 must be diminished by 60 and by 11, or by 71. All which shews us, that when 4586 is divided by 3 the remainder is 2, when divided by 15 the remainder is 11, and when divided by 105 the remainder is 71.

Hence we deduce the following Rule :--

(1) *The Quotient is found by dividing in succession by each of the factors of the Divisor:*

(2) *The final remainder at each step is found by multiplying its particular remainder by all the Divisors preceding its own, and adding the preceding final remainder*

Ex. 4. Divide 87654321 by 378, whose factors are 6, 7 and 9.

Dividing in succession by 6, 7 and 9 the particular remainders are 3, 4 and 6. The final remainder at the first step is 3; the final remainder at the second step is found thus:—multiply its remainder 4 by the preceding divisor 6 giving 24, to which add the preceding remainder 3 giving 27. The final remainder at the third step is found thus:—multiply its remainder 6 by the preceding divisors 7 and 6 giving 252, to which add the preceding final remainder 27 giving 279.

```
6)87654321
7)14609053...3
 9)2087007...4...24...27
    231889...6...252...279
```

53. Case IV. *When the Divisor is terminated by one or more ciphers.*

(1) To divide a number by 10, 100, 1000,...

Let the dividend be 345678. Now 345678 is the sum of 345670 and 8; but 345670 is the product of 34567 and 10 (40); therefore the quotient of 345670 by 10 is 34567 and therefore the quotient of 345678 by 10 is 34567 with a remainder 8. But if in 345678 we cut off one figure to the right, thus 34567,8;—the number on the left 34567 will give the quotient, and the number on the right 8, the remainder.

In like manner to divide 345678 by 100 we cut off 2 figures to the right, thus 3456,78: the quotient is 3456 and the remainder 78.

And to divide by 1000 we cut off 3 figures, thus 345,678: the quotient is 345 and the remainder 678.

(2) Divide now by 700: 700 is the product of 100 and 7, hence we first divide by 100, and then by 7: that is we first cut off two figures to the right giving the quotient 3456 and a remainder 78, and then we divide 3456 by 7 giving a quotient 493 and a remainder 5: but this remainder 5 we multiply by the first divisor giving 500, and to it add the first remainder 78 giving a final remainder 578 (52), or in other words to the particular remainder 5 bring down the figures cut off 78, giving 578.

```
7 ) 3456,78
    493......5
```

Hence we have the following Rule:—

Cut off all the ciphers on the right of the Divisor and as many figures from the right of the Dividend:—for the quotient, divide the remaining figures of the Dividend by the remaining figures of the Divisor, and for the final remainder bring down to the particular remainder the figures cut off from the Dividend.

Ex. 5. Divide 3687594 by 80 and by 53000.

```
8,0 ) 368759,4          53,000 ) 3687,594 ( 69
      46094......74              318
                                 507
                                 477
                                  30594
```

In the first example in dividing by 8 the remainder is 7, to which we bring down the figure cut off 4, giving 74 for the final remainder.

In the second example the remainder in dividing by 53 is 30, to which we bring down the figures cut off 594, giving 30594 for the final remainder.

EXERCISE 6.

1. Divide 79304684 by 8, by 12, by 7, by 9, and by 11.

2. Divide 935384767 by 37, by 43, by 348, and by 4836.

3. Divide 137876093 by 5605, by 9089, by 40857 and 57085.

4. Find the quotient by short division of 76538959 by 28, and by 64, and by 72, and by 96.

5. Divide 379060808 by 132, and by 196 (whose factors are 4, 7 and 7), and by 378 (whose factors are 6, 7 and 9).

6. Divide 78534826 by 60, by 800, by 12000, and by 3200.

7. Divide 385426973 by 310, by 5900, by 587000, and by 90900.

8. Find the quotient of Fifty-seven millions six thousand and seventy by six thousand and sixty-seven.

9. The product of two numbers is 17037006 and one of them is 4858, what is the other?

10. What number multiplied by 2951 will give 20376655?

11. If the divisor be 3857, the quotient 489, and the remainder 1305, what is the dividend?

12. What number must we subtract from 57385, so that it can be exactly divided by 387; and what number must we add?

13. How many times in succession can 3589 be subtracted from 241462? What will the final remainder be?

14. How many times in succession must 1739 be added to 83487, to make the final sum 200000?

15. If the distance of the Sun from the Earth be 95000000 miles, and light travels from one to the other in 495 seconds, what is the velocity per second?

16. Write in figures, Twelve thousand twelve hundred and twelve.

17. Write down 576987, and under it write the eighth succeeding number, and under this latter the next eighth succeeding number, and so proceed till nine numbers have been written down; find their sum.

18. Take a set of numbers each of which is greater than the one preceding, for example

67, 259, 3005, 46320, 574008, 7654321;

subtract each number from the one following, and add the remainders and the first number together; the sum ought to be the last number.

CHAPTER III.

PROPOSITIONS IN THE FUNDAMENTAL OPERATIONS.

54. THE sign of Addition is + *plus*: thus $7+3$ means that to 7 we are to add 3, and $7+3+2$ means that to 7 we are to add 3 and to the *sum* we are to add 2: although the result will be the same in whatever order we take these numbers together (25).

The sign of Subtraction is − *minus*: thus $7-3$ means that from 7 we are to subtract 3, and $7-3-2$ means that from 7 we are to subtract 3 and from the *remainder* we are to subtract 2.

The sign of Multiplication is × or . *multiplied by* or *into*: thus 7×3 or 7.3 means that 7 is to be multiplied by 3, and $7\times3\times2$ means that 7 is to be multiplied by 3 and the *product* is to be multiplied by 2; although the result will be the same in whatever order these numbers are multiplied together (61). Also $7.3.2$ is called the continued *product* of 7, 3 and 2; and each number is called a *factor* of the product.

The sign of Division is ÷ *divided by* or simply *by*: thus $24\div3$ means that 24 is to be divided by 3, and $24\div3\div2$ means that 24 is to be divided by 3 and the *quotient* is to be divided by 2. Also $24\div3\times2$ means that 24 is to be divided by 3 and then that the quotient is to be multiplied by 2.

To these four signs of *operation* we shall afterwards add two others; those of Involution and Evolution.

55. The sign of Equality is = *equals* or *is equal to*: thus $7+3=10$ means that the *sum* of 7 and 3 is equal to 10.

When two or more numbers connected by signs of operation have a line called a vinculum drawn over them, or are enclosed in a bracket, the whole is to be treated as a *single* number: thus $7+(5-3)$ means that to 7 we are to add the *difference* of 5 and 3; and $(18+7)\div5$ means that the *sum* of 18 and 7 is to be divided by 5.

It therefore follows from the last Art. that $7+3+2=(7+3)+2$; $7\times3\times2=(7\times3)\times2$: and $24\div3\times2=(24\div3)\times2$.

The signs of Addition and Subtraction, $(+)$ and $(-)$, were first introduced by Michael Stifel, in a work published by him at Nuremberg in 1544. The sign of Multiplication $(\times)$ was introduced by William Oughtred in his *Clavis Mathematica*, published in 1631. The sign of Division $(\div)$ was invented by Dr John Pell, an English mathematician of the 17th century. The sign of Equality $(=)$ was introduced by Robert Recorde in his *Whetstone of Wit*, a work on Algebra, published in 1557. The Vinculum was first used by Vieta, the Italian mathematician, and the Bracket by Albert Girarde, a Dutch writer on Algebra.

ADDITION AND SUBTRACTION.

56. *To a number we may add the* sum *of two others by adding them in succession.*

Thus $29+(7+5)=29+7+5$.

For to 29 we are to add 7 increased by 5; if therefore we first add 7 we must increase the result by 5, thus giving $29+7+5$.

In the same way we may shew that

From a number we may subtract the sum *of two numbers by subtracting them in succession; thus:—*

$$29-(7+5)=29-7-5.$$

57. *To a number we may add the* difference *of two others by adding the first and subtracting the second.*

Thus $$29+(7-5)=29+7-5.$$

For to 29 we are to add 7 diminished by 5; if therefore we first add 7 we must diminish the result by 5, thus giving $29+7-5$.

In the same way we may shew that

From a number we may subtract the difference *of two others by subtracting the first and adding the second; thus:—*

$$29-(7-5)=29-7+5.$$

58. Again, let the number to be added or subtracted be an expression made up of additions and subtractions, as $7-5+3$: then looking upon $7-5$ as a single number, we have from the preceding cases

$$29+(7-5+3)=29+(7-5)+3=29+7-5+3$$

and $$29-(7-5+3)=29-(7-5)-3=29-7+5-3.$$

59. *If an addition and a subtraction, or vice versâ, have to be performed in succession, we may invert their order, provided the resulting expression be possible.*

For $9=9-3+3$

therefore $9+5=9-3+3+5=(9-3)+3+5$ (55)

$=(9-3)+5+3,$ (25)

therefore $9+5-3=9-3+5+3-3$

$=9-3+5.$

60. Hence it is easily shewn that

(1) *Additions and subtractions may be performed in any order.*

(2) *An expression made up of additions and subtractions may be made equal to the difference of two sums; thus:—*

$$9-8+7-6+5-4=(9+7+5)-(8+6+4).$$

MULTIPLICATION AND DIVISION.

61. *The product of two or more numbers will remain the same, however we may change the order of its factors.*

(1) Let there be *two* factors; we may change their order (35), thus $7.5=5.7$.

(2) Let there be *three* factors; we may change the order of the last two, thus $8.7.5=8.5.7$. For write 8 in a horizontal line 7 times, and repeat this line 5 times. Now the sum of the numbers in each line is 8.7, and there are 5 such lines, therefore the total sum is $8.7.5$. Again the sum of the numbers in each vertical line is 8.5, and there are 7 such lines, therefore the total sum is $8.5.7$: that is $8.7.5=8.5.7$.

8	8	8	8	8	8	8
8	8	8	8	8	8	8
8	8	8	8	8	8	8
8	8	8	8	8	8	8
8	8	8	8	8	8	8

(3) Let there be *any* number of factors as in $4.2.7.5.3$; any one of these factors, 5 for example, may occupy the first place. For

$4.2.7.5=(4.2).7.5=(4.2).5.7$ (61, 2)

$=4.2.5.7,$

therefore $4.2.7.5.3=4.2.5.7.3;$

that is in the given product, 5 has changed places with the preceding factor 7 giving $4.2.5.7.3$: in like manner it may now change

places with the preceding factor 2 giving $4.5.2.7.3$; and lastly it may change places with the first factor 4 giving $5.4.2.7.3$.

In the same way any other factor may occupy the second place, any remaining factor the third place, &c.; thus the factors may occupy any places we please.

62. *If we multiply separately each of the parts which compose one number by each of the parts which compose another number, the sum of these partial products will give the product of the two numbers.*

(1) Let the Multiplicand be 11 or $4+5+2$, and the Multiplier 3; then their product is the sum of 3 numbers each equal to $4+5+2$ (34); and from the arrangement at the side, this sum is the sum of 4 taken 3 times, of 5 taken 3 times, and of 2 taken 3 times, that is of 4.3, of 5.3 and of 2.3; or

$$4+5+2$$
$$4+5+2$$
$$4+5+2$$

$$(4+5+2)\times 3=4.3+5.3+2.3.$$

(2) Let the Multiplicand be 11 and the Multiplier 13 or $6+7$, then

$$\begin{aligned} 11\times(6+7)&=(6+7)\times 11 & (35)\\ &=6.11+7.11 & (62,\ 1)\\ &=11.6+11.7. & (35)\end{aligned}$$

(3) Let the Multiplicand be $4+5+2$ and the Multiplier $6+7$, then

$$\begin{aligned}(4+5+2)\times(6+7)&=(4+5+2)\times 6+(4+5+2)\times 7 & (62,\ 2)\\ &=4.6+5.6+2.6+4.7+5.7+2.7. & (62,\ 1)\end{aligned}$$

Hence the truth of the proposition.

REMARK. The theory of Multiplication is involved in the three cases of this proposition, as may be seen by referring to Arts. 39, 40 and 41.

63. (1) In like manner if the Multiplicand be the *difference* of two numbers as $9-2$, and the Multiplier another number as 3, we may shew that

$$(9-2)\times 3=9\times 3-2\times 3,$$

and interchanging Multiplicand and Multiplier we have further

$$3\times(9-2)=9.3-2.3,$$

or

$$=3.9-3.2,$$

COR. Hence to multiply a number by 99, 999,... that is by $100-1$, $1000-1$,... we may multiply by 100, 1000,... and subtract the number: to multiply by 98, 998,... we may multiply by 100, 1000,... and subtract twice the number.

(2) Again, if the Multiplicand be an expression made up of additions and subtractions as $9-7+5-2$, and the Multiplier any number as 3, then looking on $9-7+5$ as a single number we have

$$(9-7+5-2)\times 3=(9-7+5)\times 3-2.3 \qquad (63,\ 1)$$
$$=(9-7)\times 3+5.3-2.3 \qquad (62,\ 1)$$
$$=9.3-7.3+5.3-2.3 \qquad (63,\ 1)$$

64. *If Divisor and Dividend be multiplied by the same number, the Quotient will remain the same, but the Remainder will also be multiplied by that number.*

For example, if the Quotient of 38 by 7 is 5 with Remainder 3, the Quotient of 38.4 by 7.4 will also be 5, but with Remainder 3.4.

Now $$38=7.5+3, \qquad (47)$$

and multiplying each member of this equality by 4, we get

$$38.4=7.5.4+3.4 \qquad (62,\ 1)$$
$$=(7.4).5+3.4, \qquad (60)$$

which shews that the Quotient of 38.4 by 7.4 is 5 with Remainder 3.4.

INVOLUTION.

65. INVOLUTION is the operation by which we find any power of a given number.

The second, third, fourth,...*power* of a number is the product of 2, 3, 4,... factors each equal to that number: thus the second power of 4 is 4.4, the third power of 5 is $5.5.5$. The second and third powers of a number are usually called its *square* and its *cube*.

The square, cube, fourth,... power of a number is represented by placing a small 2, 3, 4,... to the right of the number, a little above the line: thus the square of 6 is represented by 6^2. Hence the first power of a number will be the number itself, or 5^1 is the same as 5.

These small figures denoting the powers of a number are called its *indices* or *exponents*.

66. *The product of two or more powers of the same number is a power of that number whose index is the sum of the indices of the factors.*

Since 7^2 is the product of 2 factors each equal to 7, and 7^3 the product of 3 factors each equal to 7, we have

$$7^2 \times 7^3 = 7 \times 7 \times 7 \times 7 \times 7 = 7^{2+3}.$$

Again $$7^2 \times 7^3 \times 7^4 = 7^{2+3} \times 7^4 = 7^{2+3+4},$$

and so if there are more factors.

COR. *To raise a power to another power we multiply the first index by the second.*

For $$(7^4)^3 = 7^4 \times 7^4 \times 7^4 = 7^{4+4+4} = 7^{4 \times 3}.$$

Ex. 1. Find the value of

$$(1536 - 1392) \div (29 + 7).$$

This expression means that we are to divide the *difference* of 1536 and 1392 by the *sum* of 29 and 7; but the difference of 1536 and 1392 is 144, and the sum of 29 and 7 is 36, therefore

$$(1536 - 1392) \div (29 + 7) = 144 \div 36 = 4.$$

Ex. 2. Find the value of

$$(1536 - 487) - 1392 \div 29 + 7 \times 5.$$

This expression consists of 3 terms: the first is $(1536 - 487)$, the second $1392 \div 29$, and the third $7 \cdot 5$; it therefore means that from the *difference* of 1536 and 487, we are to subtract the *quotient* of 1392 by 29, and to the result we are to add the *product* of 7 and 5; therefore

$$(1536 - 487) - 1392 \div 29 + 7 \cdot 5 = 1049 - 48 + 35$$
$$= 1036.$$

Ex. 3. Find the value of

$$(194 + 65) \times 7 + (352 - 220) \div 11 - 952 \div (91 - 35).$$

This expression means that to $(194 + 65) \times 7$ we are to add $(352 - 220) \div 11$ and from the sum subtract $952 \div (91 - 35)$; hence

$$\text{given expression} = 259 \times 7 + 132 \div 11 - 952 \div 56$$
$$= 1813 + 12 - 17$$
$$= 1808$$

Exercise 7.

Find the value of

1. $27[illegible]048 + 37506 + 6709 + 209 + 740087$.

2. $67351 - 3985 - 4537 + 80359 - 57036$.

3. $(7805 + 3907) + (7805 - 3907)$; $(7805 + 3907) - (7805 - 3907)$.

4. $(7805 + 3907) \times (7805 - 3907)$; $7805 + (3907 - 1996) - 854$.

5. $7805 + (3907 - 1996) \times 854$; $324 - 19 \times 17$; $(324 - 19) \times 17$.

6. $756 \times 3 - 25 \times 16 + 3^2$; $1536 \div 8 + 9 \cdot 125 - 100$.

7. $524 \div 4 - 1392 \div (29 - 5)$; $1536 - 1392 \div 29 + 16$.

8. $5880 \div (167 - 132) \times 6$; $5880 \div (35 \div 7) + 17 \cdot 12$.

9. $(67893 - 8637) \div 823 + 7546 \times (2356 - 945) - 9870 \cdot 170$.

10. $59256 \div 72 \times 91 \div (130 - 117)$; $19 \cdot 20 \cdot 21 \div (3 \cdot 5 \cdot 7)$.

11. Express the following statement by signs:—Multiply the difference between 325 and 293 by the quotient of 306 by 17, and to the product add the sum of 1000 and 99. Find the value of the expression.

12. Express by signs:—From 34856 subtract the product of 763 and 41, and to the remainder add the quotient of 1998 by the difference between 663 and 441; and then find the value of the expression.

13. The product of 175 and 53 is 9275: how much must be added to it to give the product of 177 and 53; and to give the product of 175 and 59 (see Art. 62)?

14. The product of 3267 and 71 is 231957: how much must be subtracted from it to give the product of 3267 and 68; and to give the product of 3258 and 71 (see Art. 63)?

15. Multiply 387659 by 85672 in 3 lines.

16. Find the prime number expressed by $1251^2 + 2920^2$.

17. The following results are true when a and b stand for *any* numbers, which will allow them to be arithmetically possible. Shew that they are true in each case, when $a = 935$ and $b = 396$:—

(1) $(a+b)+(a-b)=2 \cdot a$.

(2) $(a+b)-(a-b)=2 \cdot b$.

(3) $(a+b) \times (a-b)=a^2-b^2$.

(4) $(a+b)^2-(a-b)^2=4 \cdot a \cdot b$.

(5) $a^2+2 \cdot a \cdot b+b^2=(a+b)^2$.

(6) $a^2-2 \cdot a \cdot b+b^2=(a-b)^2$.

(7) $(a+b) \cdot (a^2-a \cdot b+b^2)=a^3+b^3$.

(8) $(a-b) \cdot (a^2+a \cdot b+b^2)=a^3-b^3$.

CHAPTER IV.

GREATEST COMMON MEASURE: LEAST COMMON MULTIPLE, &c.

GREATEST COMMON MEASURE.

67. ONE number is a *measure*, or a *submultiple*, or an *aliquot part* of another, when it divides that other exactly (47): thus 4 is a measure, or a submultiple, or an aliquot part of 20, but not of 21.

One number is a *multiple* of another, when it can be divided by that other exactly: thus 20 is a multiple of 4, and of 5, but not of 6.

The *Greatest Common Measure* (G.C.M.) of two or more numbers is the greatest number that divides each of them exactly.

The *Least Common Multiple* (L.C.M.) of two or more numbers is the least number that can be divided by each of them exactly.

68. A *Prime number*, or a *Prime*, is a number divisible only by itself and by 1: thus 2, 3, 5, 7, 11...... are primes.

Two numbers are *prime* to each other, when their only common measure is 1: hence

(1) *Two primes must be prime to each other.* (2) *A prime number must be prime to every number not divisible by it.*

69. *A measure of each of several numbers is a measure of their sum.*

Let 7 be a measure of each of the numbers, then each number is composed of parts each equal to 7, and therefore their sum is composed of parts each equal to 7; that is, 7 is a measure of their sum.

COR. *A measure of any number is a measure of any multiple of it.*

70. *A measure of each of two numbers is a measure of their difference; also of the difference between the first and any multiple of the second.*

Let 7 be a measure of each of them, then each is composed of parts each equal to 7, and therefore their difference is composed of parts each equal to 7: that is 7 is a measure of their difference.

Again, since 7 is a measure of the second number, it is a measure of any multiple of the second number: and is therefore a measure of the difference between the first number and any multiple of the second.

71. *To find the* G.C.M. *of any two numbers.*

(1) The G.C.M. of Dividend and Divisor is also the G.C.M. of Divisor and Remainder, and vice versa.

The Remainder is the difference between the Dividend and a multiple of the Divisor, and therefore every common measure of Dividend and Divisor is a measure of the Remainder (70), and is therefore a common measure of Divisor and Remainder.

Again, the Dividend is the sum of a multiple of the Divisor and the Remainder, therefore every common measure of Divisor and Remainder is a measure of the Dividend (69), and therefore a common measure of Divisor and Dividend.

Hence Dividend and Divisor, and Divisor and Remainder, have the same common measures; and therefore the G.C.M. of Dividend and Divisor is the G.C.M. of Divisor and Remainder.

(2) Find the G.C.M. of 304 and 1072.

Divide 1072 by 304 giving the Remainder 160; then divide 304 by 160 giving the Remainder 144; then divide 160 by 144 giving the Remainder 16: lastly divide 144 by 16 giving no Remainder.

```
304)1072(3
     912
     160)304(1
         160
         144)160(1
             144
              16)144(9
                 144
```

Now the G.C.M. of 304 and 1072 is also the G.C.M. of 160 and 304; and the G.C.M. of 160 and 304 is also the G.C.M. of 144 and 160; and the G.C.M. of 144 and 160 is also the G.C.M. of 16 and 144; but the G.C.M. of 16 and 144 is 16; therefore the G.C.M. of 304 and 1072 is 16.

We have then the following Rule:—

Divide the greater number by the less, the less by the first remainder, the first remainder by the second remainder, and continue this process till we come to a remainder that exactly divides the preceding remainder: such last remainder is the G.C.M.

REMARK 1. We shall find it convenient to place the numbers in a horizontal line, drawing a vertical line between them, and then carry on the divisions alternately on the right and left of this line. We may neglect the quotient figures.

REMARK 2. At any step of the process we may find the difference between the greater number and the *nearest* multiple of the less, regardless whether that nearest multiple be less or greater than the greater number.

Ex. 1. Find the G.C.M. of 93883 and 166581.

93883	166581
72698	93883
21185	72698
18286	63555
2899	9143
2676	8697
223	446
	446

93883	166581
84740	187766
9143	21185
8697	18286
446	2899
446	2676
	223

$\therefore$ G.C.M. = 223.

72. *Every common measure of two numbers is a measure of their* G.C.M.

For being a measure of the two numbers it is a measure of the first remainder, and being a measure of the less number and the first remainder, it is a measure of the second remainder; and in like manner it is a measure of each succeeding remainder: it is therefore a measure of the last remainder, that is of the G.C.M. of the two numbers.

73. *To find the* G.C.M. *of three or more numbers.*

Let 304, 1072, 40 and 36 be the given numbers: and let 16 be the common measure of 304 and 1072. Now every common measure

of the given numbers is a common measure of 304 and 1072, and therefore of 16 their G.C.M. (72), and therefore of 16, 40 and 36.

Again, every measure of 16 is a common measure of 304 and 1072 (69 Cor.), and therefore every common measure of 16, 40 and 36 is a common measure of 304, 1072, 40 and 36.

Hence 304, 1072, 40 and 36 have the same common measures as 16, 40 and 36: and therefore the G.C.M. of the given numbers is the G.C.M. of 16, 40, 36.

In like manner if the G.C.M. of 16 and 40 be 8, the G.C.M. of 16, 40 and 36 is the G.C.M. of 8 and 36: and therefore the G.C.M. of the given numbers is the G.C.M. of 8 and 36.

And in this way step by step may be found the G.C.M. of as many numbers as we please. Hence we have the following Rule:-

Find the G.C.M. *of the first two numbers; then the* G.C.M. *of this* G.C.M. *and the third number; then the* G.C.M. *of this last* G.C.M. *and the fourth number; and continue this process unto the last number: the last* G.C.M. *is the* G.C.M. *of the given numbers.*

74. *If two numbers be multiplied by a third, the* G.C.M. *of the products will be the* G.C.M. *of the given numbers multiplied by the third number.*

Thus if the G.C.M. of 304 and 1072 be 16, the G.C.M. of $304 \cdot 7$ and $1072 \cdot 7$ will be $16 \cdot 7$. Referring to Art. 71 we see that the remainders in finding the G.C.M. of 304 and 1072 are successively 160, 144 and 16: hence (64) the remainders in finding the G.C.M. of $304 \cdot 7$ and $1072 \cdot 7$ will be

$$160 \cdot 7,\ 144 \cdot 7, \text{ and } 16 \cdot 7;$$

that is if the G.C.M. of 304 and 1072 is 16

the G.C.M. of $304 \cdot 7$ and $1072 \cdot 7$ is $16 \cdot 7$.

COR. 1. Conversely, since the G.C.M. of the second set of numbers is the G.C.M. of the first set *multiplied* by 7, the G.C.M. of the first set is the G.C.M. of the second set *divided* by 7.

COR. 2. If we divide two numbers by their G.C.M. the quotients will be prime to each other, for their G.C.M. will be 1.

Ex. 2. Find the G.C.M. of 285600 and 621600.

800)	285600	621600
3)	357	777
	119	259
	126	238
	7	21
		21

It is seen that the G.C.M. of 119 and 259 is 7, therefore the G.C.M. of 357 and 777 is 7 . 3, and the G.C.M. of the given numbers is 7 . 3 . 800, or 16800.

75. *If a number divide a product of two factors and be prime to one of them, it must divide the other.*

For example, if 4 divide 7 . 24 and 4 is prime to 7, then 4 must divide 24. The G.C.M. of 4 and 7 is 1, therefore the G.C.M. of 4 . 24 and 7 . 24 is 1 . 24 (74) or 24. Now 4 is a measure of 4 . 24, and by hypothesis of 7 . 24, therefore it is a measure of their G.C.M. 24 (72); that is 4 divides 24.

76. *The G.C.M. of two numbers will remain the same, if one of them be multiplied or divided by a number prime to the other.*

Thus if 5 be prime to 24, the G.C.M. of 24 and 16 is the same as the G.C.M. of 24 and 5 . 16.

Every measure of 16 is a measure of 5 . 16 (69 Cor.): therefore every common measure of 24 and 16 is a common measure of 24 and 5 . 16. Again, since 24 is prime to 5, every measure of 24 is prime to 5, therefore every measure of 24 which measures 5 . 16 must measure 16 (75), and therefore every common measure of 24 and 5 . 16 is a common measure of 24 and 16.

Hence 24 and 16, and 24 and 5 . 16 have the same common measures, and therefore the same G.C.M.

Ex. 3. Find the G.C.M. of 39835 and 162424.

39835	162424
7967	20303
7196	23901
771	3598
257	1799
	1799

Divide 39835 by 5, for 5 is prime to 162424, and divide 162424 by 8, for 8 is prime to 7967; we have therefore to find the G.C.M. of 7967 and 20303. After the first step we have to find the G.C.M. of 3598 and 7967: but we may divide 3598 by 2, for 2 is prime to 7967, &c.

Ex. 4. Find the G.C.M. of 218707, 526769 and 695822.

218707	526769	851	695822 (817
178710	437414	888	1502
39997	89355	37	6512
37444	79994		555
2553	9361		111
2553	10212		0
	851		

G.C.M. is 37.

Ex. 5. Are 5789 and 7337 prime to each other? are 2698703 and 54987261 prime to each other?

5789	7337	2698703	54987261 (20
6192	5789	3039603	53974060
403	1548	340900	1013201
384	1612	3640	1022700
19	64	234	9499
1		24	10227
		7	728
			728

Yes: for their G.C.M. is 1.

No: they have a common measure 7.

EXERCISE 8.

Find the G.C.M. of:

1. 9367 and 14501; 4833 and 6237; 12925 and 63305.
2. 24501 and 67347; 19001 and 46253; 513989 and 6790735.
3. 703037 and 5134083; 336663 and 4264731; 385529 and 7855323.
4. 3085345 and 45386655; 3876519 and 3101729671.
5. 48961183 and 997626011; 41615795893 and 87726701910 6.
6. 1617, 1871 and 4213; 14385, 20391 and 49287.
7. 9263, 1298413 and 4131416.
8. 156009, 221697 and 342171.
9. 70843288, 85170643 and 686138242.
10. 5040, 13940, 28350 and 31773.
11. 12680, 49140, 154980, 429660 and 925932.

12. Are 3375 and 5832 prime to each other? and are 49561 and 97073 prime to each other? if not, what is their G.C.M.?

13. Are 58573 and 84229 prime to each other? and are 18432, 21952 and 42875 prime to each other?

14. Two vats contain respectively 7875 and 16128 gallons: find the barrel of greatest capacity that will completely empty off both vats.

15. Find the greatest weight, in grains, that will measure both pounds Standard and pounds Troy. N. B. One pound Troy is 5760 grains.

16. Find the greatest number that will divide 739 and 916, leaving the remainders 4 and 6 respectively.

17. Find the greatest number that will divide 2293, 4245 and 5348, leaving the remainders 18, 20 and 23.

18. Is there any number that will divide 5230, 6325 and 8540, leaving the remainders 34, 33 and 32 respectively?

LEAST COMMON MULTIPLE.

77. *If a number be divisible by two others which are prime to each other, it will be divisible by their product.*

Thus if 840 be divisible by 5, and by 6, where 5 and 6 are prime to each other, it will be divisible by $5 . 6$.

Since 840 *is* divisible by 5, let the quotient be 168, so that

$$840 = 5 . 168.$$

Again, since 840 or $5 . 168$ is divisible by 6, and 6 is prime to 5, 168 must be divisible by 6 (75), let the quotient be 28, so that

$$168 = 6 . 28,$$

and therefore $$840 = 5 . 6 . 28 = (5 . 6) . 28;$$

that is, 840 is divisible by $5 . 6$.

COR. *If two numbers are prime to each other, their* L. C. M. *is their product.*

78. *To find the* L. C. M. *of any two numbers.*

Let 63 and 112 be any two numbers, and let 7 be their G. C. M., so that

$$63 = 7 . 9 \text{ and } 112 = 7 . 16;$$

therefore 9 and 16 are prime to each other (74, Cor. 2).

Every common multiple of the given numbers when divided by 7 must be further divisible by 9, and also by 16, and therefore by $9 . 16$ (77), that is every common multiple must be divisible by $7 . 9 . 16$; and therefore must either be $7 . 9 . 16$ itself, or a multiple of $7 . 9 . 16$.

But $7 . 9 . 16$ *itself* is divisible by $7 . 9$ and by $7 . 16$, that is by 63 and by 112; therefore $7 . 9 . 16$ is the L. C. M. of 63 and 112.

Now $7 . 9 . 16 = 16 . 63$ where 16 is the quotient of 112 by the G.C.M. 7, and $= 9 . 112$ where 9 is the quotient of 63 by the G.C.M.; therefore we have this Rule:—

Find the G.C.M. *of the two numbers; divide either number by this* G.C.M., *and multiply the quotient by the other number.*

COR. 1. *Also the* L.C.M. *of two numbers is their product divided by their* G.C.M.

COR. 2. *Every common multiple of two numbers is a multiple of their* L.C.M. For every common multiple of the given numbers is either $7 . 9 . 16$ itself, or a *multiple* of $7 . 9 . 16$.

Ex. 1. Find the L.C.M. of 7455 and 18744.

7455	18744 (2	213) 7455 (35	18744
7668	14910	1065	93720
213	3834 (18	0	656040
	1704		
	0		

that is G.C.M. $= 213$; and $7455 \div 213 = 35$;

$\therefore$ L.C.M. $= 35 \times 18744 = 656040$.

79. *To find the* L.C.M. *of three or more numbers.*

Let 63, 112, 40 and 36 be the given numbers, and let the L.C.M. of 63 and 112 be 1008. Now every common multiple of the given numbers is a common multiple of 63 and 112, and therefore of their L.C.M. 1008 (79, Cor. 2), and therefore of 1008, 40 and 36. Again, every multiple of 1008 is a common multiple of 63 and 112, and therefore every common multiple of 1008, 40 and 36 is a common multiple of 63, 112, 40 and 36. Hence 63, 112, 40 and 36 have the same common multiples as 1008, 40 and 36; and therefore the L.C.M. of one set is the L.C.M. of the other set.

Again, if the L.C.M. of 1008 and 40 be 5040, it follows that the L.C.M. of 1008, 40 and 36 is the L.C.M. of 5040 and 36. And so we can proceed step by step to the last number.

We have then the following Rule:—

Find the* L.C.M. *of the first two numbers, then the* L.C.M. *of that* L.C.M. *and the third number; then the* L.C.M. *of this last

L.C.M. *and the fourth number, and continue the process unto the last number: this last* L.C.M. *is the* L.C.M. *of the given numbers.*

80. *If two or more numbers be multiplied or divided by another number, the* L.C.M. *of the products or quotients is the* L.C.M. *of the given numbers, multiplied or divided by that other number.*

Thus if the L.C.M. of 63, 112 and 40 be 5040, the L.C.M. of 63 . 5, 112 . 5 and 40 . 5 will be 5040 . 5.

Let the G.C.M. of 63 and 112 be 7, and the quotients 9 and 16; then the G.C.M. of 63 . 5 and 112 . 5 will be 7 . 5 (74), and therefore the quotients will also be 9 and 16. But the L.C.M. of 63 and 112 is $7 \times 9 . 16$, or 1008; and the L.C.M. of 63 . 5 and 112 . 5 is $7 . 5 \times 9 . 16$ or 1008 . 5.

In like manner, if the L.C.M. of 1008 and 40 be 5040, the L.C.M. of 1008 . 5 and 40 . 5 will be 5040 . 5; that is (79) if the L.C.M. of 63, 112 and 40 be 5040, the L.C.M. of 63 . 5, 112 . 5 and 40 . 5 will be 5040 . 5.

Ex. 2. Find the L.C.M. of 31500 and 56000.

Now 31500 = 63 . 500 and 56000 = 112 . 500,

and the L.C.M. of 63 and 112 = 1008,

∴ the L.C.M. of 31500 and 56000 = 1008 . 500 = 504000.

81. *If there be two numbers, and one of them be multiplied or divided by a number prime to the other, the* L.C.M. *must also be multiplied or divided by that other number.*

Thus, if the L.C.M. of 63 and 112 be 1008, the L.C.M. of 63 . 5 and 112—where 5 is prime to 112—will be 1008 . 5. For the L.C.M. of two numbers is their product by their G.C.M.; but the G.C.M. of 63 and 112, and of 63 . 5 and 112, is the same (76); hence the L.C.M. of the latter set is the L.C.M. of the former multiplied by 5.

82. *If there be several numbers, and two or more of them can be divided by a number prime to the rest, the* L.C.M. *of the given numbers is the* L.C.M. *of the quotients, and of the undivided numbers, multiplied by that divisor.*

*Take the numbers 15, 18, 25, 32, and 40; 15, 25 and 40 are divisible by 5, which is prime to 18 and 32, and the quotients are 3, 5 and 8; then the L.C.M. of the given numbers is the L.C.M. of 3, 5, 8, 18 and 32 multiplied by 5.

The L.C.M. of 15, 25 and 40 is the L.C.M. of 3, 5 and 8 multiplied by 5 (80): let the L.C.M. of 3, 5 and 8 be 120, then the L.C.M. of 15, 25 and 40 will be 120 . 5. Again, the L.C.M. of 120 . 5 and 18 is the L.C.M. of 120 and 18 multiplied by 5 (81); that is (79) the L.C.M. of 15, 25, 40 and 18 is the L.C.M. of 3, 5, 8 and 18 multiplied by 5: let the L.C.M. of 3, 5, 8 and 18 be 360, then the L.C.M. of 15, 25, 40 and 18 will be 360 . 5. Lastly, the L.C.M. of 360 . 5 and 32 is the L.C.M. of 360 and 32 multiplied by 5: that is, the L.C.M. of 15, 25, 40, 18 and 32 is the L.C.M. of 3, 5, 8, 18 and 32 multiplied by 5.

83. The usual arrangement is to write down the given numbers in a line, with the divisor on the left, and the quotients and the undivided numbers in a line below, thus:

$$\begin{array}{r|l} 5 & 15,\ 18,\ 25,\ 32,\ 40 \\ \hline & 3,\ 18,\ 5,\ 32,\ 8 \end{array} \qquad\qquad \begin{array}{r|l} 5 & 15,\ 18,\ 25,\ 32,\ 40 \\ \hline 2 & 3,\ 18,\ 5,\ 32,\ 8 \\ \hline & \quad 9,\ 5,\ 16 \end{array}$$

which shews that the L.C.M. of the numbers in the first line is the L.C.M. of the numbers in the second line multiplied by 5. Again, of the numbers in the second line, 3 and 8 being factors of 18 and 32 may be suppressed; and of the others 18 and 32 are divisible by 2, which is prime to 5: hence the L.C.M. of the numbers in the second line is the L.C.M. of the numbers in the third line multiplied by 2. Lastly, there is no number that will divide any two of the numbers 9, 5 and 16; hence their L.C.M is their product 9 . 5 . 16; and therefore the L.C.M. of the numbers in the second line is 9 . 5 . 16 . 2; and of the given numbers 9 . 5 . 16 . 2 . 5.

We have then the following Rule:—

Write down the given numbers in a horizontal line, suppressing those that are divisors of any of the others. Seek out two numbers at least, that have a prime divisor 2, 3, 5, 7, 11...; divide them by

this prime, placing the quotients and the undivided numbers in the line below. Proceed in the same way with the numbers in the second, and each succeeding line, till we come to a line where no two numbers have a common divisor. The product of the numbers in the last line and of the several divisors is the L.C.M. *of the given numbers.*

Example. Find the L.C.M. of 15, 16, 18, 20, 24, 25, 27 and 30.

2) 15, 16, 18, 20, 24, 25, 27, 30
3) 8, 9, 10, 12, 25, 27, 15
5) 8, 3, 10, 4, 25, 9, 5
8, 2, 5, 9

∴ L.C.M. = 2 . 3 . 5 . 8 . 5 . 9 = 10800.

5) 15, 16, 18, 20, 24, 25, 27, 30
3) 16, 18, 4, 24, 5, 27, 6
3) 16, 6, 8, 5, 9
16, 2, 5, 3

∴ L.C.M. = 5 . 3 . 3 . 16 . 5 . 3 = 10800

In the preceding Example we might begin by taking either 2 or 3 or 5 as a divisor; but instead let us take at once 2 . 3 . 5 or 30 as the divisor: then the first number 16 can be divided by 2 giving 8: the second number 18, by 2 . 3 or 6 giving 3: the next number 20, by 2 . 5 or 10 giving 2: and so on. The work will therefore stand thus

2 . 3 . 5 or 30) 15, 16, 18, 20, 24, 25, 27, 30
8, 3, 2, 4, 5, 9,

∴ L.C.M. = 30 . 8 . 5 . 9 = 10800.

Exercise 9.

Find the L.C.M. of

1. 156 and 429; 1287 and 6281; 729 and 1681.
2. 7469 and 102025; 8975 and 25489; 12432 and 36075.
3. 7247 and 9365; 2341 and 203667; 25664 and 781033.
4. 629, 851 and 253; 1003, 2301 and 4017.
5. 2691, 6435 and 8349; 2523, 5887, 203 and 8631.
6. the first 12 numbers; the first 12 odd numbers.
7. the even numbers from 10 to 28 inclusive.
8. the odd numbers from 13 to 27 inclusive.
9. 12, 16, 18, 28, 32, 40, 42; 9, 12, 15, 18, 21, 24, 27, 30.
10. 15, 18, 24, 33, 40, 54, 55; 32, 63, 25, 36, 42, 49, 84.

11. 35, 52, 63, 77, 132, 117, 143; 18, 21, 22, 45, 99, 154, 168.

12. 25, 14, 35, 12, 21, 105, 72, 112; 27, 91, 42, 39, 63, 156, 234.

13. 187, 221, 119, 154, 385, 165, 231, 340.

14. 24, 35, 52, 60, 91, 108, 126, 156, 315.

15. 27, 87, 189, 116, 145, 110, 203, 261, 385.

16. Seven bells are tolling, and they toll at intervals of 3, 5, 7, 8, 9, 10 and 11 seconds respectively. What interval will elapse between their once tolling together, and tolling together again?

17. Two cog-wheels containing 80 and 128 cogs respectively are working together: after how many revolutions of the smaller wheel will two cogs which once touch, touch again?

18. Find the least number which divided by 6, by 8, and by 9, gives in every case the remainder 4.

19. Find the least number which divided by 675, 1050, and 4368, will leave the same remainder 32.

20. A heap of pebbles can be made up exactly into groups of 25; but when made up into groups of 18, 27, and 32, there is always a remainder of 11; find the least number of pebbles such heap can contain.

21. Three horses are running round a race-course of 5280 yards: the first horse runs 440 yards a minute, the second 352 yards, and the third 264 yards: find the time between their once coming all together, and their coming all together again.

CRITERIA OF DIVISIBILITY.

84. One number is *divisible* by another when it can be divided by that other exactly (47), that is when the former is a multiple of the latter (67).

85. ***The sum of several multiples of a given number is a multiple of that number.***

For each multiple may be considered as made up of parts each equal to the given number, therefore their sum will be made up of parts each equal to the given number, or will be a multiple of the given number.

COR. ***Any multiple of a multiple of a given number is a multiple of that number.***

86. ***Any number is divisible by 2, 4, 8,.. if the number expressed by the last one, two, three,...figures to the right be divisible by 2, 4, 8,...***

Any number 87654 is the sum of 87650 and 4; and the first part 87650 is divisible by 10 (53) and therefore by 2 (85 Cor.), for 10 is a multiple of 2: therefore the whole 87654 is divisible by 2, if 4 be divisible by 2.

Again, 87654 is the sum of 87600 and 54; and the first part 87600 is divisible by 100 (53) and therefore by 4, for 100 is divisible by 4; therefore the whole 87654 is divisible by 4, if 54 be divisible by 4.

In like manner 87654 is divisible by 8, if 654 be divisible by 8, &c.

COR. Hence those numbers only, whose units' figures are 0, 2, 4, 6 and 8, are divisible by 2: such numbers are called *even* numbers; and numbers not divisible by 2, or numbers whose units' figures are 1, 3, 5, 7 and 9, are called *odd* numbers.

87. *Any number is divisible by 5, 25, 125, ... if the number expressed by the last one, two, three,... figures to the right be divisible by 5, 25, 125,...*

Proceed exactly as in the last Article.

COR. Hence those numbers only, whose units' figures are 0 and 5, are divisible by 5.

88. *The remainder in dividing any number by 9 is the same as in dividing the sum of its figures by 9.*

Take 1 followed by any number of ciphers, as 10000; now

$$10000 = 9999 + 1 = \text{a multiple of } 9 + 1.$$

Multiply these equals by any simple number 6, then

$$60000 = \text{a multiple of } 9 + 6. \qquad (69)$$

Now 87654 is the sum of 80000, 7000, 600, 50 and 4;

$$\begin{aligned} \text{and} \qquad 80000 &= \text{a multiple of } 9 + 8, \\ 7000 &= \text{a multiple of } 9 + 7, \\ 600 &= \text{a multiple of } 9 + 6, \\ 50 &= \text{a multiple of } 9 + 5, \\ 4 &= \qquad\qquad\qquad 4; \\ \text{therefore} \qquad 87654 &= \text{a multiple of } 9 + 8 + 7 + 6 + 5 + 4; \qquad (85) \end{aligned}$$

and therefore the remainder in dividing 87654 by 9 is the same as in dividing the sum of its figures $8+7+6+5+4$ by 9.

COR. 1. *Hence any number is divisible by 9, if the sum of its figures be divisible by 9.*

COR. 2. Since 9 is a multiple of 3, any number

$$87654 = \text{a multiple of } 3 + (8+7+6+5+4);$$

therefore *any number is divisible by 3, if the sum of its figures be divisible by 3.*

89. *Any number is divisible by 11, if the difference between the sum of its figures in the odd and in the even places be 0, or be divisible by 11.*

Take 1 followed by an *even* number of ciphers as 10000;

now $\quad 10000 = 9999 + 1 = \text{a multiple of } 11 + 1,$

therefore $\quad 30000 = \text{a multiple of } 11 + 3.$ (62)

Again, take 1 followed by an *odd* number of ciphers as 1000;

now $\quad 1000 = 990 + 10 = 990 + 11 - 1 = \text{a multiple of } 11 - 1;$

therefore $\quad 3000 = \text{a multiple of } 11 - 3.$ (63)

Any number 87654 is the sum of 80000, 7000, 600, 50 and 4;

and

$$\begin{aligned} 80000 &= \text{a multiple of } 11 + 8, \\ 7000 &= \text{a multiple of } 11 - 7, \\ 600 &= \text{a multiple of } 11 + 6, \\ 50 &= \text{a multiple of } 11 - 5. \\ 4 &= 4; \end{aligned}$$

therefore

$$\begin{aligned} 87654 &= \text{a multiple of } 11 + 4 - 5 + 6 - 7 + 8 && (85) \\ &= \text{a multiple of } 11 + (4+6+8) - (5+7); && (59, 2) \end{aligned}$$

and therefore 87654 is divisible by 11 if the difference between the sum of its figures in the odd and in the even places be 0, or be a multiple of 11.

90. ***Criteria of Divisibility when the Divisor is not greater than 12.***

Since 2 and 3 are prime to each other, any number divisible by 2 and by 3 is divisible by $2 . 3$ or 6 (77): and any number divisible by 3 and by 4 is divisible by 12.

Therefore any number is divisible by

2, if its units' figure be even (86);

3, if the sum of its figures be divisible by 3 (88);

4, if the number formed by its last 2 figures to the right be divisible by 4;

5, if its units' figure be 0 or 5 (87);

6, if it be divisible by 2 and by 3 (90);

7, by trial;

8, if the number formed by the last 3 figures to the right be divisible by 8;

9, if the sum of its figures be divisible by 9;

10, if its units' figure be 0;

11, if the difference between the sum of its figures in the odd and in the even places be 0, or be divisible by 11 (89);

12, if it be divisible by 3 and by 4.

91. *Proof of Multiplication by casting out the nines.*

Casting the nines out of the Multiplicand and Multiplier, suppose the remainders to be 5 and 3, so that

$$\text{Multiplicand} = \text{a multiple of } 9 + 5,$$
$$\text{Multiplier} = \text{a multiple of } 9 + 3.$$

Now we may multiply Multiplicand and Multiplier together by multiplying each part of the first number by each part of the second (62). The first part of this product will be the product of two multiples of 9, and therefore a multiple of 9 (85 Cor.), the second part will be 5 times a multiple of 9, the third part 3 times a multiple of 9, and the fourth part $5 \cdot 3$. But the sum of the first three parts will be a multiple of 9 (85), hence

$$\text{Product} = \text{a multiple of } 9 + 5 \cdot 3,$$

and therefore the remainder in dividing the Product by 9 is the same as in dividing $5 \cdot 3$ by 9: that is, in casting the nines out of the Product and out of $5 \cdot 3$ the results ought to be the same: hence the usual Rule (43).

REMARK. This Proof will not point out an error in the product (1) if a 0 be set down for 9, or *vice versâ*; or (2) if a figure has been set down as much too high, as another has been too low;

or (3) if a partial product has been set down in a wrong place; or (4) if one or more noughts have been omitted in the product.

PRIMES AND PRIME FACTORS.

92. We shall take it for granted that every number is either a prime or the product of two or more prime factors. In the latter case the number is sometimes called *composite*.

To decompose a number into its prime factors is to find those prime numbers which when multiplied together produce the given number, as when 210 is put under the form $2 . 3 . 5 . 7$, or 504 under the form $2^3 . 3^2 . 7$.

93. *To ascertain what numbers are prime.*

(1) Every number whose units' figure is 0, 2, 4, 6, or 8 is divisible by 2 (86), and therefore every such number except 2 itself is not a prime. Every number whose units' figure is 0 or 5 is divisible by 5, and therefore every such number except 5 itself is not a prime. Hence the units' figure of every prime number, except 2 and 5, must be 1, 3, 7 or 9.

(2) If then the units' figure of the given number be 1, 3, 7 or 9, try as divisors one after another the primes 3, 7, 11, 13,... until we either obtain an exact quotient, in which case the number is not prime, or until we obtain a quotient less than the divisor, in which case it is prime; for if the given number were divisible by a prime which gives a quotient *less* than the divisor, it must also be divisible by this quotient; and this quotient is either a prime, or the product of two or more primes, and therefore the number would be divisible by a prime which gives a quotient greater than the divisor; but it is not: hence the number is a prime.

Ex. 1. Is 689 a prime number?

689 is not divisible by 3 (88, Cor. 1), nor by 7 by trial, nor by 11 (89), but *is* divisible by 13: therefore 689 is *not* a prime.

Ex. 2. Is 947 a prime?

947 is not divisible by 3, 7, 11, 13, 17, 19, 23 or 29; and when divided by the next prime 31 (93), the quotient is *less* than 31: therefore 947 *is* a prime.

94. *To decompose a number into its prime factors.*

$$\begin{array}{r} 2)\underline{44856} \\ 2)\underline{22428} \\ 2)\underline{11214} \\ 3)\underline{5607} \\ 3)\underline{1869} \\ 7)\underline{623} \\ 89 \end{array}$$

Let the number be 44856: it is divisible by 2 giving the quotient 22428; 22428 is divisible by 2 giving the quotient 11214; 11214 is divisible by 2 giving the quotient 5607; and 5607 is not divisible by 2.

Again, 5607 is divisible by 3 (88, Cor. 2) giving the quotient 1869: 1869 is divisible by 3 giving the quotient 623; but 623 is not divisible by 3.

Nor is 623 divisible by the next prime 5 (87); but it is divisible by 7 giving the quotient 89, and 89 is not divisible by 7.

Lastly, 89 is not divisible by the next prime 11; moreover any greater prime than 11 would give a quotient less than itself: therefore 89 must be prime (93). Hence

$$44856 = 2 . 2 . 2 . 3 . 3 . 7 . 89 = 2^3 . 3^2 . 7 . 89.$$

And as the same method is applicable to every number, we have the following Rule:—

Divide in succession, and in each case as often as possible, by each of the primes 2, 3, 5, 7, 11,... which can be used as divisors, until we come to a quotient which is prime (68); these divisors and the last quotient put under the form of a product express the decomposition of the given number into its prime factors.

Ex. 3. Resolve 8862777 into its prime factors.

$$\begin{array}{r} 3^2=9\,)\,8862777 \\ 3^2=9\,)\,\underline{984753} \\ 7\,)\,\underline{109417} \\ 7\,)\,\underline{15631} \\ 7\,)\,\underline{2233} \\ 11\,)\,\underline{319} \\ 29 \end{array}$$

By adding the figures together, we find that the number is divisible by 9 or 3^2: in like manner the quotient is divisible by 9 or 3^2; but the quotient 109417 is not divisible by 3. We now try 7 as many times in succession as possible, and we get the quotient 319 which is not divisible by 7: we therefore try 11, giving the quotient 29, and 29 is prime; therefore

$$8862777 = 3^2 . 3^2 . 7 . 7 . 7 . 11 . 29 = 3^4 . 7^3 . 11 . 29.$$

95. *If a number be prime to each of two others, it is prime to their product.*

Thus if 8 is prime to 7 and to 15, it is prime to 7 . 15. For if not, 8 and 7 . 15 have a common measure: let it be 2. Now since 8 is

prime to 7 and to 15, every measure of 8 is prime to 7 and to 15, and therefore 2 is prime to 7 and to 15. But again, 2 divides 7 . 15 and it is prime to 7, therefore it divides 15 (75): but it is prime to 15, which is impossible. Therefore 8 is prime to 7 . 15.

COR. 1. *If one number be prime to another, it is prime to any power of that other.* Thus if 3 be prime to 7 it is prime to 7 . 7 or 7^2, and therefore to $7^2 . 7$ or 7^3, and therefore to 7^4, 7^5,...

COR. 2. *If one number be prime to another, any power of the former is prime to any power of the latter.* Thus if 3 be prime to 7, it is prime to any power of 7 (Cor. 1), that is, any power of 7 is prime to 3, and therefore to any power of 3 (Cor. 1).

96. *The G.C.M. of two or more numbers may be found by decomposing them into their prime factors, and forming the product of the least powers of those factors which are common to all the given numbers.*

Suppose the given numbers when decomposed into their prime factors to be

$$2^3 . 3^4 . 5^2 . 7 ;\ 2^4 . 3^3 . 5^2 ;\ 2^5 . 3^2 . 5 . 7^3 ;\ \text{and}\ 2^6 . 3^3 . 5^4 . 7^3,$$

then their G.C.M. will be $2^3 . 3^2 . 5$.

For take the number in which the lowest power of 2 occurs, namely $2^3 . 3^4 . 5^2 . 7$; this number is divisible by 2^3, but since 2 is prime to 3, 5, and 7, it is prime to $3^4 . 5^2 . 7$ (95), and therefore $2^3 . 3^4 . 5^2 . 7$ is not divisible by a higher power of 2 than 2^3; and therefore a higher power of 2 than 2^3 cannot occur as a factor in the G.C.M. Similarly a higher power of 3 cannot occur as a factor in the G.C.M. than 3^2; nor of 5 than 5; and 7 cannot occur at all, for $2^3 . 3^2 . 5$ is not divisible by 7. Nor can any other prime number occur as a factor in the G.C.M. such as 11, 13,... for it would be prime to each of the given numbers (95). Therefore the G.C.M. required cannot be a number greater than $2^3 . 3^2 . 5$. But again, each of the given numbers contains all the factors of $2^3 . 3^2 . 5$, or $2^3 . 3^2 . 5$ is a common measure of those numbers; hence $2^3 . 3^2 . 5$ is the G.C.M. required.

97. *The* L.C.M. *of two or more numbers may be found by decomposing them into their prime factors, and forming the product of the highest powers of all the factors that occur in the given numbers.*

Suppose the given numbers, when decomposed into their prime factors, to be

$$2^3 . 3^4 . 5^1 . 7 ; \; 2^4 . 3^3 . 5^2 ; \; 2^2 . 3^2 . 5 . 7^3 ; \text{ and } 2^5 . 3^3 . 5^2 . 7^2 ;$$

then their L.C.M. will be $2^5 . 3^4 . 5^2 . 7^3$.

For take the number in which the highest power of 2 occurs, namely $2^5 . 3^3 . 5^2 . 7^2$; in order that the L.C.M. required may be divisible by this number, it must be divisible by 2^5, that is, by the highest power of 2 that occurs in the given numbers. In like manner, it must be divisible by the highest powers of all the other numbers that occur as factors in the given numbers, that is, by 3^4, 5^2, and 7^3; or the L.C.M. must be divisible by 2^5, 3^4, 5^2, and 7^3. And since each of these numbers is prime to all the others, the L.C.M. must be divisible by $2^5 . 3^4 . 5^2 . 7^3$ (77), and cannot therefore be less than $2^5 . 3^4 . 5^2 . 7^3$ itself. But again, $2^5 . 3^4 . 5^2 . 7^3$, containing as it does all the factors of each of the given numbers, is a common multiple of them; hence $2^5 . 3^4 . 5^2 . 7^3$ is the L.C.M. required.

EXERCISE 10.

Ascertain which of the following numbers are prime, and, when not prime, give their least divisor:—

(1) 289, 461, 667, 851. (2) 953, 1517, 1729. (3) 2501, 3389, 4717.

How many prime numbers are there between

(4) 330 and 350. (5) 556 and 580. (6) 790 and 825?

Decompose the following numbers into their prime factors:—

(7) 462, 460, 310, 1188. (8) 1309, 945, 10000.
(9) 1827, 1485, 226800. (10) 55020, 16632, 402325.
(11) 93555, 53599, 47089. (12) 88725, 98735, 190463.
(13) 508079, 4149173. (14) 259811, 73896433.

Find the G.C.M. and L.C.M. of the following numbers by decomposing them into their prime factors:—

(15) 756 and 6435. (16) 756, 350 and 9075.
(17) 735, 1575 and 2205. (18) 14553, 22869 and 53361.
(19) $13 . 17 . 19$, $17 . 19 . 21$, and $19 . 21 . 23$.

CHAPTER V.

FRACTIONS.

98. A MAGNITUDE may contain its unit a number of times exactly, or a number of times with a part remaining over less than the unit, or the magnitude itself may be less than its unit. In the latter cases we divide or *break* up the unit into a certain number of equal parts, and taking one of these equal parts or sub-multiples of the unit as our *sub-unit*, we find how many times it must be repeated to make up the remaining part, or to make up the given magnitude.

Thus if we divide the unit into 8 equal parts, the sub-unit will be one-eighth of the unit: and if this sub-unit has to be repeated 5 times, the magnitude is 5-eighths of the unit.

99. When a magnitude contains its unit a number of times exactly, the resulting number (4) is called an *integer* or *whole* number; when a magnitude contains a sub-multiple of the unit, a number of times exactly, the resulting number is called a *fraction;* thus 5 is an *integer*, 5-eighths is a *fraction*.

The sum of a whole number and a fraction is called a *mixed* number, as 7 and 5-eighths.

100. The number which points out into how many equal parts the unit has been divided is called the *denominator* of the fraction; and the number which points out how many times one of these parts has been repeated is called the *numerator*. The numerator and denominator are called the *terms* of a fraction.

101. *Notation and Numeration of Fractions.* We express a fraction in figures by writing the numerator above the denominator, and drawing a bar between them; thus the fraction 5-eighths

which has 5 for its numerator and 8 for its denominator is written $\frac{5}{8}$. And the mixed number 7 and 5-eighths is written $7+\frac{5}{8}$, or rather $7\frac{5}{8}$; for the addition-sign is almost always omitted.

Conversely, a fraction expressed in figures is read by first reading the numerator and then the denominator with the termination "ths"; thus $\frac{8}{13}$ is read eight-thirteenths. The exceptions are that fractions with denominator 2 or 3 are read as so many *halves* or *thirds*, and with denominator 4 as so many *quarters* as well as *fourths*. A mixed number is read by connecting the integer and the fraction by "and"; thus $7\frac{5}{8}$ is read seven *and* five-eighths.

102. Not only do we measure magnitudes which are less than the unit by a sub-unit, but sometimes those which are equal to or greater than the unit; hence we may have such fractions as $\frac{9}{5}$, $\frac{124}{13}$. Also if we suppose the unit to be divided into 1 part, so that the sub-unit is the same as the unit, we may have such fractions as $\frac{5}{1}$, $\frac{9}{1}$: which differ only *in form* from the integers 5, 9.

Again, in measuring a magnitude by a *sub-unit*, we may have a part remaining over less than the sub-unit, and this remaining part we may measure by a subordinate sub-unit: hence we may have such fractions as $\frac{5\frac{4}{16}}{8}$, $\frac{13\frac{1}{2}}{20}$, $\frac{\frac{1}{2}}{4}$.

And, lastly, if we take our sub-unit so that a *fractional* number of sub-units make one unit, we may have such fractions as

$$\frac{3}{5\frac{1}{2}},\ \frac{3\frac{2}{3}}{5\frac{1}{3}},\ \frac{3\frac{1}{4}}{4\frac{1}{2}}.$$

103. Again, *A fraction expresses the quotient of the numerator by the denominator.*

For to take an eighth part of 5 is to take an eighth part of each of the units which make up 5, and is therefore one-eighth repeated 5 times, or is 5-eighths.

Or we may proceed thus:—Since 1 unit is 8-eighths, therefore 5 units is 40-eighths, and therefore 5 divided by 8 is 40-eighths

divided by 8, and is therefore 5-eighths; that is

$$5 \div 8 = \tfrac{5}{8}.$$

COR. 1. Hence $\frac{5}{8}$ is not only read 5-*eighths*, but also 5 *by* 8. Sometimes, the *position* only of the figures is regarded, and $\frac{5}{8}$ is read 5 *over* 8; but this reading ought to be discouraged.

COR. 2. *If we multiply a fraction by its denominator we obtain its numerator.*

Since $\frac{5}{8}$ is the eighth part of 5, $\frac{5}{8}$ repeated 8 times gives 5, or

$$\tfrac{5}{8} \times 8 = 5.$$

104. *Fraction of a Fraction.* If we take a fractional magnitude, and regarding it as a new unit, divide it into any number of equal parts and take one or more of these parts we shall get a fraction of a fraction: as $\frac{4}{5}$ of $\frac{5}{9}$.

105. We distinguish fractions into the following kinds:

(1) A *proper* fraction is one in which the numerator is less than the denominator, as $\frac{3}{7}$.

(2) An *improper* fraction is one in which the numerator is not less than the denominator, as $\frac{8}{8}$, $\frac{10}{7}$.

(3) A *simple* fraction is one in which numerator and denominator are both whole numbers, as $\frac{3}{4}$, $\frac{10}{7}$.

(4) A *complex* fraction is one in which numerator or denominator or both are not whole numbers, as

$$\frac{\frac{5}{9}}{7},\ \frac{3}{5\frac{1}{3}},\ \frac{2\frac{1}{3}}{4\frac{3}{4}},\ \frac{3\frac{1}{5}-2\frac{3}{4}}{3\frac{1}{3}+2\frac{2}{3}}.$$

(5) A *compound* fraction is a fraction of a fraction, as $\frac{3}{5}$ of $\frac{7}{8}$.

106. The following definitions will also be required:—

(1) The *reciprocal* of a fraction is the fraction formed by interchanging its terms; thus the reciprocal of $\frac{9}{7}$ is $\frac{7}{9}$, of 6 or $\frac{6}{1}$ is $\frac{1}{6}$.

(2) Fractions whose denominators may be any number we please, whole or fractional, are called *vulgar*, that is, common or ordinary fractions; whereas fractions whose denominators are 10, 100, 1000,... are called *decimal* fractions.

REDUCTION OF MIXED NUMBERS TO FRACTIONS.

107. *To express a whole number or a mixed number as a fraction.*

(1) Take the *whole* number 7. Since 1 is equal to 2-halves, or 3-thirds, or 4-fourths,... therefore 7 is equal to 7 times 2-halves, or 7 times 3-thirds, or 7 times 4-fourths, ...; that is

$$7 = \frac{7 \times 2}{2} = \frac{7 \times 3}{3} = \frac{7 \times 4}{4} = \frac{7 \times 5}{5} = \&c.,$$

hence, *For the denominator take any integer, and for the numerator the product of the given number and the denominator.*

(2) Take the *mixed* number $7\frac{3}{5}$. By the preceding case

$$7 = \frac{7 \times 5}{5} = \frac{35}{5};$$

therefore $$7\tfrac{3}{5} = 7 + \frac{3}{5} = \frac{35}{5} + \frac{3}{5},$$

but these fractions have the same sub-unit, namely 1-fifth; and of such sub-units the first contains 35 and the second 3; therefore their sum contains 35 + 3 of them: that is

$$7\tfrac{3}{5} = \frac{35}{5} + \frac{3}{5} = \frac{35+3}{5},$$

hence, *For the denominator take the denominator of the fraction, and for the numerator the product of the whole number and the denominator increased by the numerator.*

108. Conversely, *To express an improper fraction as a mixed number.*

Take the improper fraction $\frac{38}{5}$. Since 5-fifths is equal to 1, 35-fifths is equal to 7, and 38-fifths is equal to 7 and 3-fifths: that is

$$\tfrac{38}{5} = \tfrac{35}{5} + \tfrac{3}{5} = 7 + \tfrac{3}{5} = 7\tfrac{3}{5},$$

where the division of 38 by 5 gives the quotient 7 and the remainder 3: hence we have this Rule:

Divide the numerator by the denominator; the quotient will be the integer of the mixed number, the remainder will be the numerator of its fraction, and the denominator of the given fraction its denominator.

Ex. 1. Express 325 as a fraction, with denominator 999.

$$325 = \frac{325 \times 999}{999} = \frac{325000 - 325}{999} (63, 1) = \frac{324675}{999}.$$

Ex. 2. Express $59\frac{17}{73}$ as an improper fraction.

$$59\frac{17}{73} = \frac{59 \times 73 + 17}{73} = \frac{4324}{73}.$$

$$\begin{array}{r} 59 \\ \underline{73} \\ 177 \\ 413 \\ \underline{17} \\ 4324 \end{array}$$

Ex. 3. Express $\frac{6901}{274}$ as a mixed number.

$$\begin{array}{l} 274\,)\,6901\,(\,25 \\ \quad\;\; 1421 \\ \qquad\; 51 \end{array} \qquad \text{therefore } \frac{6901}{274} = 25\frac{51}{274}.$$

EXERCISE 11.

1. What fraction do we form in dividing a unit into 17 equal parts, and taking 12 of them; into 10000 equal parts, and taking 1001?

2. Express in figures: One fifth; one quarter; nine halves; twenty-three thirds; twenty-five forty-ninths; five thousand and ninety millionths.

3. Express in figures: Three, and a half; two, and a quarter; five, and five ninths; seven, and three elevenths; sixteen, and nine twenty-oneths; three hundred, and ninety-one thousandths.

4. Express in words: $\frac{3}{4}$, $\frac{2}{3}$, $\frac{1}{6}$, $\frac{10}{11}$, $\frac{83}{100}$, $3\frac{4}{5}$, $9\frac{11}{13}$, $27\frac{7}{24}$.

5. Express as a fraction: 12 with denominator 11; 9 with den. 17; 56 with den. 37; 779 with den. 13; 87 with den. 97.

Express the following mixed numbers as fractions:

6. $11\frac{1}{6}$, $13\frac{3}{5}$, $12\frac{11}{12}$, $33\frac{7}{11}$, $46\frac{8}{13}$, $87\frac{13}{100}$, $156\frac{8}{13}$, $95\frac{1}{4}$.

7. $101\frac{12}{17}$, $19\frac{13}{16}$, $323\frac{23}{27}$, $49\frac{14}{17}$, $685\frac{111}{112}$, $9879\frac{111}{176}$.

Express the following improper fractions as mixed numbers:

8. $\frac{131}{11}$, $\frac{143}{13}$, $\frac{143}{12}$, $\frac{187}{17}$, $\frac{875}{8}$, $\frac{523}{23}$, $\frac{747}{45}$, $\frac{1000}{23}$, $\frac{19583}{144}$.

9. $\frac{3003}{17}$, $\frac{4521}{71}$, $\frac{76845}{999}$, $\frac{879641}{3125}$, $\frac{830526}{9891}$, $\frac{3854271}{3769}$.

10. Express the reciprocals of the following fractions as mixed numbers:

$$\frac{7}{15}, \frac{15}{49}, \frac{17}{65}, \frac{100}{5874}, \frac{87}{2515}, \frac{99}{4567}, \frac{113}{100000}.$$

11. Find as mixed numbers: The seventh part of 1000; the seventeenth part of 2345; the eighty-ninth part of 3567.

109. *If we multiply the numerator and denominator of a simple fraction by the same whole number, the value of the fraction is unaltered.*

For example, $\frac{5}{8}=\frac{5\cdot 3}{8\cdot 3}$. For $\frac{5}{8}$ means that a unit has been divided into 8 equal parts, and that 5 of these parts have been taken; if now we divide each of these 8 parts into 3 equal parts the unit will be divided into 8 times 3 equal parts, and the 5 parts previously taken will give 5 times 3 of these new parts: hence, to divide the unit into 8 equal parts and to take 5, is equivalent to dividing it into $8\cdot 3$ equal parts and taking $5\cdot 3$ of them: that is,

$$\frac{5}{8}=\frac{5\cdot 3}{8\cdot 3}.$$

COR. *If we divide the numerator and denominator of a simple fraction by the same whole number, supposing both to be divisible by that number, the value of the fraction is unaltered.*

110. *To multiply a simple fraction by a whole number, we may multiply the numerator by that number: or, if the denominator be divisible by that number, we may divide the denominator.*

(1) The parts composing the fractions $\frac{5}{8}$ and $\frac{5\cdot 3}{8}$ are each equal to one-eighth, and the number of parts taken in the second fraction is 3 times the number taken in the first, therefore the second fraction is 3 times the first, or $\frac{5}{8}\times 3=\frac{5\cdot 3}{8}$.

(2) By the preceding case, $\frac{5}{4\cdot 3}\times 3=\frac{5\cdot 3}{4\cdot 3}$; but $\frac{5\cdot 3}{4\cdot 3}=\frac{5}{4}$ (109), therefore $\frac{5}{4\cdot 3}\times 3=\frac{5}{4}$.

111. *To divide a simple fraction by a whole number, we may divide the numerator, if it be divisible, by that number; or we may multiply the denominator.*

(1) The parts composing the fractions $\frac{5\cdot 3}{8}$ and $\frac{5}{8}$ are each equal to one-eighth, and the number of parts taken in the first

fraction is 3 times the number taken in the second, therefore the second fraction is the quotient of the first by 3, or $\frac{5\cdot 3}{8}\div 3=\frac{5}{8}$.

(2) But if the numerator be not divisible by 3, as in $\frac{5}{8}$, multiply numerator and denominator by 3, then $\frac{5}{8}=\frac{5\cdot 3}{8\cdot 3}$ (109); and by the preceding case, $\frac{5\cdot 3}{8\cdot 3}\div 3=\frac{5}{8\cdot 3}$; therefore $\frac{5}{8}\div 3=\frac{5}{8\cdot 3}$.

112. *To express a compound fraction as a simple one.*

Take the compound fraction $\frac{8}{9}$ of $\frac{11}{12}$: it means that regarding $\frac{11}{12}$ as a whole we are to divide it into 9 equal parts, and take 8 of those parts; but when we divide $\frac{11}{12}$ into 9 equal parts, each part will be equal to $\frac{11}{12}\div 9$ or to $\frac{11}{12\cdot 9}$ (111, 2), and 8 of these parts will give $\frac{11}{12\cdot 9}\times 8$ or $\frac{11\cdot 8}{12\cdot 9}$ (110); therefore

$$\frac{8}{9}\text{ of }\frac{11}{12}=\frac{11\cdot 8}{12\cdot 9}=\frac{8\cdot 11}{9\cdot 12}. \qquad (61)$$

In like manner $\frac{3}{4}$ of $\frac{8}{9}$ of $\frac{11}{12}=\frac{3}{4}$ of $\frac{8\cdot 11}{9\cdot 12}$

$$=\frac{3\cdot 8\cdot 11}{4\cdot 9\cdot 12};$$

hence we have this Rule:—

Multiply all the numerators together for the numerator required and all the denominators for the denominator.

REMARK 1. We must express mixed numbers as improper fractions before applying the Rule.

REMARK 2. If there are factors common to numerator and denominator, they may be *cancelled*, or struck out, before obtaining the final result. Thus, in $\frac{3\cdot 8\cdot 11}{4\cdot 9\cdot 12}$, rejecting the factor 3 common to numerator and denominator, we have $\frac{1\cdot 8\cdot 11}{4\cdot 3\cdot 12}$: again, rejecting the common factor 4 we have $\frac{1\cdot 2\cdot 11}{1\cdot 3\cdot 12}$; lastly, rejecting the com-

mon factor 2 we have $\frac{1 \cdot 1 \cdot 11}{1 \cdot 3 \cdot 6}$ or $\frac{11}{18}$. The work usually stands thus:

$$\frac{3}{4} \text{ of } \frac{8}{9} \text{ of } \frac{11}{12} = \frac{\cancel{3} \cdot \overset{2}{\cancel{8}} \cdot 11}{\cancel{4} \cdot \underset{3}{\cancel{9}} \cdot \underset{6}{\cancel{12}}} = \frac{11}{18}.$$

Remark 3. In cancelling, if a quotient be 1, it is usually omitted: thus, over the cancelled factors 3 and 2 in the above numerator no number appears.

Ex. 1. Express $\frac{7}{12}$, $\frac{9}{16}$ and $\frac{8}{27}$ as fractions with denominator 432.

Now $432 \div 12 = 36$ and $\frac{7}{12} = \frac{7 \cdot 36}{12 \cdot 36} = \frac{252}{432}$;

$432 \div 16 = 27$ $\quad \frac{9}{16} = \frac{9 \cdot 27}{16 \cdot 27} = \frac{243}{432}$;

$432 \div 27 = 16$ $\quad \frac{8}{27} = \frac{8 \cdot 16}{27 \cdot 16} = \frac{128}{432}$.

Ex. 2. $\frac{7}{8} \times 9 = \frac{63}{8} = 7\frac{7}{8}$. $\qquad 3\frac{7}{8} \times 4 = \frac{31}{8} \times 4 = \frac{31}{2} = 15\frac{1}{2}$.

$\frac{35}{64} \div 7 = \frac{5}{64}$. $\qquad 9\frac{10}{11} \div 11 = \frac{109}{11} \div 11 = \frac{109}{121}$.

Ex. 3. $3\frac{5}{7}$ of $\frac{12}{13}$ of $\frac{11}{24}$ of $2\frac{6}{11}$ of $\frac{5}{12} = \frac{26}{7}$ of $\frac{12}{13}$ of $\frac{11}{24}$ of $\frac{28}{11}$ of $\frac{5}{12}$

$$= \frac{\overset{2}{\cancel{26}} \times \cancel{12} \times \cancel{11} \times \overset{4}{\cancel{28}} \times 5}{\cancel{7} \times \cancel{13} \times \underset{\underset{3}{6}}{\cancel{24}} \times \cancel{11} \times \cancel{12}} = \frac{5}{3}$$

$= 1\frac{2}{3}$.

EXERCISE 12.

1. Find fractions equal to $\frac{5}{7}$, $\frac{11}{12}$, and $\frac{13}{14}$, having 84 for their denominator.

2. Transform $\frac{7}{13}$, $\frac{16}{25}$, and $\frac{49}{65}$, into equal fractions whose den^r shall be 325.

3. Find fractions equal to $\frac{14}{17}$, $\frac{18}{25}$, $\frac{21}{32}$, and $\frac{27}{35}$, having 756 for their common numerator.

4. Transform $\frac{112}{91}$, $\frac{425}{357}$, and $\frac{3341}{5140}$, into equivalent fractions whose denominators shall be 13, 21 and 20 respectively.

5. Express $17\frac{2}{3}$, $25\frac{5}{6}$ and $13\frac{5}{9}$ as fractions, with denominator 18.

6. Express $41\frac{7}{8}$, $23\frac{11}{12}$, $7\frac{13}{15}$ and $9\frac{7}{16}$ as fractions, with denominator 240.

Find the value of:

7. $\frac{17}{24}\times 8$; $\frac{12}{13}\times 13$; $\frac{8}{11}\times 12$; $5\frac{13}{18}\times 7$; $9\frac{4}{17}\times 17$; $6\frac{3}{4}\times 11$.

8. $\frac{45}{49}\div 9$; $\frac{15}{19}\div 7$; $\frac{117}{110}\div 13$; $\frac{143}{100}\div 11$; $7\frac{3}{7}\div 12$; $9\frac{11}{13}\div 15$.

Reduce the following compound fractions to simple ones:

9. $\frac{8}{9}$ of $\frac{15}{32}$; $\frac{15}{28}$ of $\frac{35}{51}$; $\frac{42}{55}$ of $\frac{65}{28}$; $\frac{3}{4}$ of $6\frac{2}{7}$ of $\frac{16}{25}$.

10. $\frac{2}{9}$ of $6\frac{3}{7}$ of $\frac{14}{15}$; $1\frac{2}{5}$ of $\frac{14}{45}$ of $\frac{27}{91}$; $\frac{4}{9}$ of $\frac{12}{25}$ of $\frac{21}{16}$ of $4\frac{8}{9}$.

11. $2\frac{1}{4}$ of $\frac{15}{16}$ of $7\frac{3}{7}$ of $\frac{42}{65}$; $\frac{7}{31}$ of $6\frac{1}{5}$ of $17\frac{1}{2}$ of $10\frac{2}{3}$ of $9\frac{7}{15}$.

12. $\frac{19}{3}$ of $\frac{14}{15}$ of $9\frac{1}{9}$ of $3\frac{1}{7}$ of $\frac{25}{42}$ of $\frac{98}{171}$.

REDUCTION OF A FRACTION TO ITS LOWEST TERMS.

113. A simple fraction is said to be in its *lowest terms*, when no fraction of equal value can be found whose terms are less than those of the given fraction. Thus $\frac{6}{8}$ is not in its lowest terms, for it is equal to $\frac{3}{4}$; but $\frac{3}{4}$ is in its lowest terms, for no equal fraction can be found whose numerator and denominator are respectively less than 3 and 4.

114. *If the numerator and denominator of a fraction be prime to each other, the numerator and denominator of any fraction of equal value will be equimultiples of the numerator and denominator of the given fraction.*

Take any fraction $\frac{4}{5}$ whose numerator and denominator are prime to each other, and suppose $\frac{4}{5}$ is equal to the fraction $\frac{a}{b}$ where a and b stand for two whole numbers; then a and b are equimultiples of 4 and 5.

If we multiply each of the equal fractions $\frac{a}{b}$ and $\frac{4}{5}$ by b the products will be equal; but $\frac{a}{b}$ multiplied by b gives a (103), and

$\frac{4}{5}$ multiplied by b gives $\frac{4 \times b}{5}$ (110);

therefore $$a = \frac{4 \times b}{5};$$

and since a is a whole number, $4 \times b$ must be divisible by 5: but 5 is prime to 4, therefore it must divide b (75). In dividing b by 5, let the quotient be denoted by the whole number c, therefore we have $$b = 5 \times c$$

and therefore $$a = \frac{4 \times 5 \times c}{5} = 4 \times c;$$

thus a is the same multiple of 4 that b is of 5; or a and b are equimultiples of 4 and 5.

COR. Hence all the fractions which are equal to a given fraction, whose numerator and denominator are prime to each other, are found by multiplying its numerator and denominator by the numbers 2, 3, 4......

115. *For a fraction to be in its lowest terms, it is necessary and sufficient that numerator and denominator be prime to each other.*

(1) It is necessary,—for if they are not prime to each other, they have a common measure; divide them by this common measure, and we have a fraction equal to the proposed in *lower* terms.

(2) It is sufficient,—for if they are prime to each other, every equal fraction must have its terms equimultiples of the terms of the proposed fraction; and must therefore be in *higher* terms than that fraction.

COR. If we divide the numerator and denominator of any fraction by their G.C.M., the numerator and denominator of the new fraction will be prime to each other, and therefore it will be in its lowest terms. Hence, to reduce a fraction to its lowest terms, *we divide the numerator and denominator by their* G.C.M.

Example. Reduce $\frac{2415}{7035}$ to its lowest terms.

2415	7035	105) 2415 (23	105) 7035 (67
2310	7245	315	735
105	210	0	0
	210		

$\therefore \frac{2415}{7035} = \frac{23}{67}$ fraction required.

116. In practice, however, we seldom need to find the G. C. M. of numerator and denominator : we see by inspection, or find by trial, some factor common to both; and, having expelled that factor, we proceed again in the same way, and continue the process until the terms are prime to each other.

Thus in the preceding example we see that 5 is a factor of both terms (86), hence we have $\frac{483}{1407}$: we now see that 3 is a factor of both (87), hence we have $\frac{161}{469}$: again we find that 7 is a factor of both, hence we have $\frac{23}{67}$: but 23 is a prime, and does not divide 67, therefore 23 and 67 are prime to each other (68), and $\frac{23}{67}$ is in its lowest terms.

The work usually stands thus :

$$\frac{2415}{7035}=\frac{483}{1407}=\frac{161}{469}=\frac{23}{67}.$$

EXERCISE 13.

Reduce the following fractions to their lowest terms, without finding the G.C.M. of numerator and denominator (116):

1. $\frac{119}{168}$; $\frac{208}{864}$; $\frac{304}{1072}$.
2. $\frac{539}{637}$; $\frac{660}{1155}$; $\frac{672}{1056}$.
3. $\frac{1584}{5940}$; $\frac{4235}{7315}$; $\frac{3276}{4914}$.
4. $\frac{5929}{9163}$, $\frac{10395}{19635}$.
5. $\frac{12540}{21945}$; $\frac{99715}{113960}$.
6. $\frac{30429}{88641}$; $\frac{475200}{639936}$.

Reduce the following fractions to their lowest terms by finding the G.C.M. of numerator and denominator :

7. $\frac{1261}{1649}$; $\frac{8991}{10989}$; $\frac{10307}{94637}$.
8. $\frac{18577}{20006}$; $\frac{24293}{48569}$; $\frac{61013}{63529}$.
9. $\frac{42237}{75582}$; $\frac{95469}{359784}$.
10. $\frac{180194}{1973594}$; $\frac{333567}{436203}$.
11. $\frac{70609}{231001}$; $\frac{356417}{701011}$.
12. $\frac{1842051}{2592525}$; $\frac{496606401}{1006110363}$.

Express the following fractions as mixed numbers, reducing the fractional part to its lowest terms :

13. $\frac{69}{9}$; $\frac{371}{21}$; $\frac{6244}{112}$.
14. $\frac{16191}{257}$; $\frac{15157}{161}$; $\frac{75648}{864}$.

LEAST COMMON DENOMINATOR.

117. To reduce fractions to others having a *least common denominator* is to find equal fractions having a *common* denominator, and that denominator the *least* that can be taken.

Let the given fractions be in their lowest terms, then the terms of the equal fractions must be equimultiples of the terms of the given fractions (114), and therefore every common denominator of these equal fractions must be a common multiple of the given denominators.

Again, take any common multiple of the given denominators; find the factors by which we must multiply each denominator to produce this common multiple, multiply the terms of each fraction by their factor, and we have equal fractions with this common multiple for each denominator.

Hence every common denominator of the given fractions is a common multiple of their denominators, and every common multiple of the denominators is a common denominator of them: therefore the L.C.M. of the denominators is the least common denominator of the given fractions.

Therefore, to reduce fractions to others having the least common denominator, we have this Rule:

Find the L.C.M. of the given denominators, and take it for the least common denominator: divide it by the denominator of the first fraction, and multiply the terms of this fraction by the quotient; and do the same with all the other given fractions.

Ex. 1. Find fractions equivalent to $\frac{3}{8}$, $\frac{11}{12}$, $\frac{8}{15}$ and $\frac{17}{21}$ having the least common denominator.

$$2 \times 3 \text{ or } 6 \,)\; 8,\quad 12,\quad 15,\quad 21$$
$$\qquad\qquad 4,\quad 2,\quad 5,\quad 7$$

$\therefore$ Least common denominator $= 8 \times 3 \times 5 \times 7 = 840$.

Now $\quad 840 \div 8 = 105$, therefore $\frac{3}{8} = \frac{3 \times 105}{8 \times 105} = \frac{315}{840}$;

$\quad 840 \div 12 = 70$, $\qquad \frac{11}{12} = \frac{11 \times 70}{12 \times 70} = \frac{770}{840}$;

$$840 \div 15 = 56, \text{ therefore } \frac{8}{15} = \frac{8 \times 56}{15 \times 56} = \frac{448}{840};$$

$$840 \div 21 = 40, \qquad \frac{17}{21} = \frac{17 \times 40}{21 \times 40} = \frac{680}{840}.$$

Ex. 2. Arrange in order of magnitude the fractions $\frac{3}{8}, \frac{11}{12}, \frac{8}{15}$ and $\frac{17}{21}$.

Reduce these fractions to equivalent ones with the least common denominator; these by Ex. 1 we find to be

$$\frac{315}{840}, \frac{770}{840}, \frac{448}{840} \text{ and } \frac{680}{840};$$

and, as the parts composing each of these fractions are all equal, the one in which the greatest number of parts is taken will be greatest; hence, arranged in order of magnitude, we have

$$\frac{315}{840}, \frac{448}{840}, \frac{680}{840} \text{ and } \frac{770}{840};$$

that is, $\qquad \frac{3}{8}, \frac{8}{15}, \frac{17}{21}$ and $\frac{11}{12}$

are in order of magnitude; $\frac{3}{8}$ being least and $\frac{11}{12}$ greatest.

EXERCISE 14.

Reduce to equivalent fractions with the least common denominator:

1. $\frac{5}{6}, \frac{7}{9}, \frac{8}{15}, \frac{9}{10}$; $\qquad \frac{6}{7}, \frac{7}{8}, \frac{8}{9}, \frac{11}{12}$.

2. $\frac{3}{5}, \frac{5}{7}, \frac{7}{9}, \frac{8}{21}, \frac{24}{35}$; $\qquad \frac{3}{4}, \frac{9}{10}, \frac{11}{15}, \frac{17}{12}$.

3. $\frac{23}{27}, \frac{17}{18}, \frac{19}{24}, \frac{47}{54}$; $\qquad \frac{3}{10}, \frac{57}{1000}, \frac{459}{10000}, \frac{8756}{100000}$.

4. $\frac{73}{60}, \frac{11}{15}, \frac{13}{24}, \frac{31}{40}, \frac{117}{100}$; $\qquad \frac{19}{36}, \frac{17}{8}, \frac{13}{54}, \frac{125}{48}, \frac{25}{32}$.

5. $\frac{19}{21}, \frac{11}{12}, \frac{8}{15}, \frac{25}{27}, \frac{81}{35}, \frac{27}{40}$; $\qquad \frac{14}{15}, \frac{15}{16}, \frac{17}{18}, \frac{19}{20}, \frac{23}{24}, \frac{26}{27}$.

Express the following numbers as fractions having the least common denominator:

6. $3\frac{7}{10}, \frac{37}{40}, 1\frac{11}{12}, \frac{16}{27}, 13\frac{11}{12}$; $\qquad 8\frac{11}{18}, 10\frac{5}{18}, 6\frac{4}{15}, \frac{137}{45}, 9\frac{11}{15}$.

7. Arrange in order of magnitude:

$$\frac{7}{15},\ \frac{10}{21},\ \frac{16}{35};\qquad \frac{18}{35},\ \frac{31}{47},\ \frac{14}{19};\qquad \frac{15}{4},\ 3\tfrac{1}{4},\ \frac{2}{7}\text{ of }9\tfrac{1}{2}.$$

8. Which is greater, $\frac{15}{19}$ or $\frac{15+8}{19+8}$? $\frac{15}{19}$ or $\frac{15-8}{19-8}$?

9. Arrange in order of magnitude

$$\frac{8}{17},\ \frac{9}{25},\ \frac{8+9}{17+25};\qquad \frac{13}{14},\ \frac{15}{16},\ \frac{13+15}{14+16};$$

$$\frac{5}{6},\ \frac{6}{7},\ \frac{7}{8},\ \frac{9}{10},\ \frac{5+6+7+9}{6+7+8+10}.$$

10. Find the least and the greatest of the following numbers:

$$\frac{3}{7},\ \frac{5}{13},\ \frac{7}{16},\ \frac{9}{20};\qquad \frac{10}{18},\ \frac{15}{24},\ \frac{19}{27},\ \frac{37}{35};\qquad \frac{101}{13},\ 7\tfrac{12}{13},\ 7\tfrac{9}{11},\ \frac{1091}{143}.$$

11. Find a fraction intermediate in value to $\frac{5}{6}$ and $\frac{6}{7}$ whose denominator is 84; to $\frac{2}{5}$ and $\frac{29}{72}$ whose denominator is 720.

12. Of $\frac{27}{87}$, $\frac{118}{341}$ and $\frac{145}{428}$ which is intermediate in value?

13. Reduce $\frac{2}{9}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{12}{19}$, $\frac{21}{23}$ to equal fractions with the least common numerator.

ADDITION, SUBTRACTION, &c. OF FRACTIONS.

118. The definitions that have been given of Addition, Subtraction, Multiplication and Division have been given with reference to *whole numbers* only: it will now be necessary to extend them, that they may be applicable and consistent whether we are operating on whole numbers or on fractions.

ADDITION.

119. ADDITION is the operation by which we find a single number that is equal to two or more fractional numbers put together.

This single number is called the *sum* of the given numbers.

120. *To find the sum of two or more given fractions.*

(1) Let the given fractions have the *same* denominator: for example, find the sum of $\frac{4}{7}$, $\frac{5}{7}$ and $\frac{8}{7}$. These fractions have all the

same sub-unit, one-seventh; and of such sub-units the first fraction contains 4, the second 5, and the third 8; therefore their sum contains $4+5+8$ of them; or

$$\frac{4}{7}+\frac{5}{7}+\frac{8}{7}=\frac{4+5+8}{7};$$

that is,—*We add the numerators of the given fractions together for the numerator of the sum, and take their denominator for its denominator.*

(2) Let the given fractions have *different* denominators: for example, find the sum of $\frac{2}{3}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}$. Since the parts composing these fractions are all different, the first step will be to find fractions equal to them composed of the same parts, that is, which have the same denominator; and of all common denominators the *least* will be the most convenient.

The least common denominator is found to be 24, so that

$$\begin{aligned}\frac{2}{3}+\frac{3}{4}+\frac{5}{6}+\frac{7}{8}&=\frac{16}{24}+\frac{18}{24}+\frac{20}{24}+\frac{21}{24}\\ &=\frac{16+18+20+21}{24} \qquad (120.\ 1)\\ &=\frac{75}{24}=\frac{25}{8}\\ &=3\tfrac{1}{8}.\end{aligned}$$

We have then this Rule,—*Reduce the given fractions to their least common denominator; add the numerators together for the numerator of their sum, and take the least common denominator for its denominator.*

REMARK 1. The sum should always be expressed in its lowest terms; and, if an improper fraction, should be reduced to a mixed number.

REMARK 2. Compound fractions should be reduced to simple ones, before the application of the Rule.

REMARK 3. *Numbers, whether whole or fractional, may be added together in any order we please;* for reduce all of them to a common denominator, then their sum is found by finding the sum of their numerators; but these, being all integers, may be added in any order we please (25), and therefore the numbers themselves may be added in any order we please. Hence, instead of reducing

mixed numbers to improper fractions, we may add separately the integral and the fractional parts of such mixed numbers; thus

$$8\tfrac{3}{4}+3\tfrac{4}{7}+2\tfrac{5}{6}=8+\tfrac{3}{4}+3+\tfrac{4}{7}+2+\tfrac{5}{6}$$
$$=8+3+2+\tfrac{3}{4}+\tfrac{4}{7}+\tfrac{5}{6}.$$

Ex. 1. Find the sum of $\frac{25}{19}$, $\frac{18}{19}$, $\frac{58}{19}$ and $\frac{17}{19}$.

$$\frac{25}{19}+\frac{18}{19}+\frac{58}{19}+\frac{17}{19}=\frac{25+18+58+17}{19}$$
$$=\frac{118}{19}$$
$$=6\tfrac{4}{19}.$$

Ex. 2. *Find the sum of* $5\frac{7}{8}$, $8\frac{7}{9}$, $13\frac{1}{3}$ *of* $\frac{9}{25}$, $\frac{101}{12}$ *and* $17\frac{17}{24}$.

$$13\tfrac{1}{3}\text{ of }\tfrac{9}{25}=\frac{\cancel{40}^{\,8}}{\cancel{3}}\text{ of }\frac{\cancel{9}^{\,3}}{\cancel{25}_{\,5}}=\frac{24}{5}=4\tfrac{4}{5};\qquad \frac{101}{12}=8\tfrac{5}{12};$$

$$\begin{array}{r|llllll} 3 & 8, & 9, & 5, & 12, & 24 \\ \hline & & 3, & 5, & & 8 \end{array}$$

∴ least common denominator $=3\times3\times5\times8=360$,

hence,

$$5\tfrac{7}{8}+8\tfrac{7}{9}+13\tfrac{1}{3}\text{ of }\tfrac{9}{25}+\tfrac{101}{12}+17\tfrac{17}{24}=5\tfrac{7}{8}+8\tfrac{7}{9}+4\tfrac{4}{5}+8\tfrac{5}{12}+17\tfrac{17}{24}$$
$$=42+\tfrac{7}{8}+\tfrac{7}{9}+\tfrac{4}{5}+\tfrac{5}{12}+\tfrac{17}{24}$$
$$=42+\tfrac{315}{360}+\tfrac{280}{360}+\tfrac{288}{360}+\tfrac{150}{360}+\tfrac{255}{360}$$
$$=42+\tfrac{1288}{360}$$
$$=42+\tfrac{161}{45}$$
$$=42+3\tfrac{26}{45}$$
$$=45\tfrac{26}{45}.$$

EXERCISE 15.

Find the sum of

1. $\frac{8}{17}$, $\frac{11}{17}$, $\frac{9}{17}$, $\frac{13}{17}$; 7[illegible], 5[illegible], $\frac{127}{47}$, $\frac{18}{47}$; 6[illegible], $\frac{1325}{529}$, $\frac{6041}{529}$, 13[illegible].

2. $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$; $\frac{7}{12}$, $\frac{11}{18}$, $\frac{17}{20}$, $\frac{14}{15}$; $\frac{17}{21}$, $\frac{5}{11}$, $\frac{13}{15}$, $\frac{8}{9}$, $\frac{29}{35}$.

3. $\frac{113}{226}$, $\frac{226}{339}$, $\frac{565}{678}$, $\frac{133}{182}$, $\frac{235}{325}$, $\frac{261}{351}$; $\frac{3}{4}$ of $\frac{5}{8}+\frac{7}{12}$ of $\frac{21}{16}+\frac{9}{16}$ of $4\frac{4}{5}$.

4. $4\frac{2}{3}$, $8\frac{3}{5}$, $3\frac{2}{7}$, $8\frac{3}{4}$; $1\frac{2}{21}$, $2\frac{2}{15}$, $3\frac{3}{26}$, $4\frac{4}{27}$; $\frac{7}{10}$, $\frac{57}{100}$, $\frac{567}{1000}$, $\frac{5673}{10000}$.

5. $175\frac{1}{4}$, $62\frac{11}{120}$, $1031\frac{1}{2}$, $\frac{7}{8}$ of $4150\frac{1}{7}$; $\frac{13}{15}$, $\frac{17}{20}$, $\frac{19}{21}$, $\frac{23}{25}$, $\frac{29}{30}$.

Find the value of

6. $3\frac{5}{12}+7\frac{11}{14}+2\frac{1}{4}$ of $\frac{3}{5}+6\frac{7}{12}$; $\frac{7}{11}+\frac{2}{19}$ of $2\frac{3}{8}+\frac{8}{15}+8\frac{2}{14}$ of $4\frac{2}{3}+30\frac{1}{36}$.

7. $\frac{1}{3}$ of $4\frac{2}{5}+7\frac{1}{4}+\frac{9}{10}$ of $6\frac{2}{3}+\frac{4}{7}$ of $5\frac{1}{4}+\frac{3}{5}$ of $9\frac{1}{6}$.

8. $\frac{3}{4}$ of $\frac{5}{7}$ of $51\frac{4}{15}+\frac{2}{5}$ of $\frac{1}{9}$ of $506\frac{1}{4}+\frac{3}{5}$ of $\frac{3}{8}$ of 1864.

9. $\frac{1}{7}$ of $\frac{5}{7}\div\frac{4}{5}$ of $\frac{1}{10}+\frac{3}{5}$ of $\left(\frac{1}{2}\div\frac{11}{14}\right)\div\frac{3}{70}$ of $\left(\frac{2}{7}+\frac{4}{5}\right)$.

10. Of the numbers $1\frac{4}{55}$, $\frac{2}{3}\div\frac{3}{4}$, and $\frac{1}{2}\div\frac{1}{5}+\frac{5}{7}$, which is greatest?

SUBTRACTION.

121. SUBTRACTION is the operation by which we find what number is left when a smaller fractional number is taken from a greater.

The number left is called the *Remainder*.

122. As in integers, the number left is the *difference* between the two given numbers, is the *excess* of the greater number over the less, and is the number which must be added to the less number to make it equal to the greater.

123. *To subtract one fraction from another fraction.*

(1) Let the given fractions have the same denominator: for example, subtract $\frac{4}{17}$ from $\frac{9}{17}$. Each of these fractions has the same sub-unit, one-seventeenth; and of such sub units the Minuend contains 9 and the Subtrahend 4, therefore the Remainder contains $9-4$ of them; or

$$\frac{9}{17}-\frac{4}{17}=\frac{9-4}{17};$$

that is,—*We subtract the numerators of the given fractions for the numerator of the Remainder, and take their denominator for its denominator.*

(2) Let the given fractions have *different* denominators: for example, subtract $\frac{3}{8}$ from $\frac{11}{12}$. Since the parts composing these fractions are different, the first step will be to find fractions equal to them composed of the same parts, that is, which have the same denominator; and of all common denominators the *least* will be the most convenient.

The least common denominator is found to be 24, so that

$$\frac{11}{12}-\frac{3}{8}=\frac{22}{24}-\frac{9}{24}$$
$$=\frac{22-9}{24}$$
$$=\frac{13}{24}.$$

We have then this Rule,—*Reduce the given fractions to others having the least common denominator, subtract their numerators for the numerator of the Remainder, and take the least common denominator for its denominator.*

REMARK 1. The Remainder should always be expressed in its lowest terms; and, if an improper fraction, should be reduced to a mixed number.

REMARK 2. Compound fractions should be reduced to simple ones, before the application of the Rule.

ADDITION AND SUBTRACTION.

124. The propositions relating to the addition and subtraction of *expressions* made up of additions and subtractions enunciated in Arts. 54—57 are equally applicable to fractions as to whole numbers: thus

(1) *To a fraction we may add the sum of two others by adding them in succession* (54).

$$\text{For } 8\tfrac{3}{4}+\left(3\tfrac{4}{7}+2\tfrac{5}{6}\right)=\frac{735}{84}+\left(\frac{300}{84}+\frac{238}{84}\right) \qquad (117)$$
$$=\frac{735}{84}+\frac{300+238}{84}$$
$$=\frac{735+(300+238)}{84} \qquad (120)$$
$$=\frac{735+300+238}{84} \qquad (54)$$

$$=\frac{735}{84}+\frac{300}{84}+\frac{238}{84} \qquad (120)$$
$$=8\tfrac{3}{4}+3\tfrac{4}{7}+2\tfrac{5}{6}.$$

(2) Again, *From a fraction we may subtract the sum of two others, by subtracting them in succession.*

$$\text{For } 8\tfrac{3}{4}-\left(3\tfrac{4}{7}+2\tfrac{5}{6}\right)=\frac{735}{84}-\left(\frac{300}{84}+\frac{238}{84}\right)=\frac{735}{84}-\frac{300+238}{84}$$
$$=\frac{735-(300+238)}{84}=\frac{735-300-238}{84} \qquad (54)$$
$$=\frac{735-300}{84}-\frac{238}{84} \qquad (121)$$
$$=\frac{735}{84}-\frac{300}{84}-\frac{238}{84}$$
$$=8\tfrac{3}{4}-3\tfrac{4}{7}-2\tfrac{5}{6}.$$

(3) And in precisely the same way may the other propositions in Arts. 55, 56 be shewn to hold true of fractions.

125. We may also shew in the same way that

(1) *Additions and subtractions of fractions may be performed in any order.* Hence instead of reducing mixed numbers to improper fractions, we may subtract separately the integral and the fractional parts of the mixed numbers; thus

$$8\tfrac{3}{4}-3\tfrac{4}{7}=8+\tfrac{3}{4}-\left(3+\tfrac{4}{7}\right)=8+\tfrac{3}{4}-3-\tfrac{4}{7} \qquad (124,\ 2)$$
$$=5+\tfrac{3}{4}-\tfrac{4}{7}.$$

(2) *An expression made up of additions and subtractions of fractions may be made equal to the difference of two sums;* thus

$$8\tfrac{3}{4}-3\tfrac{4}{7}+2\tfrac{5}{6}-\tfrac{3}{5}+\tfrac{2}{3}-\tfrac{1}{2}=\left(8\tfrac{3}{4}+2\tfrac{5}{6}+\tfrac{2}{3}\right)-\left(3\tfrac{4}{7}+\tfrac{3}{5}+\tfrac{1}{2}\right).$$

Ex. 1.
$$1-\tfrac{17}{19}=\tfrac{19}{19}-\tfrac{17}{19}=\tfrac{2}{19}.$$
$$1\tfrac{12}{19}-\tfrac{17}{19}=\tfrac{12}{19}+1-\tfrac{17}{19}=\tfrac{12}{19}+\tfrac{2}{19}=\tfrac{14}{19}.$$
$$8\tfrac{12}{19}-3\tfrac{17}{19}=5\tfrac{12}{19}-\tfrac{17}{19}=4\tfrac{12}{19}+\tfrac{2}{19}=4\tfrac{14}{19}.$$

Ex. 2. Subtract $3\frac{7}{11}$ from $8\frac{4}{5}$; and $3\frac{4}{5}$ from $8\frac{7}{11}$.

(1) $8\frac{4}{5}-3\frac{7}{11}=5\frac{44}{55}-\frac{35}{55}=5\frac{9}{55}$.

(2) $8\frac{7}{11}-3\frac{4}{5}=5\frac{35}{55}-\frac{44}{55}=4\frac{55}{55}+\frac{35}{55}-\frac{44}{55}=4\frac{46}{55}$.

Or we may adopt the following arrangement :—

$$\text{(1)}\quad \begin{array}{ll} 8\frac{4}{5} & \ldots 44 \\ \underline{3\frac{7}{11}} & \ldots 35 \\ 5\frac{9}{55} & \end{array} \qquad\qquad \text{(2)}\quad \begin{array}{ll} 8\frac{7}{11} & \ldots 35 \\ \underline{3\frac{4}{5}} & \ldots 44 \\ 4\frac{46}{55} & \end{array}$$

In the second example as we cannot subtract $\frac{44}{55}$ from $\frac{35}{55}$ we borrow 1 or $\frac{55}{55}$, and then say $\frac{44}{55}$ from $\frac{55}{55}$ leave $\frac{11}{55}$, and $\frac{11}{55}$ and $\frac{35}{55}$ is $\frac{46}{55}$: or simply 44 from 55 11, and 35 46.

Ex. 3. $8\frac{3}{4}-3\frac{13}{14}+2\frac{7}{9}$ of $\frac{3}{14}$ of $4\frac{4}{5}-\left(5\frac{11}{12}-2\frac{7}{8}\right)$

$$=8\frac{3}{4}-3\frac{13}{14}+\frac{25}{9}\text{ of }\frac{3}{14}\text{ of }\frac{24}{5}-5\frac{11}{12}+2\frac{7}{8}$$

$$=8\frac{3}{4}-3\frac{13}{14}+2\frac{6}{7}-5\frac{11}{12}+2\frac{7}{8}$$

$$=4+\frac{3}{4}+\frac{6}{7}+\frac{7}{8}-\left(\frac{13}{14}+\frac{11}{12}\right)$$

$$=4+\frac{126+144+147}{168}-\frac{156+154}{168}$$

$$=4+\frac{417}{168}-\frac{310}{168}=4+\frac{107}{168}$$

$$=4\frac{107}{168}.$$

EXERCISE 16.

Find the difference between

1. $\frac{12}{13}$ and $\frac{5}{13}$; 4 and $\frac{11}{19}$; 8[illegible] and $\frac{8}{9}$; 3[illegible] and $\frac{11}{12}$.

2. 9[illegible] and 2[illegible]; 37[illegible] and 19[illegible]; $\frac{14}{15}$ and $\frac{11}{12}$; $\frac{17}{39}$ and $\frac{11}{26}$.

3. $\frac{31}{56}$ and $\frac{13}{98}$; 8[illegible] and 5[illegible]; 3[illegible] and 2[illegible].

In the following examples write down the remainders

4. 25[illegible] 8[illegible]; 18[illegible] 12[illegible]; 5[illegible] 3[illegible]; 22[illegible] 8[illegible]; 10[illegible] 8[illegible]; 6[illegible] 3[illegible]

Find the value of

5. $\frac{2}{3}+\frac{1}{2}-\frac{5}{6}+\frac{7}{8}-\frac{5}{24}$; $\frac{2}{3}+\frac{5}{6}+\frac{8}{9}-\frac{11}{12}-\frac{17}{18}$.

6. $\frac{3}{20}+\frac{5}{12}+\frac{4}{15}-\frac{5}{6}$; $3\frac{1}{3}+2\frac{3}{4}-6\frac{5}{8}+1\frac{5}{12}$; $\frac{1}{12}+\frac{1}{81}+\frac{1}{36}-\frac{1}{64}$.

7. $\frac{3}{8}$ of $\frac{4}{7}-\frac{2}{11}$ of $3\frac{1}{2}+\frac{5}{9}$ of $3\frac{3}{5}$; $\frac{15}{16}-\frac{14}{15}+\frac{13}{14}-\frac{11}{12}$.

8. $\left(\frac{11}{12}-\frac{5}{6}\right)+\left(\frac{4}{5}-\frac{2}{3}\right)$; $\left(\frac{11}{12}+\frac{5}{6}\right)-\left(\frac{4}{5}+\frac{2}{3}\right)$; $\frac{11}{12}+\left(\frac{5}{6}-\frac{4}{5}\right)+\frac{2}{3}$.

9. $6\frac{3}{4}$ of $2\frac{3}{11}-(6\frac{3}{4}-2\frac{2}{11})$; $2\frac{7}{8}$ of $\frac{11}{75}$ of $3\frac{3}{4}-\frac{11}{12}+\frac{15}{16}$.

10. Of the fractions $\frac{3}{16}$, $\frac{5}{24}$, $\frac{7}{36}$, $\frac{11}{54}$ find how much the sum of the greatest and least exceeds the difference of the other two.

11. Find the least fraction which added to the sum of $\frac{7}{8}$, $\frac{9}{10}$ and $\frac{29}{12}$ will make the result an integer.

12. From the sum of $25\frac{7}{8}$ and $16\frac{9}{25}$ take the difference between $18\frac{7}{12}$ and $5\frac{11}{12}$.

13. Subtract $\frac{2}{3}$ of $\frac{5}{17}$ of $6\frac{3}{8}$ from $\frac{7}{8}$ of $5\frac{2}{3}$; and $\frac{2}{3}$ of $\frac{5}{11}$ of 1089 from $\frac{4}{7}$ of $\frac{2}{13}$ of $4140\frac{1}{2}$.

14. Add together $\frac{5}{12}$, $1\frac{2}{3}$, $\frac{3}{16}$ and $\frac{14}{13}$ and subtract the result from $4\frac{1}{2}$.

15. By how much does $3\frac{1}{7}+9\frac{2}{3}-5\frac{3}{4}$ fall short of $8\frac{7}{16}$ of $2\frac{7}{15}$?

16. Find the sum of the greatest and least of the numbers $\frac{3}{8}$, $\frac{5}{12}$, $\frac{7}{20}$, the sum of the other two, and the difference of these sums.

MULTIPLICATION.

126. MULTIPLICATION is the operation by which we do to one given number called the Multiplicand, what we do to unity to obtain another given number called the Multiplier. The result is called their *Product*.

Since 5 is 1 repeated 5 times, to multiply a number by 5 is to repeat that number five times: hence the above definition includes the multiplication of integers.

127. *To multiply any number by a fraction.*

For example, multiply $\frac{3}{4}$ by $\frac{5}{7}$. Now $\frac{5}{7}$ is obtained by dividing 1 into 7 equal parts and taking 5 of those parts; hence, to multiply a number by $\frac{5}{7}$ (126), we must divide the number into 7 equal parts and take 5 of those parts; that is, we must first divide by 7, and then multiply by 5. But $\frac{3}{4}$ divided by 7 gives $\frac{3}{4 \times 7}$ (111), and this result multiplied by 5 gives $\frac{3 \times 5}{4 \times 7}$ (110); that is

$$\frac{3}{4} \times \frac{5}{7} = \frac{3 \times 5}{4 \times 7};$$

hence we have this Rule,—*Multiply the numerators for the numerator of the Product, and the denominators for its denominator.*

REMARK 1. If the Multiplicand or Multiplier be an integer, we may consider it as a fraction with denominator 1 (101).

REMARK 2. If Multiplicand or Multiplier, or both, be mixed numbers, we may reduce them to improper fractions, and apply the preceding Rule, thus

$$4\tfrac{3}{8} \times 3\tfrac{7}{15} = \frac{35}{8} \times \frac{52}{15} = \frac{35 \times 52}{8 \times 15}.$$

REMARK 3. Before obtaining the final result, we may cancel any factor common to numerator and denominator, thus

$$4\tfrac{3}{8} \times 3\tfrac{7}{15} = \frac{\overset{7}{\cancel{35}}}{\underset{2}{\cancel{8}}} \times \frac{\overset{13}{\cancel{52}}}{\underset{3}{\cancel{15}}} = \frac{91}{6} = 15\tfrac{1}{6}.$$

128. Comparing the process of the last Art. with that of Art. 112, it is seen that to take a fractional part of any number is equivalent to multiplying that number by the fraction: thus, $\frac{5}{7}$ of $\frac{3}{4}$ is equivalent to $\frac{3}{4} \times \frac{5}{7}$.

Again, if we multiply $\frac{3}{4}$ by 5, and then divide by 7, we shall obtain the same result as in first dividing by 7 and then multiplying by 5; and, as the same remark holds good whatever fraction we take for Multiplier, we may say that

To multiply by a fraction, *we may multiply by the numerator and then divide by the denominator*, or *we may divide by the de-*

nominator and then multiply by the numerator; the *order* of the operations being indifferent to the result.

129. *To find the continued product of three or more fractions.*

For example, find the continued product of $\frac{3}{4}$, $\frac{5}{7}$ and $\frac{8}{15}$. Now

$$\frac{3}{4} \times \frac{5}{7} = \frac{3 \times 5}{4 \times 7} \qquad (127)$$

therefore

$$\frac{3}{4} \times \frac{5}{7} \times \frac{8}{15} = \frac{3 \times 5}{4 \times 7} \times \frac{8}{15}$$

$$= \frac{3 \times 5 \times 8}{4 \times 7 \times 15};$$

hence we have this Rule,—*Multiply all the numerators together for the numerator of the continued Product, and all the denominators for its denominator:* cancelling all the factors common to numerator and denominator before obtaining the final result.

130. *The Product of two or more fractions remains the same, however we change the order of the factors.*

For the product has for its numerator the product of the numerators of the factors, and for its denominator the product of the denominators, and the order of the factors in this new numerator and new denominator may be changed in any way we please (61).

COR. This proposition carries with it the following:

To multiply by a number which is the product of two or more fractions we may multiply by each of those fractions in succession; thus

$$\frac{3}{4} \times \left(\frac{5}{7} \times \frac{8}{15}\right) = \frac{3}{4} \times \frac{5}{7} \times \frac{8}{15}.$$

131. (1) *If the Multiplicand be the sum of two or more fractions, we may multiply each separately by the Multiplier, and take their sum.*

For

$$\left(\frac{2}{3} + \frac{4}{5}\right) \times \frac{6}{7} = \frac{10+12}{15} \times \frac{6}{7} = \frac{(10+12) \times 6}{15 \times 7} \qquad (127)$$

$$= \frac{10 \times 6 + 12 \times 6}{15 \times 7} \qquad (62)$$

$$= \frac{10 \times 6}{15 \times 7} + \frac{12 \times 6}{15 \times 7} \qquad (120)$$

$$= \frac{10}{15} \times \frac{6}{7} + \frac{12}{15} \times \frac{6}{7} \qquad (127)$$

$$= \frac{2}{3} \times \frac{6}{7} + \frac{4}{5} \times \frac{6}{7}.$$

(2) In like manner we may shew that,—*If the Multiplicand be the difference of two fractions, we may multiply each separately by the Multiplier, and subtract:* thus

$$\left(\tfrac{4}{5}-\tfrac{2}{3}\right)\times\tfrac{6}{7}=\tfrac{4}{5}\times\tfrac{6}{7}-\tfrac{2}{3}\times\tfrac{6}{7}.$$

(3) And, as in Art. 63, 2, we may shew that the same principle applies, if the Multiplicand is made up of any number of additions and subtractions of fractions; thus

$$\left(\tfrac{4}{5}-\tfrac{2}{3}+\tfrac{8}{9}-\tfrac{3}{4}\right)\times\tfrac{6}{7}=\tfrac{4}{5}\times\tfrac{6}{7}-\tfrac{2}{3}\times\tfrac{6}{7}+\tfrac{8}{9}\times\tfrac{6}{7}-\tfrac{3}{4}\times\tfrac{6}{7}.$$

REMARK. The above Propositions equally apply whether the numbers be fractional, or whole, or mixed; for whole numbers and mixed numbers can always be expressed as improper fractions: thus

$$359\tfrac{4}{5}\times\tfrac{6}{7}=\left(359+\tfrac{4}{5}\right)\times\tfrac{6}{7}=359\times\tfrac{6}{7}+\tfrac{4}{5}\times\tfrac{6}{7},$$

and $$359\tfrac{4}{5}\times\tfrac{6}{7}=\left(360-\tfrac{1}{5}\right)\times\tfrac{6}{7}=360\times\tfrac{6}{7}-\tfrac{1}{5}\times\tfrac{6}{7}.$$

Ex. 1. Multiply $3\tfrac{24}{25}$ by $2\tfrac{37}{54}$; and $5\tfrac{4}{9}$ by $2\tfrac{3}{7}$ of $\tfrac{15}{17}$.

(1) $$3\tfrac{24}{25}\times2\tfrac{37}{54}=\frac{\overset{11}{\cancel{99}}}{\underset{5}{\cancel{25}}}\times\frac{\overset{29}{\cancel{145}}}{\underset{6}{\cancel{54}}}=\frac{319}{30}=10\tfrac{19}{30}.$$

(2) $$5\tfrac{4}{9}\times2\tfrac{3}{7}\text{ of }\tfrac{15}{17}=\tfrac{49}{9}\times\left(\tfrac{17}{7}\times\tfrac{15}{17}\right)=\frac{\overset{7}{\cancel{49}}}{\underset{3}{\cancel{9}}}\times\frac{\cancel{17}}{\cancel{7}}\times\frac{\overset{5}{\cancel{15}}}{\cancel{17}}\quad\text{(130 Cor.)}$$
$$=\frac{35}{3}$$
$$=11\tfrac{2}{3}.$$

Ex. 2. Multiply $159\tfrac{3}{8}$ by 12; and $1727\tfrac{2}{7}$ by 34.

(1) $$\begin{array}{r} 159\tfrac{3}{8} \\ 12 \\ \hline 1912\tfrac{1}{2} \end{array}$$

(2) $$\begin{array}{r} 1727\tfrac{2}{7} \\ 34 \\ \hline 6917\tfrac{5}{7} \\ 5181 \\ \hline 58727\tfrac{5}{7} \end{array}$$

$$34\times\tfrac{2}{7}=\tfrac{68}{7}=9\tfrac{5}{7}.$$

In (1) multiplying separately the fractional and the integral parts (131) we say 12 times $\tfrac{3}{8}$ is $\tfrac{36}{8}$ or $4\tfrac{4}{8}$ or $4\tfrac{1}{2}$, 12 times 9 is 108 and 4 is 112, &c.

In (2) we say 34 times $\tfrac{2}{7}$ is $\tfrac{68}{7}$ or $9\tfrac{5}{7}$; then multiplying by 4 we add in $9\tfrac{5}{7}$.

Ex. 3. $74\frac{26}{27}\times 43=\left(75-\frac{1}{27}\right)\times 43=75\cdot 43-\frac{43}{27}$ (131 Rk.)

$$=3225-1\frac{16}{27}$$

$$=3223\frac{11}{27}.$$

Ex. 4. $7\frac{13}{14}\times 5\frac{4}{9}-\left(8\frac{2}{3}-6\frac{3}{4}\right)\times 2\frac{2}{5}+3\frac{4}{5}\text{ of }1\frac{1}{6}$

$$=\frac{\overset{37}{\cancel{111}}}{\underset{2}{\cancel{14}}}\times\frac{\overset{7}{\cancel{49}}}{\underset{3}{\cancel{9}}}-\left(\frac{8}{3}-\frac{3}{4}\right)\times\frac{12}{5}+\frac{19}{5}\times\frac{7}{6}$$

$$=\frac{259}{6}-\frac{23}{\cancel{12}}\times\frac{\cancel{12}}{5}+\frac{133}{30}$$

$$=43\frac{1}{6}-4\frac{3}{5}+4\frac{13}{30}$$

$$=43+\frac{5}{30}-\frac{18}{30}+\frac{13}{30}$$

$$=43.$$

EXERCISE 17.

Multiply

1. $\frac{11}{12}$ by $\frac{15}{18}$; $\frac{35}{72}$ by $\frac{27}{49}$; $\frac{36}{65}$ by $\frac{52}{99}$; $6\frac{3}{5}$ by $2\frac{1}{2}$; $8\frac{5}{11}$ by $6\frac{4}{7}$; $15\frac{5}{8}$ by $3\frac{3}{5}$.

2. $17\frac{5}{12}$ by $25\frac{1}{7}$; $54\frac{1}{11}$ by $23\frac{5}{8}$; $4\frac{4}{5}$ of $2\frac{1}{8}$ by $\frac{5}{12}$; $19\frac{4}{7}$ by $\frac{3}{5}$ of $\frac{10}{23}$.

3. $\frac{8}{9}$ of $3\frac{3}{17}$ by $\frac{7}{12}$ of $15\frac{5}{7}$; $5\frac{1}{4}$ of $6\frac{2}{3}$ by $3\frac{4}{21}$ of $10\frac{1}{3}$; $3\frac{1}{8}$ by $(3\frac{1}{4}-\frac{2}{3})$ of $\frac{6}{7}$.

4. $\frac{2}{3}+\frac{3}{4}+\frac{4}{5}$ by $\frac{5}{6}+\frac{7}{8}-\frac{8}{9}$; $\frac{5}{6}\cdot\frac{5}{6}-\frac{5}{6}\cdot\frac{3}{4}+\frac{3}{4}\cdot\frac{3}{4}$ by $\frac{5}{6}+\frac{3}{4}$.

*5. $\frac{3}{4}+\frac{7}{6}$ of $5\frac{1}{2}$ by $\frac{5}{6}+\frac{2}{3}+3\frac{3}{8}$; $3-\frac{1}{5}+\frac{1}{42}-\frac{1}{3570}$ by $2+\frac{1}{2}+\frac{1}{3}$.

Find the continued product of

6. $\frac{5}{9}$, $1\frac{9}{11}$, $\frac{6}{11}$, $5\frac{4}{5}$; $6\frac{1}{4}$, $3\frac{4}{5}$ of $1\frac{1}{6}$, $7\frac{4}{5}$, $\frac{17}{57}$; $\frac{428}{515}$, $\frac{5253}{1819}$, $\frac{615}{492}$.

7. $\frac{5687}{319}$, $\frac{667}{22011}$, $\frac{221}{629}$, $\frac{72816}{8528}$; $12\frac{1}{4}$, $8\frac{1}{3}$, $\frac{51}{143}$, $6\frac{1}{2}$ of $\frac{14}{17}$ of $2\frac{1}{7}$.

Find the value of

8. $10\frac{4}{11}\times 11$; $36\frac{5}{6}\times 12$; $324\frac{1}{6}\times 6$; $1625\frac{7}{11}\times 23$; $3589\frac{11}{47}\times 47$.

9. $(3\frac{4}{5}+2\frac{1}{4})\times 10\frac{2}{3}$; $3\frac{4}{5}+2\frac{1}{4}\times 10\frac{2}{3}$; $3\frac{4}{5}\times 5\frac{1}{4}\times\frac{7}{9}-\frac{1}{5}$ of $2\frac{1}{3}$.

10. $(19\frac{3}{5}-3\frac{3}{4})\times(3\frac{1}{2}-2\frac{1}{3})$; $19\frac{3}{5}-3\frac{3}{4}\times 3\frac{1}{2}-2\frac{1}{3}$; $19\frac{3}{5}-3\frac{3}{4}\times(3\frac{1}{2}-2\frac{1}{3})$.

Find the value of

11. 6[illegible] $\times 5$[illegible] $- 4$[illegible] $\times 2$[illegible] $+ 1$[illegible]; 6[illegible] $\times (5$[illegible] $- 4$[illegible]$) \times 2$[illegible] $+$ [illegible].

12. Multiply 99[illegible] by 324; 999[illegible] by 999; and to 479 add 1[illegible] and repeat the addition 6 times. See Ex. 3, p. 79.

13. Multiply 49[illegible] by 50[illegible] and add [illegible] to the result.

14. Multiply the sum of $1\frac{1}{4}$ and [illegible] of $\frac{1}{3}$ by 1[illegible] of the difference of [illegible] and $\frac{1}{8}$.

15. To the sum of 3[illegible] and 4[illegible] add the difference between 4[illegible] and 5[illegible] and multiply the result by 11[illegible].

16. From 11[illegible] take the sum of $2\frac{1}{3}$, 3[illegible] and 4[illegible], and multiply the difference by $2\frac{1}{3}$ of [illegible] of $6\frac{1}{3}$.

DIVISION.

132. DIVISION is the operation by which, having given the product of two numbers and one of the numbers, we find the other number.

The first of these numbers is called the *Dividend*, the second the *Divisor*, and the number to be found the *Quotient*.

When the division of one whole number by another is *exact*, we have seen (47) that the above definition is applicable to whole numbers; and when the Division is *not exact*, as in dividing 29 by 8, we do not actually divide 29 by 8, but a number under 29 divisible by 8, namely 24, and the remainder is not operated on at all. Hence we may say generally, that the above definition is applicable to the Division of whole numbers.

133. *To find the complete quotient in dividing one whole number by another.*

When 29 is divided by 8 the quotient must be greater than 3 and less than 4; that is, there is some number greater than 3 and less than 4 which multiplied by 8 gives 29; and this number we call the *complete* quotient of 29 by 8.

Now $\frac{29}{8} \times 8 = 29$, therefore the quotient of 29 by 8 is $\frac{29}{8}$, or

$$29 \div 8 = \frac{29}{8}$$
$$= 3\frac{5}{8}; \qquad (108)$$

hence (108) we have this Rule,—*Divide in the usual way, and to the integral quotient add the fraction whose numerator is the remainder, and denominator the divisor.*

Cor. Since $29 \div 8 = \frac{29}{8}$ we shall use either notation indifferently.

Ex. 5. Divide 868 by 37.

$$\begin{array}{l} 37\,)\,868\,(\,23 \\ \quad\ 128 \\ \quad\ \ \ 17 \end{array} \qquad \text{therefore } 868 \div 37 = 23\tfrac{17}{37}.$$

134. *To divide any number by a fraction.*

For example divide $\frac{3}{4}$ by $\frac{5}{7}$. Here we have to find a quotient which when multiplied by $\frac{5}{7}$ shall give the product $\frac{3}{4}$; hence (128)

$$\tfrac{5}{7} \text{ of this quotient is equal to } \tfrac{3}{4};$$

$$\text{therefore } \tfrac{1}{7} \ \ldots\ldots\ldots\ldots\ldots\ldots \ \tfrac{3}{4} \div 5 \text{ or to } \tfrac{3}{4\times 5},$$

$$\text{and therefore} \quad \text{this quotient} \ldots\ldots\ldots\ldots \ \tfrac{3}{4\times 5} \times 7 \text{ or to } \tfrac{3\times 7}{4\times 5};$$

$$\text{but } \tfrac{3\times 7}{4\times 5} = \tfrac{3}{4} \times \tfrac{7}{5}; \qquad (127)$$

$$\text{therefore } \tfrac{3}{4} \div \tfrac{5}{7} = \tfrac{3}{4} \times \tfrac{7}{5}.$$

Or we may proceed thus:

$$\text{quotient } \times \tfrac{5}{7} = \tfrac{3}{4} \text{ by definition};$$

multiply each term of this equality by $\frac{7}{5}$,

$$\text{therefore} \qquad \text{quotient} \times \tfrac{5}{7} \times \tfrac{7}{5} = \tfrac{3}{4} \times \tfrac{7}{5},$$

$$\textit{or} \qquad \tfrac{5}{7} \times \tfrac{7}{5} \times \textit{quotient} = \tfrac{3}{4} \times \tfrac{7}{5}, \qquad (130)$$

$$\text{that is} \qquad \text{quotient} = \tfrac{3}{4} \times \tfrac{7}{5};$$

the same result as before; that is, to divide a number by $\frac{5}{7}$ we multiply the number by $\frac{7}{5}$; but $\frac{7}{5}$ is the *reciprocal* of $\frac{5}{7}$; hence to divide a number by a fraction we have this Rule :—

Multiply the number by the reciprocal of the Divisor.

REMARK 1. If the Dividend be a whole number, or if Dividend or Divisor or both be mixed numbers, we reduce them to improper fractions, and then apply the Rule.

REMARK 2. Before obtaining the final result, we may cancel any factors common to numerator and denominator; thus

$$1\tfrac{1}{14}\div 5\tfrac{5}{7}=\tfrac{15}{14}\div\tfrac{40}{7}=\frac{\overset{3}{\cancel{15}}}{\underset{2}{\cancel{14}}}\times\frac{\cancel{7}}{\underset{8}{\cancel{40}}}=\tfrac{3}{16}.$$

COR. To divide by $\frac{5}{7}$ is to multiply by $\frac{7}{5}$; hence, to divide a number by a fraction,—*We multiply by the denominator and then divide by the numerator, or we divide by the numerator and then multiply by the denominator* (128).

Ex. 1. Divide $3\frac{3}{4}$ by $1\frac{17}{18}$; and $7\frac{7}{8}$ by $3\frac{3}{14}$ of $2\frac{1}{10}$.

(1) $3\frac{3}{4}\div 1\frac{17}{18}=\frac{15}{4}\div\frac{35}{18}=\dfrac{\overset{3}{\cancel{15}}}{\underset{2}{\cancel{4}}}\times\dfrac{\overset{9}{\cancel{18}}}{\underset{7}{\cancel{35}}}=\frac{27}{14}=1\frac{13}{14}.$

(2) $7\frac{7}{8}\div 3\frac{3}{14}$ of $2\frac{1}{10}=\frac{63}{8}\div\dfrac{\overset{9}{\cancel{45}}}{\underset{2}{\cancel{14}}}$ of $\dfrac{\overset{3}{\cancel{21}}}{\underset{2}{\cancel{10}}}=\frac{63}{8}\div\frac{27}{4}=\dfrac{\overset{7}{\cancel{63}}}{\underset{2}{\cancel{8}}}\times\dfrac{\cancel{4}}{\underset{3}{\cancel{27}}}=1\frac{1}{6}.$

Ex. 2. Divide $4164\frac{7}{8}$ by 11, and by 132.

$$\begin{array}{r}11\,)\,4164\tfrac{7}{8}\\ 12\,)\;\;378\tfrac{5}{8}\\ \hline 31\tfrac{53}{96}\end{array}$$

Dividing by 11 the integral remainder is 6 and the full remainder $6\frac{7}{8}$ or $\frac{55}{8}$: but $\frac{55}{8}\div 11$ is $\frac{5}{8}$.

Again, dividing by 12 the full remainder is $6\frac{5}{8}$ or $\frac{53}{8}$, and $\frac{53}{8}\div 12$ is $\frac{53}{96}$.

Ex. 3. $10\frac{2}{3}\times 3\frac{3}{4}\div 4\frac{4}{5}-(2\frac{2}{3}+3\frac{3}{4})\div 4\frac{2}{5}+7\frac{1}{8}\div\ 3\frac{3}{4}+4\frac{4}{5})$

$$=\tfrac{32}{3}\times\tfrac{15}{4}\div\tfrac{24}{5}-(\tfrac{8}{3}+\tfrac{15}{4})\div\tfrac{22}{5}+\tfrac{57}{8}\div(\tfrac{15}{4}+\tfrac{24}{5})$$

$$=\frac{\overset{4}{\cancel{32}}}{\cancel{3}}\times\frac{\overset{5}{\cancel{15}}}{\cancel{4}}\times\frac{5}{\underset{3}{\cancel{24}}}-\frac{\overset{7}{\cancel{77}}}{12}\times\frac{5}{\underset{2}{\cancel{22}}}+\frac{\cancel{57}}{\underset{2}{\cancel{8}}}\times\frac{\overset{5}{\cancel{20}}}{\underset{3}{\cancel{171}}}$$

$$=\tfrac{25}{3}-\tfrac{35}{24}+\tfrac{5}{6}$$

$$=\frac{200-35+20}{24}=\frac{185}{24}$$

$$=7\tfrac{17}{24}.$$

EXERCISE 18.

1. Find the complete quotient in dividing 567 by 13; 8768 by 45; $845\frac{11}{12}$ by 12; $6739\frac{13}{18}$ by 37 and by 73.

Divide :

2. 143 by $\frac{11}{12}$; 35 by $3\frac{1}{3}$; $\frac{11}{12}$ by $\frac{47}{56}$; $24\frac{3}{8}$ by $5\frac{5}{12}$; $\frac{108}{135}$ by $\frac{130}{441}$.

3. $7\frac{8}{11}$ by $5\frac{5}{6}$; $\frac{2}{3}$ of $\frac{9}{14}$ by $3\frac{1}{7}$; $2\frac{7}{8}$ by $\frac{3}{5}$ of $4\frac{1}{3}$; $\frac{4}{5}$ of $4\frac{2}{7}$ by $7\frac{6}{11}$.

4. $3\frac{11}{13}$ by $2\frac{1}{3}$ of $3\frac{1}{2}$; $3\frac{2}{3}$ of $2\frac{3}{16}$ by $5\frac{5}{8}$ of $8\frac{2}{7}$; $3\frac{1}{2}$ of $\frac{4}{5}$ by $2\frac{1}{10}$ of $\frac{4}{13}$.

5. $31\frac{1}{11} \times 3\frac{2}{7}$ by $\frac{47}{315} \times 9$; $\frac{41}{162} - \frac{9}{49} - \frac{3}{54}$ by $\frac{4}{9} + \frac{1}{2} - \frac{13}{14}$.

6. $3\frac{1}{4} - \frac{5}{6}$ of $\frac{4}{15}$ by $21\frac{1}{3} + \frac{3}{10} + 4\frac{1}{3}$ of $\frac{2}{5}$; $\frac{4}{5} - \frac{16}{25} + \frac{24}{125}$ by $\frac{2}{3} - \frac{2}{9} + \frac{52}{81}$.

7. $\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \frac{1}{10}$ by $\frac{1}{3} + \frac{1}{5} + \frac{1}{9} + \frac{1}{15}$; $2\frac{7}{15}$ of $2\frac{4}{9}$ by $2\frac{2}{5} - 2\frac{1}{4}$.

8. $2 + \frac{1}{3} + \frac{1}{8} + \frac{1}{12} + \frac{1}{15} + \frac{1}{18}$ by $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}$.

Find the value of

9. $3245\frac{4}{7} \div \frac{7}{9}$; $\left(\frac{2}{3} + \frac{3}{4} - \frac{5}{6}\right) \div 4\frac{2}{3}$; $\left(\frac{5}{7} \times \frac{2}{9} \times 13\frac{1}{2}\right) \div \left(\frac{1}{9} \times \frac{3}{7} + 40\right)$.

10. $6\frac{7}{8} + (1\frac{3}{8} + 3\frac{7}{15}) \div 6\frac{3}{7}$; $6\frac{7}{8} + 1\frac{3}{8} + 3\frac{7}{15} \div 6\frac{3}{7}$; $\frac{3}{7} \times 1\frac{2}{3} \times 12\frac{1}{2} \div 6\frac{2}{3}$.

11. $\left(\frac{2}{19} + \frac{1}{3}\right) \div \left(3 - \frac{1}{3}\right) \times \left(\frac{1}{3} + \frac{1}{5}\right)$; $\left(\frac{1}{7} + \frac{3}{4} \text{ of } \frac{136}{153}\right) \div \left(\frac{4}{11} \text{ of } 2\frac{5}{12} - \frac{14}{63}\right)$.

12. $\frac{2247}{1017} \div \frac{301}{339} \times \frac{774}{615} \div \frac{1926}{565}$; $3\frac{3}{4} \div 4\frac{2}{3} - 3\frac{1}{2} \div 4\frac{1}{4} + \frac{4}{7} \div 2\frac{1}{3}$.

13. Multiply the sum of $\frac{1}{2}$, $1\frac{3}{4}$ and $\frac{5}{6}$ by the difference of $\frac{4}{15}$ and $\frac{3}{20}$, and divide the product by $\frac{11}{18}$ of $1\frac{14}{15}$.

14. Of the fractions $\frac{1}{21}$ of $2\frac{1}{3}$, $\frac{1}{24}$ of $3\frac{1}{3}$, $\frac{1}{28}$ of $4\frac{8}{11}$, divide the sum of the greatest and least by the intermediate one.

15. Add $\frac{2}{3}$ of $\frac{3}{7}$ to $\frac{3}{7}$ of $2\frac{1}{3}$ and multiply the result by $\left(\frac{2}{3} \text{ of } \frac{5}{6}\right) \div \left(\frac{5}{4} + \frac{4}{5}\right)$.

16. Divide the sum of $2\frac{1}{3}$, $3\frac{2}{3}$ and $5\frac{1}{8}$ by the sum of $4\frac{1}{3}$ and $8\frac{2}{9}$, and to the quotient add the difference of $10\frac{1}{16}$ and $5\frac{7}{8}$.

17. Divide the sum of $\frac{5}{6}$ and $\frac{8}{9}$ by $7\frac{2}{3}$ and subtract the quotient from $3\frac{1}{4}$ of $6\frac{1}{3}$.

18. To the sum of $2\frac{1}{2}$ and $3\frac{2}{3}$ add the difference between $4\frac{3}{4}$ and $5\frac{4}{5}$, and multiply the result by the quotient of $7\frac{2}{7}$ by $6\frac{1}{4}$.

GREATEST COMMON MEASURE, &c.

135. We have said (67) that

The G.C.M. of two or more numbers is the greatest number that divides each of them exactly; and

The L.C.M. of two or more numbers is the least number that can be divided by each of them exactly;

and these definitions will be applicable when the given numbers are fractions, provided that we understand by *exactly*, that the complete quotients must be integers.

Take any fraction in its lowest terms, as $\frac{8}{9}$, and suppose it to be *multiplied* by another fraction so that the product is an integer; then, since 9 is prime to 8, the numerator of this second fraction must be divisible by 9, and must therefore be 9 itself or a multiple of 9, and the denominator must be 1 or a measure of 8 including 8 itself; thus

$$\frac{8}{9}\times\frac{9}{1};\ \frac{8}{9}\times\frac{9}{2};\ \frac{8}{9}\times\frac{9}{4};\ \frac{8}{9}\times\frac{9}{8}$$

all give integral results. If therefore $\frac{8}{9}$ is to be *divided* by a fraction and the quotient is to be an integer, its denominator must be 9 or a multiple of 9, and its numerator 1 or a measure of 8.

Let us now take any number of fractions in their lowest terms, then any fraction by which we can divide each of them so that all the quotients shall be integers, must be one whose denominator is a common multiple of their denominators and whose numerator is a common measure of their numerators; and of all such fractions the greatest is the one that has the least denominator and greatest numerator: hence

The G.C.M. *of two or more fractions is a fraction whose denominator is the* L.C.M. *of their denominators, and whose numerator is the* G.C.M. *of their numerators.*

And in like manner,

The L.C.M. *of two or more fractions is a fraction whose numerator is the* L.C.M. *of their numerators, and whose denominator is the* G.C.M. *of their denominators.*

Ex. Find the G.C.M. and L.C.M. of $\frac{8}{15}$, $3\frac{1}{5}$ and $\frac{12}{25}$.

These fractions are equal to $\frac{8}{15}$, $\frac{16}{5}$ and $\frac{12}{25}$, and L.C.M. of their denominators is 75 and G.C.M. of their numerators is 4;

$$\therefore \text{ their G.C.M.} = \frac{4}{75}.$$

Again, L.C.M. of their numerators is 48, and G.C.M. of their denominators is 5;

$$\therefore \text{ their L.C.M.} = \frac{48}{5} = 9\frac{3}{5}.$$

EXERCISE 19.

Find the G.C.M. and L.C.M. of

(1) $\frac{2}{3}$, $\frac{4}{5}$, $\frac{6}{7}$, $\frac{8}{9}$. (2) $\frac{2}{3}$, $\frac{5}{6}$, $\frac{8}{9}$, $\frac{11}{12}$.

(3) $\frac{3}{4}$, $\frac{1}{6}$, $\frac{5}{8}$, $\frac{1}{7}$. (4) $1\frac{5}{28}$, $2\frac{2}{21}$, $3\frac{1}{7}$.

(5) $\frac{15}{28}$, $3\frac{15}{16}$, $1\frac{1}{20}$, $3\frac{3}{8}$. (6) $1\frac{14}{25}$, $5\frac{17}{20}$, $14\frac{3}{10}$, $6\frac{1}{15}$.

(7) $1\frac{23}{55}$, $2\frac{2}{77}$, $3\frac{31}{33}$, $8\frac{19}{22}$. (8) $1\frac{37}{75}$, $3\frac{5}{52}$, $2\frac{54}{85}$, $7\frac{28}{38}$, $5\frac{45}{53}$.

COMPLEX FRACTIONS.

136. As yet we have treated of *simple* fractions only, the main propositions of which we have explained and illustrated. We shall now treat of complex fractions, and in doing so we shall make use of the following general definition, which has been already shewn to hold for *simple* fractions (103):

A Fraction expresses the quotient of the numerator by the denominator.

With this definition we experience no difficulty in considering fractions whose terms are themselves fractions, or mixed numbers, or the sum or difference of two fractions or mixed numbers, or any arithmetical expression whatever we have as yet become acquainted with.

137. A complex fraction is read by inserting the word *by*, for *divided by*, between the readings of numerator and denominator, thus $\dfrac{3\frac{1}{2}}{2\frac{3}{4}}$ is read $3\frac{1}{2}$ *by* $2\frac{3}{4}$.

In the *sum* of a whole number and a fraction, when the fraction is complex as well as simple (106), the sign is sometimes omitted, as in $5\frac{2\frac{1}{4}}{6}$, which means $5+\frac{2\frac{1}{4}}{6}$; and in a *product* when one of the factors is enclosed in a bracket the sign is often omitted, as in $\frac{2}{3}\left(\frac{5}{8}-\frac{4}{7}\right)$, which means $\frac{2}{3}\times\left(\frac{5}{8}-\frac{4}{7}\right)$.

138. Complex fractions can always be reduced to simple ones; thus,

$$(1)\quad \frac{3\frac{1}{2}}{2\frac{2}{3}}=\frac{\frac{7}{2}}{\frac{8}{3}}=\frac{7}{2}\div\frac{8}{3}\ (\text{Def.})=\frac{7}{2}\times\frac{3}{8}\ (134)=\frac{21}{16}.$$

$$(2)\quad \frac{4\frac{1}{2}-2\frac{3}{4}}{4\frac{1}{2}+2\frac{3}{4}}=\frac{\frac{9}{2}-\frac{11}{4}}{\frac{9}{2}+\frac{11}{4}}=\frac{\frac{18}{4}-\frac{11}{4}}{\frac{18}{4}+\frac{11}{4}}=\frac{\frac{7}{4}}{\frac{29}{4}}=\frac{7}{4}\times\frac{4}{29}=\frac{7}{29}.$$

$$(3)\quad \frac{4\frac{2}{11}}{5\frac{7}{8}+\frac{3\frac{1}{2}}{2\frac{2}{3}}}=\frac{\frac{46}{11}}{\frac{47}{8}+\frac{21}{16}}\ (\text{Ex. 1})=\frac{\frac{46}{11}}{\frac{115}{16}}=\frac{46}{11}\times\frac{16}{115}=\frac{32}{55}.$$

139. Complex fractions are subject to the same rules as simple fractions; thus,

(1) *If we multiply or divide numerator and denominator of a complex fraction by* any *number, the value of the fraction is unaltered* (109).

For take any complex fraction and reduce its numerator and denominator to simple fractions, and reduce the multiplier also to a simple fraction: suppose the numerator to be $\frac{2}{3}$, the denominator $\frac{4}{5}$, and the multiplier $\frac{6}{7}$; then, since $\frac{6}{7}\times\frac{7}{6}=1$, we have

$$\frac{\frac{2}{3}}{\frac{4}{5}}=\frac{2}{3}\times\frac{5}{4}=\frac{2}{3}\times\frac{5}{4}\times\frac{6}{7}\times\frac{7}{6}=\frac{2}{3}\times\frac{6}{7}\times\frac{5}{4}\times\frac{7}{6}\ (130)$$

$$=\frac{2\times6}{3\times7}\times\frac{5\times7}{4\times6}$$

$$=\frac{\frac{2\times6}{3\times7}}{\frac{4\times6}{5\times7}}=\frac{\frac{2}{3}\times\frac{6}{7}}{\frac{4}{5}\times\frac{6}{7}};$$

and putting now for numerator, denominator, and multiplier their original values, the truth of the proposition is established.

(2) Again,—*To multiply a complex fraction by* any *number, we may multiply the numerator or divide the denominator by that number* (110).

For, making the same reductions and suppositions as in (1), we have

$$\frac{\frac{2}{3}}{\frac{4}{5}} \times \frac{6}{7} = \frac{2}{3} \times \frac{5}{4} \times \frac{6}{7} = \frac{2}{3} \times \frac{6}{7} \times \frac{5}{4} = \frac{\frac{2}{3} \times \frac{6}{7}}{\frac{4}{5}};$$

or

$$= \frac{2}{3} \times \frac{5 \times 6}{4 \times 7} = \frac{\frac{2}{3}}{\frac{4 \times 7}{5 \times 6}} = \frac{\frac{2}{3}}{\frac{4}{5} \times \frac{7}{6}} = \frac{\frac{2}{3}}{\frac{4}{5} \div \frac{6}{7}};$$

and putting for numerator, denominator, and multiplier their original values, the truth of the proposition is established.

In precisely the same way we may shew

(3) *To divide a complex fraction by* any *number we may divide the numerator or multiply the denominator by that number* (111).

(4) *If we multiply a complex fraction by its denominator we obtain its numerator* (103 Cor. ii).

(5) *If we multiply a complex fraction by its reciprocal the product is unity.*

(6) *To multiply two or more complex fractions together we multiply their numerators for the numerator of the product, and their denominators for its denominator* (127).

(7) *To divide one complex fraction by another we multiply the first by the reciprocal of the second* (134).

Ex. 1. Simplify $\dfrac{5\frac{5}{8}}{3\frac{3}{4}}$, and $\dfrac{8\frac{5}{6}-4\frac{2}{3}}{3\frac{1}{4}+7\frac{5}{12}}$.

(1) Multiplying numerator and denominator of first fraction by 8, we have

$$\frac{5\frac{5}{8}}{3\frac{3}{4}} = \frac{45}{30} = \frac{3}{2}.$$

(2) In the second fraction, the L.C.M. of the denominators of the fractional parts is 12, and multiplying numerator and denominator by 12, we have

$$\frac{8\frac{5}{6}-4\frac{2}{3}}{3\frac{3}{4}+7\frac{5}{12}}=\frac{106-56}{45+89}=\frac{50}{134}=\frac{25}{67}.$$

Ex. 2. $$5+\cfrac{1}{3+\cfrac{1}{5+\cfrac{1}{7}}}=5+\cfrac{1}{3+\cfrac{7}{35+1}}=5+\cfrac{1}{3+\cfrac{7}{36}}=5+\frac{36}{108+7}=5\tfrac{36}{115}.$$

Ex. 3. $$\frac{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}}{\frac{1}{2\frac{1}{2}}+\frac{1}{3\frac{1}{2}}+\frac{1}{4\frac{1}{2}}}=\frac{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}}{\frac{2}{5}+\frac{2}{7}+\frac{2}{9}}=\frac{\frac{6}{12}+\frac{4}{12}+\frac{3}{12}}{\frac{126}{315}+\frac{90}{315}+\frac{70}{315}}$$

$$=\frac{\frac{13}{12}}{\frac{286}{315}}=\frac{\cancel{13}}{\underset{4}{\cancel{12}}}\times\frac{\overset{105}{\cancel{315}}}{\underset{22}{\cancel{286}}}$$

$$=\frac{105}{88}=1\tfrac{17}{88}.$$

Ex. 4. $$\left\{2\tfrac{3}{4}+2\tfrac{1}{2}\text{ of }\frac{7}{3\frac{4}{5}}-\frac{1\frac{2}{3}}{2\frac{1}{2}}\right\}\div 1\tfrac{77}{228}=\left(\frac{11}{4}+\frac{5}{2}\times\frac{35}{19}-\frac{10}{15}\right)\div\frac{305}{228}$$

$$=\left(\frac{11}{4}+\frac{175}{38}-\frac{2}{3}\right)\times\frac{228}{305}$$

$$=\frac{627+1050-152}{\cancel{228}}\times\frac{\cancel{228}}{305}$$

$$=\frac{1525}{305}=5.$$

Ex. 5. From $17\frac{5}{12}$ subtract the difference of $9\frac{3}{4}$ and $4\frac{5}{7}$, and divide the remainder by the product of $\frac{5}{9}$ and $9\frac{2}{7}$.

$$\text{Quotient required}=\frac{17\frac{5}{12}-(9\frac{3}{4}-4\frac{5}{7})}{\frac{5}{9}\times 9\frac{2}{7}}=\frac{17\frac{5}{12}-9\frac{3}{4}+4\frac{5}{7}}{\frac{5}{9}\times\frac{65}{7}}\qquad(124,\ 3)$$

$$=\frac{12+\frac{35}{84}-\frac{63}{84}+\frac{60}{84}}{\frac{325}{63}}=12\tfrac{32}{84}\times\frac{63}{325}$$

$$=12\tfrac{8}{21}\times\frac{63}{325}=\frac{\overset{4}{\cancel{260}}}{\cancel{21}}\times\frac{\overset{3}{\cancel{63}}}{\underset{5}{\cancel{325}}}$$

$$=\frac{12}{5}=2\tfrac{2}{5}.$$

EXERCISE 20.

Reduce to simple fractions

1. $\dfrac{3\frac{3}{5}}{6}$; $\dfrac{18}{5\frac{1}{7}}$; $\dfrac{2\frac{2}{3}}{3\frac{5}{9}}$; $\dfrac{9\frac{1}{11}}{13\frac{3}{11}}$; $\dfrac{\frac{2}{3}\text{ of }\frac{8}{9}}{5\frac{7}{15}}$; $\dfrac{\frac{3}{4}+\frac{5}{6}}{8\frac{7}{12}}$; $\dfrac{5\frac{4}{9}}{1\frac{5}{6}+7\frac{5}{18}}$; $\dfrac{2\frac{2}{3}\text{ of }3\frac{3}{4}}{2\frac{2}{3}+3\frac{3}{4}}$.

2. $\dfrac{\frac{4}{7}+\frac{3}{11}}{1-\frac{4}{7}\text{ of }\frac{3}{11}}$; $\dfrac{\frac{120}{119}-\frac{1}{239}}{1+\frac{1}{239}\times\frac{120}{119}}$; $\dfrac{\frac{2}{3}\text{ of }\frac{4}{5}+\frac{4}{5}\text{ of }\frac{6}{7}}{\frac{4}{5}\text{ of }\frac{6}{7}-\frac{1}{3}\text{ of }\frac{2}{5}}$; $\dfrac{\frac{2}{3}+\frac{4}{5}\text{ of }\frac{5}{9}-\frac{8}{21}}{1+\frac{2}{3}\times\frac{5}{7}-\frac{5}{9}}$.

3. $\dfrac{2\frac{6}{11}}{2\frac{2}{5}}\div\dfrac{2\frac{7}{11}}{8\frac{7}{10}}$; $\dfrac{18}{\frac{8}{9}\text{ of }4\frac{2}{7}}\div 17\frac{1}{2}$; $\dfrac{7\frac{3}{4}-3\frac{11}{12}}{18\frac{1}{2}\div\frac{5}{7}}$; $\dfrac{3}{14}$ of $\dfrac{4\frac{5}{6}}{6\frac{1}{3}}$ of $\dfrac{6\frac{9}{11}}{11\frac{1}{7}}$.

4. $\dfrac{1+2\times\frac{4}{3}+\frac{4}{3}\text{ of }\frac{4}{3}}{\frac{4}{3}\times\frac{4}{3}-1}$; $\dfrac{3\frac{3}{4}\times3\frac{3}{4}-2\frac{1}{4}\times2\frac{1}{4}}{3\frac{3}{4}-2\frac{1}{4}}$; $\dfrac{1+6\frac{7}{8}\times(1+6\frac{7}{8})}{1+5\frac{4}{5}\times(1+5\frac{4}{5})}$; $\dfrac{4\frac{1}{3}\times4\frac{1}{3}\times4\frac{1}{3}-1}{4\frac{1}{3}\times4\frac{1}{3}-1}$.

5. $\dfrac{3\frac{2}{5}+12\frac{5}{8}-3\frac{7}{10}}{7\frac{4}{5}-4\frac{1}{3}\text{ of }\frac{5}{5\frac{2}{5}}}$; $\dfrac{7\frac{1}{3}-3\frac{1}{4}}{6+4\frac{2}{5}}\div\dfrac{5\frac{1}{3}+1\frac{1}{7}}{6\frac{1}{7}-2\frac{1}{3}}$; $\dfrac{11\frac{3}{4}-10\frac{1}{2}}{11\frac{3}{4}+10\frac{1}{2}}\div\dfrac{10\frac{3}{8}+11\frac{1}{8}}{10\frac{1}{8}-9\frac{7}{8}}\times\dfrac{\frac{2}{7}+\frac{3}{11}}{\frac{2}{7}-\frac{3}{11}}$.

6. $\dfrac{3}{1+\dfrac{2}{5+\frac{7}{9}}}$; $\dfrac{1}{13}$ of $\dfrac{1}{1+\dfrac{1}{3+\frac{1}{4}}}$; $2\frac{1}{3}\times\dfrac{1}{3\frac{1}{3}+\dfrac{1}{4\frac{1}{3}}}$; $\dfrac{\frac{5}{7}-\frac{1}{3}\times\left(\frac{1}{3}+\frac{1}{7}\right)}{1+\dfrac{1}{2-\frac{1}{4}}}$.

*7. $\dfrac{3+\frac{5}{3}\text{ of }\frac{21}{7\frac{1}{2}}-\frac{1}{4}-\frac{1\frac{3}{4}}{2\frac{1}{2}}}{10-\frac{151}{228}\text{ of }5}$; $\dfrac{5\frac{1}{3}+4\frac{2}{3}}{3\frac{1}{6}+2\frac{1}{6}}\times\dfrac{5\frac{1}{3}-4\frac{2}{3}}{3\frac{1}{6}-2\frac{1}{6}}\div\dfrac{28\frac{1}{3}-22\frac{8}{15}}{14\frac{35}{36}-8\frac{17}{24}}$.

8. $\dfrac{\frac{3}{11}+\frac{1}{21}+\frac{8}{77}-\frac{1}{5}}{\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}}$ of $\dfrac{\frac{4}{1\frac{1}{4}}\text{ of }\frac{3}{1\frac{1}{8}}-\frac{4}{3\frac{1}{4}}\text{ of }\frac{2\frac{1}{2}}{1\frac{1}{3}}}{\frac{3}{209}\text{ of }\frac{11}{7}\text{ of }3\frac{1}{3}+2\frac{1}{18}\text{ of }1\frac{1}{4}\text{ of }\frac{1}{19}}$.

Find the value of

9. $\dfrac{\frac{1}{9}\text{ of }1\frac{1}{4}\text{ of }4\frac{1}{2}}{\frac{5}{56}\text{ of }1\frac{1}{8}\text{ of }3\frac{1}{2}}-\dfrac{3\frac{1}{2}+4\frac{1}{3}}{6\frac{1}{3}+1\frac{1}{12}}$; $\dfrac{14\frac{1}{4}-6\frac{1}{2}}{3\frac{1}{3}+6\frac{11}{12}}\div\dfrac{4\frac{1}{12}+6\frac{1}{3}}{9\frac{1}{4}-3\frac{3}{4}}+(30\frac{7}{12}-22\frac{2}{3})$.

10. $\dfrac{\frac{3}{5}}{\frac{8}{9}}+\dfrac{\frac{5}{8}}{2\frac{1}{2}\times 1\frac{1}{3}}\times\dfrac{1}{80}$; $\quad\dfrac{1}{3\frac{1}{4}}-\dfrac{2\frac{1}{4}}{9}+\dfrac{3\frac{1}{6}}{2}-\dfrac{\frac{4}{7}}{4\frac{2}{7}}$; $\quad\dfrac{2\frac{1}{2}}{3\frac{1}{4}}+\dfrac{1\frac{1}{2}-\frac{5}{6}}{1\frac{1}{4}+\frac{5}{6}}-1\frac{1}{3}$.

11. $\dfrac{3}{5}$ of $\dfrac{13}{16}-\dfrac{1\frac{1}{8}}{6\frac{1}{4}}$ of $\dfrac{19}{20}+\dfrac{3}{7}$ of $\dfrac{6\frac{3}{11}}{3\frac{1}{3}}$; $\quad\dfrac{2}{3}$ of $\left(\dfrac{2}{3}\text{ of }\dfrac{2\frac{1}{4}+1\frac{1}{2}}{3\frac{1}{2}-1}+\dfrac{1}{2}\right)$.

12. $(3\frac{7}{11}\div 4\frac{4}{7})$ of $(10\frac{4}{7}\div 7\frac{1}{2})$ of $\dfrac{77}{540}$; $\quad 3\frac{7}{11}\div(4\frac{4}{7}\div 10\frac{4}{7})\div\left(7\frac{1}{2}\div\dfrac{77}{540}\right)$.

13. $\dfrac{5\frac{5}{8}\div 2\frac{1}{2}}{2\frac{1}{4}\text{ of }7\frac{3}{5}\div\frac{4}{5}}+2\frac{3}{11}-\dfrac{2}{3}$ of $\dfrac{13}{20}$; $\quad 2+\dfrac{1}{3+\dfrac{1}{5+\dfrac{1}{7}}}$; $\quad\dfrac{3}{8-\dfrac{7}{2-\dfrac{3}{4}}}+\dfrac{5}{6-\dfrac{5}{2-\dfrac{5}{6}}}$.

14. $2\frac{2}{3}+\dfrac{4}{5+\dfrac{6}{7+\dfrac{8}{9}}}$; $\quad 3\frac{1}{5}+\dfrac{5\frac{1}{3}}{7\frac{1}{5}+\dfrac{8\frac{7}{10}}{10\frac{1}{2}+\dfrac{13}{9\frac{3}{4}}}}$; $\quad 6\frac{2}{5}+\dfrac{3\frac{4}{5}}{\dfrac{2}{3\frac{1}{2}+\dfrac{3}{4\frac{1}{4}}}}$.

15. $\dfrac{\dfrac{1}{1-\dfrac{1}{2-\dfrac{1}{2}}}+\dfrac{1}{3}+\left(1+\dfrac{1}{2}\right)}{3\left(1+\dfrac{1}{3\frac{1}{2}}\right)-4}$; $\quad 1\frac{1}{4}\times 12\frac{1}{4}$ of $\dfrac{3}{8}-\dfrac{6\frac{1}{4}+7\frac{3}{4}}{12-6\frac{1}{4}}-\dfrac{4+\dfrac{1}{4+\dfrac{1}{4-\dfrac{1}{4}}}}{4}$.

16. Find what fraction the sum of $\frac{1}{24}$, $\frac{1}{56}$, $\frac{1}{21}$ and $\frac{1}{12}$ is of $2\frac{4}{5}$ of $1\frac{1}{8}$ of $\dfrac{2\frac{2}{3}}{3}$.

17. Of the fractions $\frac{8}{33}$, $\frac{39}{161}$ and $\frac{47}{194}$, express the difference of the first two as a fraction of the difference of the last two.

18. To the continued product of $6\frac{1}{2}$, $7\frac{2}{3}$ and $8\frac{4}{5}$, add $\dfrac{3\frac{1}{7}}{4\frac{1}{8}}$, and divide the sum by $9\frac{1}{3}$ of $10\frac{2}{3}$ of $11\frac{1}{8}$.

19. From $\dfrac{12\frac{3}{5}}{3\frac{1}{7}}$ take the sum of $\frac{2}{3}$ of $\dfrac{3\frac{1}{2}}{5\frac{1}{4}}$ and $\frac{5}{6}$ of $\dfrac{13\frac{1}{2}}{7\frac{7}{10}}$, and divide the result by $2\frac{1}{11}$.

20. Subtract $\frac{1}{3}$ of $\dfrac{3\frac{1}{2}}{\frac{1}{4}\text{ of }33\frac{3}{5}}+\frac{1}{2}$ of $\dfrac{\frac{3}{4}}{1+\frac{9}{16}}+\dfrac{\frac{3}{5}\times\frac{1}{12}}{\frac{7}{9}\text{ of }7\frac{1}{7}}$ from 101 times the sum of $\frac{3}{10}$ and $\frac{1}{2}$ of $\frac{7}{15}$ of $\frac{7}{20}$.

CHAPTER VI.

DECIMALS.

140. In the ordinary system of notation (16) a figure immediately to the right of another represents units of an order ten times less than that other: thus, if a certain figure in a number represents hundreds, the next figure to the right will represent tens, and the next simple units; and if by a natural extension of this system we agree to carry our orders beyond simple units, the next figure will represent *tenths*, the next *hundredths*, the next *thousandths*, &c. But then the order of some one figure in a number must be pointed out, from which we can derive the orders of all the others; and it has been agreed that the figure to whose right a point (·), called *decimal point*, is placed shall be the units' figure, and to distinguish it from the sign of Multiplication, it is placed towards the *top* of the figure. Thus, if we wish to represent 25 units, 3 tenths, 4 hundredths, 7 thousandths, 8 ten-thousandths, we write

25·3478.

If any of the decimal orders are wanting we supply their places by ciphers (16, 2); thus 25 units, 4 hundredths and 8 ten-thousandths is written

25·0408.

Lastly, if there be *no* units, we may suppose a cipher to occupy the units' place; thus 4 hundredths, 8 ten-thousandths is written

0·0408 or simply ·0408.

141. A number thus expressed, composed of units and *decimal* orders of unity, or of decimal orders of unity only, is called a decimal number, or simply a *decimal*. The part to the left of the point is called the *integral*, and to the right the *decimal* part of the given number.

142. *Numeration of Decimals.*

Take the decimal 25·3478. This number represents 25 and 3 tenths, 4 hundredths, 7 thousandths, and 8 ten-thousandths: but 1 of any order is equal to 10 of the next lower order (140), therefore

3 tenths and 4 hundredths is 34 hundredths,

34 hundredths and 7 thousandths is 347 thousandths,

347 thousandths and 8 ten-thousandths is 3478 ten-thousandths, and therefore 25·3478 is 25 and 3478 ten-thousandths:

In 3·141592 the last decimal figure 2 represents *millionths*, therefore as before the number is 3 and 141592 *millionths*:

In like manner ·00036 is 36 *hundred-thousandths*:

that is: — *We read off the decimal part as an integer annexing that decimal order of unity which the last figure represents.*

REMARK. In practice, however, we do not annex the decimal order, but saying (*decimal*) *point* read off the figures of the decimal *separately* in order: thus 25·3478 is read 25, point 3, 4, 7, 8; 3·141592 is read 3, point 1, 4, 1, 5, 9, 2;—·00036 is read point 0, 0, 0, 3, 6. Sometimes instead of point, we say *decimal* or *decimal point*; but this would require that in every different scale we should use a different word; thus in the scale of 12 we should have to say *duodecimal* or *duodecimal point*, and in the scale of 20 *vigesimal* or *vigesimal point.*

143. *The value of a decimal is not changed by writing ciphers to the right of the last figure.*

Thus ·307 is equal to ·30700:—for the ciphers written on, do not alter the position of the other figures relatively to the decimal point, and therefore do not alter their value: and of themselves they have no value.

COR. An integer may be expressed as a decimal by writing ciphers in the decimal part, thus 307 is equal to 307·000.

144. *To multiply a decimal by* 10, 100, 1000,... *we remove the decimal point* 1, 2, 3,... *places to the right; to divide a decimal by* 10, 100, 1000,... *we remove the decimal point* 1, 2, 3,... *places to the left.*

For in removing the decimal point one place to the right, the value of each of the figures composing the number is increased ten-fold, or the number (62) is increased ten-fold, that is, the number is multiplied by 10:—and in the same way the rest of the proposition may be proved.

Ex. 1. $25\cdot04089 \times 100 = 2504\cdot089$; $25\cdot04089 \div 100 = \cdot2504089$.

Ex. 2. $25\cdot04 \times 10000 = 250400$; $25\cdot04 \div 10000 = \cdot002504$.

145. *To convert a decimal to a decimal fraction and vice versâ.*

(1) Take the decimal $25\cdot3478$:—this number represents 25 and 3 tenths, 4 hundredths, 7 thousandths and 8 ten-thousandths; therefore

$$25\cdot3478 = 25 + \frac{3}{10} + \frac{4}{100} + \frac{7}{1000} + \frac{8}{10000}$$

$$25 + \frac{3478}{10000} = 25\frac{3478}{10000},$$

or

$$= \frac{253478}{10000},$$

where the numerator is the given number with the decimal point taken away, and the denominator represents the *decimal* order of the last decimal figure and is therefore 1 followed by as many ciphers as there are decimal figures: hence we have this Rule:—

Write down the given number suppressing the decimal point for the numerator, and for the denominator write 1 *followed by as many ciphers as there are figures in the decimal part.* To express the given decimal as a mixed number, apply the Rule to the decimal part only.

(2) Conversely. To convert a decimal fraction to a decimal.—*Write down the numerator and cut off from its right by the decimal point as many figures as there are ciphers in the denominator.* If the number of figures be less than the number of ciphers, prefix in the numerator the requisite number of ciphers.

Ex. 5. Express $206\cdot0875$ as a decimal fraction, and then as a vulgar fraction.

$$206\cdot0875 = \frac{2060875}{10000} = \frac{82435}{400} = \frac{16487}{80},$$

or

$$= 206\frac{875}{10000} = 206\frac{35}{400} = 206\frac{7}{80}.$$

Ex. 6. $\cdot00036 = \frac{36}{100000} = \frac{9}{25000}$.

Ex. 7. $\frac{678934}{10000} = 67\cdot8934$; $\frac{73}{1000} = \frac{073}{1000} = \cdot073$.

Ex. 8. $26\frac{875}{100000} = 26\frac{00875}{100000} = 26\cdot00875$.

Exercise 21.

Express as decimals

1. Three and seven tenths. Five and forty-three hundredths.
2. Four tenths seven hundredths and six thousandths.
3. Eight tenths five thousandths and three millionths.
4. Nine hundredths. Seven ten-thousandths. Five millionths.
5. Twenty-one and four tenths and four hundred-thousandths.
6. 65 and 8 hundredths 9 hundred-thousandths and 7 ten-millionths.

Read off the following decimals, annexing the decimal order of the last decimal figure (142).

7. 5·37; ·0015; ·56789; ·001405.
8. 9·87654321; 35·00000456765.
9. Multiply 8·003056 by 100, by 10000, and by 10000000.
10. Multiply ·01728 by 10, and by 1000; ·005136 by 100,000,000.
11. Divide 73·56 by 10, and by 1000; 3·7165 by 100, and by 10000.
12. Divide 57324 by 10000000; ·1 by 100; ·001 by 1000.

Express as decimal fractions, and then as vulgar fractions in their lowest terms,

13. 4·375; ·8125; ·37875; 23·04096.
14. ·0006875; 5·0096875; ·212464.

Express as mixed numbers with the fractional parts in their lowest terms,

15. 13·0675; 9·2221875; 23·006875.
16. 89·0131072; 12·08056640625.

Express the following decimal fractions as decimals:

17. $\frac{56}{100}$; $\frac{3456}{1000}$; $\frac{785}{10}$; $\frac{2}{1000}$; $\frac{48725}{10000}$; $\frac{37}{100000}$.
18. $\frac{300507}{10000000}$; $\frac{78539}{100000000}$; $\frac{20304005}{100000}$.
19. 325 millionths; 4 ten-thousandths; 79 hundred-millionths.

ADDITION AND SUBTRACTION OF DECIMALS.

146. The Rules for the Addition and Subtraction of Decimals proceed on precisely the same principles as the rules for the Addition and Subtraction of whole numbers, and may be enunciated in the same words. But in Decimals, we conveniently provide that units of the same order may be in the same vertical column (24, 1) by placing *the decimal points of the given numbers under one another;* and then the decimal point of the sum or difference will be under the other decimal points.

REMARK. In the Minuend we may if necessary *suppose* ciphers to be written to the right of the last decimal figure (143).

Ex. 1. Find the sum of 3·1415926; 2·71828; ·434294 and 144.

Ex. 2. From 35·006 subtract ·067835.

Ex. 3. Find the complement (32) of ·4771213.

(1)	3·1415926	(2)	35·006000	(3)	1·
	2·71828		·067835		·4771213
	·434294		34·938165		·5228787
	144·				
	150·2941666				

In Ex. 1 we have *supposed* the vacant decimal places in the second, third, and fourth numbers to be occupied by ciphers; in Ex. 2 we have placed ciphers in the Minuend, but in Ex. 3 we have *supposed* them to be placed there.

147. The Propositions relating to expressions made up of additions and subtractions, which have been shewn to hold for integers (56—59) and for fractions (124, 125), also hold for decimals. For the decimals in any such expressions may be replaced by their equivalent decimal fractions, these may be subjected to the required operations, and then reconverted to the original decimals.

EXERCISE 22.

Add together

1. 47·6054; 6752; ·0543; 75·572 and ·987654.
2. ·1; ·00095; 84·0563; 7·3 and 345·65432.
3. 573·45; ·008742; 0·064065; 47·83504 and 961.
4. 37·045; 6·3; ·0098; 8·6943; 617·241 and ·01.

5. Subtract 8·23456 from 37·12; ·987604 from 7·0123.

6. Subtract ·99999 from 9; 36·00051 from 45.

Find the value of

7. 3·584 + 387·6 + 5·894003 + ·00397 + 8·889.

8. 8939 + 8·939 + 89·39 + ·8939 + ·0008939 + 893·9.

9. 36·73 − 5·894; 56 − ·07359; ·001 − ·00001.

10. 5·0009 − ·089898; 0·4763 − ·387387.

11. 7·654327 − ·3793086 + 9·06996 − ·00999 + ·345.

12. 16·945 − 2·994387 − ·06735 − ·0007 + ·953 + 0·8.

13. Find the sum of 27 tenths, 345 hundredths, 17 thousandths and 4256 millionths.

14. Express as decimals 347 ten-thousandths and 347 millionths, and subtract the latter from the former.

15. Find the complement of ·7781513; ·9542425; ·000356; 97·654321; and 998·899.

16. Whether is 3·1415926535 more accurately represented by 3·1415926 or 3·1415927, and why?

MULTIPLICATION OF DECIMALS.

148. Take any two decimals and replace them by their equivalent decimal fractions; their numerators are the given numbers with their decimal points suppressed and their denominators are respectively 1 followed by as many ciphers as there are decimal places in the given numbers (141). The product of these decimal fractions is a decimal fraction whose numerator is the product of their numerators and whose denominator is 1 followed by as many ciphers as there are ciphers in the two denominators together (42): and this result becomes a decimal by cutting off from the right of the numerator as many decimal places as there are ciphers in the denominator, that is, as there are decimal places in the two given numbers (145, 2). Hence we have this Rule:

Multiply the given numbers as if they were integers, and cut off from the product as many decimal places as there are in the two given numbers together. N.B. If the number of figures in the product is less than the number of figures to be cut off, first prefix the requisite number of ciphers (145, 2).

Ex. Multiply 25·347 by 2·69; and 75 by ·00008.

(1)
```
  25·347
    2·69
  228123
 152082
 50694
 68·18343
```

(2)
```
    75
 ·00008
 ·00600 = ·006.   (143)
```

In Ex. 1, the number of decimal places in Multiplicand and Multiplier is 3 and 2 respectively, therefore the number in the product is $3+2$ or 5.

In Ex. 2, multiplying by 8 we get 600, but as there must be 5 decimal places in the Product we first prefix two ciphers, and the product becomes ·00600 or ·006.

149. The Propositions relating to Multiplication which have been shewn to hold for integers (60 – 63), and for fractions (130), also hold for decimals. See Art. 147.

DIVISION OF DECIMALS.

150. In multiplying two decimals we proceed as if they were integers, and cut off in the product as many decimal places as there are in the two numbers (148): hence, conversely, having given the product of two decimals, and one of these decimals, we find the other by dividing as if they were integers, and cutting off in the quotient as many decimal places as the number in the product exceeds the number in the divisor; or, using the terms of Division, we have this Rule:

Divide, as if Dividend and Divisor were integers, and cut off in the Quotient as many decimal places as the number in the Dividend exceeds the number in the Divisor. N.B.—If necessary, write ciphers to the right of the Dividend, so that the number of its decimal places shall be equal to the number in the Divisor, and be careful to bring down all these ciphers in the division. Also, if the number of figures in the Quotient be less than the number of places to be cut off, we must first prefix the requisite number of ciphers.

COR. If the decimal points be *equally* removed in Dividend and Divisor, the Decimal point in the Quotient will be unaltered; that is, the quotient will be unaltered. We can therefore always

arrange that the Divisor shall be a whole number, and then division of decimals proceeds on the same principles as division of whole numbers; and in *short* division of decimals we shall find it convenient to adopt this method.

Remark. The above Rule with its corollary, like the Rule in Multiplication, may be established by replacing Dividend and Divisor by their equivalent decimal fractions.

Ex. 1. Divide ·003456 by 1·1, and 26·78508 by ·072.

```
(1)     11 ) ·03456            (2)        { 8 ) 26785·08
             ·0031418                  72 { 9 ) 3348·135
                                                 372·015
```

In (1) we remove the decimal point 1 place to the right in Dividend and Divisor (150, Cor.), and in (2) 3 places to the right, so that the Divisors become 11 and 72 respectively; and then we proceed precisely as in integers.

Ex. 2. Divide 3·1415926535 by 987·543.

```
987·543 ) 3·1415926535 ( ·003181221
                 1789636
                  8020935
                   1205913
                    2183705
                     2086190
                      1111040
                       123497
```

Here there are 10 places of decimals in the Dividend, and we have brought down 2 ciphers giving 12, and there are 3 in the Divisor, therefore there must be 12 − 3 or 9 in the Quotient, and therefore we must prefix two ciphers (00) before writing the decimal point.

Exercise 23.

Multiply

1. 35·603 by 27·61;	421·619 by ·547;	1·0759 by 3·16.
2. ·01385 by 61·37;	13·676 by ·00048;	·0204 by 40·2.
3. ·346875 by ·119808;	·015625 by ·0064;	·0701 by 700·01.
4. 1·23 by ·0011;	·007853 by ·00476;	·57689 by 1·32.
5. 4·03721 by ·01107;	29000 by ·01;	56·875 by ·0144.

6. 325 tenths by 547 millionths; 128 thousandths by 78125 ten-millionths.

Find the value of

7. (37·1 − 19·08) × ·703; 37·1 − 19·08 × ·703; $(\cdot05)^2 + (\cdot025)^2 + \cdot00025$.

8. ·4 × ·05 × ·006 × ·0007 × 800000; ·845 × ·0017 × 7·4 × ·09 × 10000.

Divide

9. ·1 by ·01; ·01001 by ·001; 92·7 by ·06; 99 by ·0009.

10. 1422·3 by ·011; ·90804 by 1·2; 4·068 by ·0018.

11. ·0007672 by ·00056; ·08748 by 10·8; 418·25 by ·128.

12. 8·79462 by ·084; ·376809 by ·132; 3·14159 by 14·4.

13. ·000144 by ·012; 1·0665 by ·00135; 345·6 by 3·78.

14. 8886·66 by ·00037; 145·817 by ·0563; 1114·869145005 by ·385.

15. 7006·652 by 12·34; ·2219904 by ·3854; 1065·855558 by 7695·708.

16. ·0003738028 by ·0476; ·0064096 by 2·003; ·014904 by $3\frac{8}{21}$.

17. ·213419596 by ·0100103; $(6\cdot25)^2$ by $(\cdot025)^3$; ·001 by ·1001.

18. Express 2 and 22 hundredths, and 74 ten-thousandths, as decimals, and find the quotient of the first by the second.

Find, to the number of places of decimals indicated, the value of

19. ·765439 ÷ 359·21 to 5 places; ·5 ÷ 76·91342 to 6 places.

20. ·046 ÷ ·00762089 to 4 places; ·32165 ÷ ·0035216 to 3 places.

21. 4·00654 ÷ 329·265 to 7 places; 314159·26 ÷ ·008597 to 4 places.

22. ·01385 × 61·37 ÷ 2·77; 3833336 ÷ (8·99 × 20·8).

23. ·399 × ·007 ÷ ·000019; $(2\cdot05)^2 \times 2\cdot24 \div \cdot0041$.

24. 15·8402 + 3·689 ÷ 672·4 to 6 places; 206·59 ÷ 1872 × ·001 to 5 places.

25. Simplify: $\dfrac{\cdot0075 \times 2\cdot1}{\cdot0175}$; $\dfrac{4\cdot255 \times \cdot0064}{\cdot00032}$; $\dfrac{5\frac{5}{8}}{7\frac{1}{2}}$ of $\dfrac{21\cdot25}{\cdot046875}$.

26. Add together 1·465, ·0095, 37·15, 28·457 and 16·1685, and divide the sum by ·0296.

27. Find the sum, difference, product and two quotients of 30·33 and ·0337; and find the sum of all the results.

28. Simplify $\dfrac{1\cdot18}{\cdot152} \times \dfrac{3\cdot04}{2\cdot95}$, and divide the result by ·00125.

29. Express as mixed numbers $999\frac{23}{28} \times 2\cdot3$ and $10000\frac{1}{2316} \times \cdot5909$.

30. Express 4578 thousandths and 397 millionths as decimals, and find the quotient of the first by the second to 5 places of decimals.

CONTRACTED ADDITION AND SUBTRACTION.

151. Sometimes there are a great number of figures in the decimal parts of the numbers to be added together, and yet we only wish their *sum* to be accurate within a certain limit, as, for instance, a thousandth. But as thousandths occupy the third place of decimals, we require that the sum should be correct to the third place, and if we neglect all the remaining figures in it, the error will not amount to one thousandth. We therefore proceed thus: write down each number as far as 3 places of decimals, and then 1 or 2 places more to be sure that in making the addition we are *carrying* the correct figure to the third place, and then proceed in the usual way. The same remarks are applicable to Subtraction.

Ex. (1). Find the sum of 3·14159265, 27·789789, 54·5678678 and 543·777777 correct to 2 places of decimals. (2) Find within one ten thousandth the difference between 52·34563456 and 7·6666666.

```
(1)     3·14|15          (2)   52·3456|34
       27·78|97                 7·6666|6
       54·56|78                44·6789
      543·77|77
      -----------
      629·27
```

CONTRACTED MULTIPLICATION.

152. In multiplying two numbers we may begin with the left-hand figure of the Multiplier, provided we place each succeeding partial product one place farther to the right than the preceding one: and adopting this method we shall find it convenient to write the figures of the Multiplier in the reverse order. Thus, if it be required to multiply 32·52678 by 957·34, the work will stand as at (A):

```
       32·52678              32·52678              32·52678
        43759                 4 3759                4 3759
     ----------            ----------            ---------
      2927410|2             2927410|2             2927410
(A)    162633|90      (B)    162633|90      (C)    162634
        22768|746             22768|746             22768
          975|8034              975|8034              976
          130|10712             130|10712             130
     --------------        --------------        ---------
     31139·18|75652        31139·18|75652        31139·18
```

Now if we wish to retain (for example) only two places of decimals in the product, we must dispense with, as far as possible, all the work to the right of the vertical line at (A). But the figures in the column to the left of this line 0, 3, 8, 5, 0 are respectively the units' figures of the products of 9 in the Multiplier and 7 in the Multiplicand (with 7 carried), of 5 in the Multiplier and 6 in the Multiplicand (with 3 carried), &c.: hence, if we remove all the figures of the Multiplier one place farther to the left, these products will be formed by the figures of the Multiplier and the figures of the Multiplicand immediately *above* them (as at B): an arrangement which secures that the units' figure of the Multiplier shall be under the *second* place of decimals in the Multiplicand. But, again, in dispensing with all the work to the right of this vertical line, we lose the figure *carried* in finding the sum of the first column on the left; to compensate for this loss, instead of carrying to the first figure of each partial product the preceding tens' figure, we carry the nearest ten: thus, for any number from 5 to 14 we carry 1, from 15 to 24 we carry 2, from 25 to 34 we carry 3, &c. Thus, in multiplying by 5 (at C) we say 5 times 6 is 30 and 4 carried is 34; where the 4 carried is from 5 times 7, or 35: and in multiplying by 3 we say 3 times 5 is 15 and 1 carried (from 6) is 16.

We have then the following Rule:—

Count off from the decimal part in the Multiplicand as many figures as we are required to retain decimal places in the Product; under the last of these figures place the units' figure of the Multiplier, writing its other figures in a reverse order: and if any figure of the Multiplier has not a figure above it in the Multiplicand place a cipher there.

Begin the multiplication with the right-hand figure of the Multiplier and multiply in succession by each of the others, setting down in the product from the figure above the one we are multiplying by, but carrying to it the nearest ten from its product with the preceding figure.

Place the first figure of all these partial products in the same vertical line; add, and cut off from the result the required number of decimal places.

REMARK. The last figure in the product may not be quite correct, but to ensure its accuracy we must carry the process one place farther than is required to be retained.

Ex. Multiply ·43429448 by ·6931472 so as to retain 7 places of decimals; and 5947·183 by ·093187 so as to retain 4 places of decimals.

·43429448	5947·1830
27413960	781 3900
2605766	5352465
390865	178415
13029	5947
434	4758
174	416
30	554·2001
1	
·3010299	

CONTRACTED DIVISION.

153. The following Rule for Contracted Division requires no explanation beyond what has been given in Contracted Multiplication, or will be afforded in the Examples given below.

Determine first of all—by inspection, or by an equal removal of the decimal points in the Dividend and Divisor (150, Cor.), *or by taking one step in the ordinary way—the highest order of units in the Quotient, and then the number of figures in the Quotient: from the left of the Divisor cut off one more than this number of figures, and strike out the rest. Proceed one step with this new divisor, but in multiplying its first figure by the quotient figure, carry the nearest ten from the preceding figure. Instead of bringing down a figure to the remainder, strike off another figure from the Divisor, and proceed as before.*

If the number of figures in the Divisor be less than the number of figures to be cut off, proceed in the ordinary way until the number of figures still to be found is one less than the number of figures in the Divisor, and then apply the Rule.

Ex. 1.* Divide 496·94325 by ·17614352 so as to retain integers only in the quotient.

```
17,6,1,4352 ) 49694325 ( 2821
              14465
                374
                 22
                  4
```

Now 496 divided by ·17 or 4969 divided by 1·7 gives thousands, therefore there must be 4 figures in the Quotient: retain 5 figures in the Divisor and strike out the rest.

Ex. 2. Divide ·549532676 by 9·312167 so as to retain 5 places of decimals in the Quotient.

```
93,1,2,167 ) 549532,676 ( ·05901
             83924
               115
                22
```

The highest order of units in the Quotient is manifestly hundredths; therefore there must be 4 significant figures in the Quotient, and the first must be 0: hence we cut off 5 figures to the left of the Divisor and strike out the rest.

Ex. 3. Divide 5·78564327 by ·8345 so as to retain 5 places of decimals in the Quotient.

```
8,3,4,5 ) 578564327 ( 6·93306
           77864
           27593
            2558
              54
               4
```

The first figure of the Quotient will represent simple units, therefore there must be 6 figures in all: but as there are only 4 figures in the Divisor, we must take 3 steps in the usual way and then apply the Rule.

EXERCISE 24.

1. Find within a hundredth, the sum of 27·035035, 3·7676, ·2596596 and ·003451; and the difference of 315·857142 and 47·950375.

2. Find within a thousandth, the sum of ·0795, 617·34833, ·08391 and 15·808080; and the difference of 3·183546 and ·93681.

3. Find correct to five places of decimals the sum of ·385385, 19·777777, ·05 and 6·7897897; and the difference of 23·34534534 and 7·8888888.

4. Multiply 375·76843 by 3·14159, retaining 4 places of decimals.

5. Multiply 65·00763 by ·9876, retaining 5 places of decimals.

6. Multiply 583·26784 by ·00985, retaining 2 places of decimals.

7. Multiply 678·3089 by 45·657, retaining integers only.

8. Multiply ·86858896 by 1·0986123, retaining 5 places of decimals.

9. Multiply ·0008127 by 483·2716, retaining 6 places of decimals.

10. Divide 3789·436 by 265·5984 so as to retain 2 places of decimals.

11. Divide 742·876315 by 4967·358 so as to retain 4 places of decimals.

12. Divide 59·3264 by ·09352 so as to retain 1 place of decimals.

13. Divide 17·389267 by ·6574 so as to retain 6 places of decimals.

14. Divide 2 by 15·314865 so as to retain 5 places of decimals.

15. Divide 10·926954 by ·3547808034 so as to retain 3 places of decimals.

16. Find, in each case within a millionth, the reciprocals of 2·3025851 and 3·14159265 35.

Find, to the number of places of decimals indicated, the value of

17. $1{\cdot}050625^2$, $1{\cdot}050625^3$, $1{\cdot}050625^6$; in each case to 4 places.

18. $1{\cdot}0375^4$ and $987{\cdot}625 \times 1{\cdot}0375^4$ to 5 places of decimals.

19. $1{\cdot}0425^2$ and $357{\cdot}6 \div 1{\cdot}0425^3$ to 4 places of decimals.

20. $\dfrac{325{\cdot}75}{{\cdot}045}$; $\dfrac{1}{1{\cdot}045^8}$; $\dfrac{325{\cdot}75}{{\cdot}045}\left\{1 - \dfrac{1}{1{\cdot}045^8}\right\}$; in each case to 5 places.

REDUCTION OF FRACTIONS TO DECIMALS. REPEATING DECIMALS.

154. To reduce a vulgar fraction to its equivalent decimal fraction, we find how often the denominator is contained in some power of 10, and we multiply numerator and denominator by the quotient: thus, if we take the fraction $\frac{5}{16}$, the multiplying factor is $\frac{100\ldots}{16}$, and therefore

$$\frac{5}{16} = \frac{5 \times \frac{100\ldots}{16}}{100\ldots} = \frac{\frac{500\ldots}{16}}{100\ldots}.$$

But we have still to find *what* power of 10 must be taken, or, which is the same thing, how many ciphers must be annexed to the numerator 5; and this is found by actual division, thus:

$$\begin{array}{r} 16\,)\,\underline{50000} \\ 3125 \end{array}$$

where we see that the division terminates on bringing down 4 ciphers; therefore

$$\frac{5}{16} = \frac{3125}{10000} = {\cdot}3125\,;$$

hence, to reduce a vulgar fraction to a decimal we have this Rule:—

Write down the numerator, annexing ciphers, divide by the denominator, and cut off as many decimal places in the quotient as we have brought down ciphers.

COR. Again, a fraction represents the quotient of the numerator by the denominator (103), and by *division of decimals* we can express this quotient as a decimal. A comparison of the two processes will shew that they are in reality identical.

155. But it will often happen that however many ciphers we may bring down the division will not terminate, and consequently that the given fraction cannot be expressed exactly as a decimal. Take any fraction in its lowest terms, as $\frac{5}{12}$, then the numerator of the equivalent decimal fraction is

$$\frac{500\ldots}{12} \text{ or } \frac{5 \times 100\ldots}{12};$$

and, as 12 is prime to 5 (68), this result can only be an integer if 100... is divisible by 12 (75), that is, if some power of 10 be divisible by the given denominator. But 10 is the product of 2 and 5, therefore the second power of 10 is the product of two 2s and two 5s, the third power of three 2s and three 5s,... hence, if the prime factors of the denominator be 2 and 5 only, however often they may be repeated, there is always a power of 10 that can be divided by that denominator. But if the denominator contain any other prime factor than 2 or 5, this factor will be prime to 10, and therefore to every power of 10 (95), and consequently however far we carry on the division it will never terminate. We conclude then that

If the denominator of the given vulgar fraction (in its lowest terms) be composed of the prime factors 2 and 5 only, the fraction can be expressed as an exact or terminating decimal; otherwise, it can not.

156. But though a given vulgar fraction may not be capable of being expressed as a decimal exactly, yet it can be expressed to any degree of accuracy we please. Take the fraction $\frac{3}{11}$;

$$\begin{array}{r} 11\,\underline{)\ 30000000} \\ \cdot 2727272 \end{array}$$

and we see that $\frac{3}{11}$ is greater than ·2 but less than ·3; is greater than ·27 but less than ·28; is greater than ·272 but less than ·273;... that is, if the decimals

·2; ·27; ·272; ·2727; ·27272;...

be taken to represent $\frac{3}{11}$, the errors are respectively less than

$$\frac{1}{10}, \frac{1}{100}, \frac{1}{1000}, \frac{1}{10.000}, \frac{1}{100.000}, \ldots$$

and thus by taking a proper number of figures in the decimal part, we can represent $\frac{3}{11}$ to any required degree of accuracy.

157. *In non-terminating decimals the figures of the quotient must recur.*

Take the fraction $\frac{4}{7}$: to convert it to a decimal we annex ciphers to 4 and divide by 7. Since the division does not terminate, we cannot have the remainder 0, and the only other remainders that can arise are 1, 2, 3, 4, 5 and 6, and consequently after 6 steps at most we must come to the proposed numerator or to a remainder that has occurred before, and therefore from that point we must have a recurrence of the remainders, and therefore of the quotient figures.

158. (1) When, starting from a certain point in the decimal part of a number, the figures repeat themselves indefinitely and in the same order, the number is said to be a *repeating*, *recurring* or *circulating* decimal.

(2) When the recurrence takes place from the first figure of the decimal part, the number is a *pure repeating* decimal; otherwise it is a *mixed repeating* decimal: thus ·272727... is a *pure*, and 25·34567567... is a *mixed* repeating decimal.

(3) The whole body of figures which constantly repeat themselves in the same order is called the *period*.

(4) In writing a repeating decimal we usually stop at the end of the first period and place dots over its first and last figure. Thus ·272727... is written $\cdot\dot{2}\dot{7}$; and 25·34567567... is written $25\cdot34\dot{5}6\dot{7}$.

(5) The period may be supposed to begin at any point we please after the first repeating figure: thus, 25·34567567... may be written $25\cdot34\dot{5}6\dot{7}$ or $25\cdot345\dot{6}7\dot{5}$ or $25\cdot3456\dot{7}5\dot{6}$ or ....

(6) Sometimes the period is made to commence in the *integral* part, as in $76\dot{5}\cdot3\dot{4}$, which is the same as $765\cdot\dot{3}4\dot{5}$ or $765\cdot3\dot{4}5\dot{3}$ or ....

(7) In converting a fraction to a repeating decimal we may often shorten the work by expressing the remainder at some step as a fraction: thus

$$\tfrac{1}{7} = \cdot 142\tfrac{6}{7}, \ \therefore \tfrac{6}{7} = \cdot 857\tfrac{1}{7} \text{ and } \therefore \tfrac{1}{7} = \cdot 142857\tfrac{1}{7} = \cdot\dot{1}4285\dot{7}.$$

(8) In converting $\frac{1}{7}$, the remainders in order are 3, 2, 6, 4, 5, 1: therefore starting from the second figure we get the decimal for $\frac{3}{7}$, from the third figure for $\frac{2}{7}$, and so on: thus

$$\cdot\dot{1}4285\dot{7} = \tfrac{1}{7}, \ \cdot\dot{4}2857\dot{1} = \tfrac{3}{7}, \ \cdot\dot{2}8571\dot{4} = \tfrac{2}{7}, \ \&c.$$

159. *To convert a repeating decimal to a vulgar fraction.*

(1) Let the repeating decimal be *pure*. It is seen at sight that $\frac{1}{9} = \cdot 111\ldots$, $\frac{1}{99} = \cdot 0101\ldots$, $\frac{1}{999} = \cdot 001001\ldots$, $\frac{1}{9999} = \cdot 00010001\ldots$, &c.; therefore $\dot{2} = \cdot 222\ldots = 2 \times \cdot 111\ldots = 2 \times \frac{1}{9} = \frac{2}{9}$,

$$\cdot\dot{2}\dot{3} = \cdot 2323\ldots = 23 \times \cdot 0101\ldots = 23 \times \tfrac{1}{99} = \tfrac{23}{99};$$

$$\cdot\dot{2}3\dot{4} = \cdot 234234\ldots = 234 \times \cdot 001001\ldots = 234 \times \tfrac{1}{999} = \tfrac{234}{999},$$

$$\cdot\dot{2}34\dot{5} = \cdot 23452345\ldots = 2345 \times \cdot 00010001\ldots = \tfrac{2345}{9999}, \ \&c.;$$

that is,—*We write down the period for the numerator, and for the denominator as many 9s as there are figures in the period.*

(2) Let the repeating decimal be *mixed*: as for example, $\cdot 23\dot{5}4\dot{8}$. Remove the decimal point so as to be before the first figure of the period, as $23\cdot\dot{5}4\dot{8}$; then

$$23\cdot\dot{5}4\dot{8} = 23\tfrac{548}{999} \qquad (159, 1)$$

$$= \frac{23 \times 999 + 548}{999} \qquad (107, 2)$$

$$= \frac{23(1000-1)+548}{999} = \frac{23000-23+548}{999} \qquad (63, 1 \text{ Cor.})$$

$$= \frac{23548-23}{999}; \qquad (59)$$

divide each of these equal quantities by 100 and we get (144)

$$\cdot 23\dot{5}4\dot{8} = \frac{23548-23}{99900};$$

and, as the same method may be pursued with every other mixed repeating decimal, we have this Rule :—

For the numerator take the difference between the given decimal and the non-repeating part, both considered as integers: and for the denominator write down as many 9s as there are figures in the period followed by as many 0s as there are non-repeating figures in the decimal part.

COR. Since $\cdot\dot{9} = \frac{9}{9} = 1$, it follows that $\cdot 0\dot{9} = \cdot 1$ and $\cdot 00\dot{9} = \cdot 01$; and therefore $27\cdot\dot{9} = 28$, $27\cdot 3\dot{9} = 27\cdot 4$, $27\cdot 634\dot{9} = 27\cdot 635$, &c.

Ex. 1. Can $\frac{31}{1250}$ be expressed as a terminating decimal?
Yes: for $1250 = 2 \cdot 5^4$ and involves factors of 2 and 5 only (155).

Ex. 2. Can $\frac{223}{576}$ be expressed as a finite decimal?
No: for $576 = 2^6 \cdot 3^2$ and involves other factors than 2 and 5.

Ex. 3. Express $\frac{5}{8}$, $\frac{9}{25}$ and $\frac{17}{320}$ as decimals.

$$(1)\quad 8\,)\,\underline{5000} \qquad (2)\quad 5\,)\,\underline{9} \qquad (3)\quad 8\,)\,\underline{17}$$
$$\cdot 625 \qquad\qquad 5\,)\,\underline{1\cdot 8} \qquad\qquad 40\,)\,\underline{2\cdot 125}$$
$$\cdot 36 \qquad\qquad \cdot 053125$$

In Ex. 1 we annexed ciphers to the numerator; in Ex. 2 we *supposed* them to be annexed; and in Ex. 3 we proceeded as in division of Decimals.

Ex. 4. Express $\frac{4}{7}$, $\frac{11}{72}$ and $\frac{21}{37}$ as decimals.

$$(1)\quad 7\,)\,\underline{4} \qquad (2)\quad 8\,)\,\underline{11} \qquad (3)\quad 37\,)\,210\,(\,\cdot\dot{5}6\dot{7}$$
$$\cdot\dot{5}7142\dot{8} \qquad 9\,)\,\underline{1\cdot 375} \qquad 250$$
$$\cdot 152\dot{7} \qquad 280$$
$$21$$

In Ex. 2 we first divide by 8, and then by 9, and not *vice versâ*, because the division by 8 gives a terminating quotient and by 9 does not.

Ex. 5. Express the following repeating decimals as vulgar fractions:

(1) $\cdot\dot{3} = \frac{3}{9} = \frac{1}{3}$. (2) $\cdot\dot{2}9\dot{6} = \frac{296}{999} = \frac{8}{27}$.

(3) $\cdot 3\dot{1}4\dot{8} = \frac{3148-3}{9990} = \frac{3145}{9990} = \frac{17}{54}$.

(4) $23\cdot 45\dot{2}7\dot{6} = \frac{2345276-2345}{99900} = \frac{2342931}{99900} = \frac{3471}{148}$;

or $= 23\frac{45276-45}{99900} = 23\frac{45231}{99900} = 23\frac{67}{148}$.

EXERCISE 25.

Express the following vulgar fractions as decimals:

1. $\frac{7}{8}$, $\frac{47}{125}$, $\frac{13}{16}$, $\frac{27}{800}$, $\frac{91}{32}$, $\frac{201}{625}$, $\frac{97}{128}$.

2. $\frac{4096}{12500}$, $\frac{175}{256}$, $\frac{347}{25600}$, $\frac{3476}{15625}$, $\frac{1025}{1024}$, $\frac{34567}{20480}$.

3. $\frac{3^4}{10^8}$, $\frac{361}{2^3 \cdot 5^4}$, $\frac{287297}{2^8 \cdot 5^6}$, $\frac{3 \cdot 9 \cdot 27 \cdot 81}{2 \cdot 4 \cdot 8 \cdot 16}$, $\frac{5 \cdot 6 \cdot 7 \cdot 8}{1 \cdot 2 \cdot 3 \cdot 4} \cdot \frac{1}{5^4}$.

Express as decimals and find the sum of

4. $3\frac{4}{25}$, $1\frac{7}{8}$, $\cdot 648$, $\frac{5}{32}$.

5. $\frac{5}{16}$, $\frac{7}{32}$, $\frac{3}{8}$, $\cdot 046875$, $1 \cdot 23$.

6. $17\frac{8}{25}$, $25\frac{3}{8}$, $6\frac{25}{32}$, $13\frac{4}{25}$, and $20\frac{73}{10000}$.

7. Express as decimals and find the difference of

$$\frac{586}{625} \text{ and } \frac{15}{16}; \quad \frac{31}{32} \text{ and } \frac{15001}{15625}.$$

Find a decimal which shall not differ from

8. $\frac{4}{7}$ by a ten-thousandth; $1\frac{13}{17}$ by a thousandth.

9. $\frac{41 \times 37}{17}$ by a millionth; $\frac{27}{14} - \frac{21}{17 \times 31}$ by a hundred-millionth.

10. Find the difference between $\frac{355}{113}$ and $3 \cdot 14159265$ to 6 places of decimals.

11. Which of the following fractions can be expressed as finite decimals?

$$\frac{19}{64}, \frac{85}{192}, \frac{79}{405}, \frac{91}{560}, \frac{167}{625}, \frac{512}{875}, \frac{231}{288}.$$

12. Write down those numbers between 1 and 20, of which if any one be the denominator of a fraction in its lowest terms, that fraction can be converted into a terminating decimal.

Express the following vulgar fractions as repeating decimals:

13. $\frac{5}{3}$, $\frac{44}{9}$, $3\frac{8}{9}$, $\frac{209}{450}$, $\frac{103}{180}$.

14. $\frac{343}{375}$, $\frac{21}{11}$, $9\frac{13}{33}$, $\frac{401}{352}$.

15. $18\frac{44}{111}$, $\frac{5}{396}$, $\frac{314}{37}$, $\frac{233}{185}$.

16. $\frac{809}{296}$, $5\frac{28}{101}$, $\frac{506}{505}$, $\frac{625}{576}$.

17. $\frac{997}{1375}$, $3\frac{413}{1111}$, $\frac{189}{925}$, $\frac{907}{1010}$.

18. $\frac{80}{41}$, $\frac{200}{271}$, $\frac{425}{328}$, $\frac{541}{1084}$.

19. $\frac{5}{7}$, $\frac{97}{13}$, $4\frac{11}{13}$, $\frac{8}{63}$, $7\frac{34}{111}$.

20. $\frac{725}{52}$, $\frac{135}{143}$, $\frac{1000}{407}$, $\frac{313}{416}$.

21. $\frac{3542}{1025}$, $\frac{661}{728}$, $\frac{53}{259}$, $\frac{1000}{1001}$.

22. $\frac{7624}{4329}$, $\frac{10000}{5291}$, $\frac{16}{17}$, $\frac{17}{19}$.

Convert the following repeating decimals to vulgar fractions in their lowest terms:

23. $\cdot\dot{6}$, $\cdot\dot{5}\dot{6}$, $\cdot\dot{7}\dot{2}$, $\cdot\dot{2}5\dot{9}$, $8\cdot\dot{9}6\dot{1}$. 24. $\cdot\dot{7}22\dot{7}$, $3\cdot780\dot{4}\dot{8}$, $5\cdot\dot{3}535\dot{6}$.

25. $\cdot\dot{5}7142\dot{8}$, $\cdot\dot{6}2026\dot{8}$, $\cdot\dot{4}833\dot{9}$. 26. $\cdot0\dot{5}$, $2\cdot0\dot{8}\dot{1}$, $\cdot004\dot{9}\dot{5}$, $3\cdot\dot{5}4\dot{5}$.

27. $\cdot011\dot{3}\dot{6}$, $\cdot00\dot{7}2\dot{9}$, $42\cdot1083\dot{7}$. 28. $\cdot\dot{5}91\dot{5}$, $8\cdot431\dot{4}\dot{7}$, $\cdot25\dot{4}62\dot{9}$.

29. $37\cdot4018\dot{5}$, $\cdot7\dot{5}434\dot{7}$, $\cdot0\dot{3}013\dot{5}$. 30. $3\cdot864\dot{3}01\dot{8}$, $5\cdot7893\dot{0}\dot{6}$.

31. $2\cdot\dot{6}1904\dot{7}$, $18\cdot7621951$. 32. $\cdot00849713\dot{3}$, $13\cdot94\dot{2}3076\dot{9}$.

ADDITION AND SUBTRACTION OF REPEATING DECIMALS.

160. The sum of two or more repeating decimals may be found to a limited degree of accuracy by the method pointed out in Art. 145.

If we wish to find the sum *exactly*, we must bear in mind that after a decimal has begun to repeat, the repetition may be supposed to begin at any subsequent figure (158, 5), and that a period of 2 figures is the same as one of 4, 6, 8,... figures, thus $\cdot\dot{2}\dot{7}$ is the same as $\cdot\dot{2}72\dot{7}$, $\cdot\dot{2}7272\dot{7}$..., that a period of 3 figures is the same as one of 6, 9, 12... figures, &c.: hence, if several decimals have to be added, and one period consists of 2 figures, another of 3 figures, and another of 4 figures, we may consider each of them to consist of a period of 12 figures, where 12 is the L.C.M. of 2, 3, and 4; and as the whole body of figures within these 12 places will constantly be repeated, their sum will be constantly repeated. We have then this Rule:—

Extend each decimal as far as the farthest non-repeating figure in any of them: find the L.C.M. *of the number of figures in each period, and extend each repeating decimal so many places further, and one or two places more to make sure we are carrying the correct figure to the last place of the second extension: add in the usual way, then in the sum the first extension will give the non-repeating part, and the second the repeating part.*

If in this Rule we write subtract for add and remainder for sum, we shall have the Rule for the Subtraction of Repeating Decimals.

Ex. 1. Find the sum of $3{\cdot}1416$, $8{\cdot}25\dot{1}4285\dot{7}$, ${\cdot}03\dot{4}$, $23{\cdot}257\dot{6}3\dot{5}$ and $5{\cdot}456\dot{2}\dot{7}$; (1) correct within one ten-thousandth, (2) quite correctly.

$$\text{(1)}\quad \begin{array}{r|l} 3{\cdot}1416 & \\ 8{\cdot}2514 & 28 \\ {\cdot}0344 & 44 \\ 23{\cdot}2576 & 35 \\ 5{\cdot}4562 & 72 \\ \hline 40{\cdot}1413 & \end{array} \qquad \text{(2)}\quad \begin{array}{r|l|l} 3{\cdot}1416 & & \\ 8{\cdot}2514 & 285714 & 28 \\ {\cdot}0344 & 444444 & 44 \\ 23{\cdot}2576 & 356356 & 35 \\ 5{\cdot}4562 & 727272 & 72 \\ \hline 40{\cdot}1413 & \dot{8}1378\dot{7} & \end{array}$$

Ex. 2. Find the difference between $27{\cdot}0354\dot{7}\dot{1}$ and $5{\cdot}98\dot{7}6\dot{5}$ (1) correct to one millionth, and (2) absolutely correct.

$$\text{(1)}\quad \begin{array}{r|l} 27{\cdot}035471 & 71 \\ 5{\cdot}987657 & 65 \\ \hline 21{\cdot}047814 & \end{array} \qquad \text{(2)}\quad \begin{array}{r|l|l} 27{\cdot}0354 & 717171 & 71 \\ 5{\cdot}9876 & 576576 & 57 \\ \hline 21{\cdot}0478 & \dot{1}4059\dot{5} & \end{array}$$

MULTIPLICATION AND DIVISION OF REPEATING DECIMALS.

161. If the Multiplicand be a repeating decimal, and the Multiplier an integer or finite decimal, the Product will be a repeating decimal of the same kind as the Multiplicand, but will sometimes admit of simplification. To form the Product we proceed in the usual way, carrying out the work one or two places beyond the period, to make sure that the figures carried are always correct.

$$\begin{array}{r} 37{\cdot}83\dot{4}5\dot{9}_{45} \\ 7 \\ \hline 264{\cdot}84\dot{2}1\dot{6} \end{array} \qquad \begin{array}{r} 37{\cdot}82\dot{3}\dot{6}_{36} \\ 11 \\ \hline 416{\cdot}05\dot{9}\dot{9} \end{array} \ \text{or}\ \begin{array}{r} 37{\cdot}82\tfrac{4}{11} \\ 11 \\ \hline 416{\cdot}06 \end{array} \qquad \begin{array}{r} 37{\cdot}6\dot{2}8571\dot{4}_{285} \\ 7 \\ \hline 263{\cdot}3\dot{9}9999\dot{9} \end{array} = 263{\cdot}4$$

$$\text{(A)}\quad \begin{array}{r} 37{\cdot}83\dot{4}5\dot{9} \\ 537 \\ \hline 26484216 \\ 11350378 \\ 19917297 \\ \hline \end{array} \qquad \text{(B)}\quad \begin{array}{r|l|l} & 37{\cdot}83\dot{4}5\dot{9} & \\ & 537 & \\ \hline 26484 & 216 & 21 \\ 113503 & 783 & 78 \\ 1991729 & 729 & 72 \\ \hline 21317{\cdot}17 & 72\dot{9} & \end{array}$$

In the last Ex. preceding, as at (A), we see that the periods in the partial dividends are 216, 378 and 297; if therefore we draw or suppose to be drawn vertical lines under the period of the Multiplicand and extend the above periods we obtain the product required, as at (B).

REMARK. If the Multiplier had been $53\cdot 7$ or $5\cdot 37$ or $\cdot 537$,... the only difference would have been in placing the decimal point in the Product.

162. The *Product* or *Quotient* of two *repeating* decimals may be found—

(1) to a limited degree of accuracy by contracted Multiplication or Division of decimals (152, 153); and

(2) *exactly*, by converting them into fractions, performing the required operation, and reducing the resulting fraction to a decimal.

Ex. 1. Multiply $27\cdot 54\dot{3}\dot{6}$ by $8\cdot 34\dot{7}$, (1) retaining 5 places of decimals, and (2) exactly.

$$\begin{array}{r}
27\cdot 5436363 \\
777\;777438 \\
\hline
220\;349090 \\
8\;263091 \\
1\;101745 \\
192805 \\
19280 \\
1928 \\
193 \\
19 \\
2 \\
\hline
229\cdot 92815.
\end{array}$$

$$27\cdot 54\dot{3}\dot{6} \times 8\cdot 34\dot{7} = \frac{275436 - 2754}{9900} \times \frac{8347 - 834}{900}$$

$$= \frac{\overset{30298}{\cancel{272682}}}{\underset{900}{\cancel{9900}}} \times \frac{\overset{683}{\cancel{7513}}}{\underset{100}{\cancel{900}}}$$

$$= \frac{20693534}{90000}$$

$$= 229\cdot 9281\dot{5}.$$

Ex. 2. Divide $66\cdot 02\dot{0}3\dot{7}$ by $248\cdot 7\dot{2}\dot{2}$, (1) correct to 5 places of decimals and (2) exactly.

$$66\cdot 02\dot{0}3\dot{7} \div 248\cdot 7\dot{2}\dot{2} = \frac{6602037 - 6602}{99900} \div \frac{248722 - 2487}{990}$$

$$\begin{array}{r}
24,8,7,2,2\;)\;6602037\;(\;\cdot 26543 \\
162759 \\
13526 \\
1090 \\
95 \\
21
\end{array}$$

$$= \frac{6595435}{99900} \times \frac{990}{246235}$$

$$= \frac{3241}{6} \times \frac{1}{2035}$$

$$= \frac{3241}{12210}$$

$$= \cdot 2\dot{6}5438\dot{1}.$$

Exercise 26.

Find the sum of

1. 6·3, 26·43 and 375·8543; ·7843, 12·4718 and ·0632.
2. 23·076, 19·245 and 31·263; 36·25142857 and 63·74857142.
3. 27·64235, 9·2642037, 5·4925, 1·498 and ·603306.
4. 3·807, 6·76435, 8·5486, ·0037, ·6571428 and 87·689.

Find the difference between

5. ·328 and ·328; 25·47 and 16·8578; 753·6726 and 19.004259.
6. 17·573 and 14·57; 6·73459 and 3·0726; ·714285 and ·001136.
7. Find the complement of ·04563; ·0789; 25·6420376.

Multiply

8. ·37642 by 9; ·37642 by 11; ·37642 by 37; 2·3857142 by 5·6.
9. ·4322443218 by 88; 3·54268 by ·144; ·635427 by 13·2.
10. 27·38443 by 26·7; ·7853981 by 3457; 9·3850787 by 7·659.
11. Divide 3·457954 by 8; 37·635842 by 7; 8·9854 by 12.
12. Divide 235·47 by 24 × 20; 539·63436 by 112.

Find the value of

13. 4·8 × 24; 1·18 × ·538461; 43·291 × 6·24; 6·36 × ·571428.
14. 7·857142 × 4·6; ·27 × 4·90; 7·63 × 8·83; 19·72 × 29·45.
15. 22·917 × ·560; 5598·9243 × 8·247; 44·20645 × 1·5823707.
16. ·5 ÷ ·25; 3·8 ÷ 2·73; 1·90 ÷ ·583; 3·6 ÷ ·017; 60·45 ÷ 7·38.
17. 43·291 ÷ 6·23; ·37 ÷ ·148; ·01236 ÷ ·051; 28·583 ÷ 37·12.
18. 6·891 ÷ 15·45; 9·53 ÷ 3·2083; 411·3519 ÷ 19·5881;
    77·67027 ÷ 9·486; 14·476190 ÷ 2·1590.
19. $\dfrac{2\frac{2}{3}}{3\frac{1}{2}}$ of ·0006 of $\dfrac{·45}{·0024}$; 1·83 of ·954 of ·428571 of 2·25.
20. $\dfrac{2·5 + 1·25 - 2·125}{3·75 + 2·3 - 4·25}$; $\dfrac{1 + 5·4 \times 6·4}{1 + 2·3 \times 3·3}$; 5·09 of 11·16 ÷ 4·2592 of ·36.
21. $\dfrac{2·6 \text{ of } 2·83}{6·2 \text{ of } ·857142} + \dfrac{4\frac{1}{2} \text{ of } 4·036}{3·75 \text{ of } 1·7}$; $\dfrac{·03 - ·03}{·123}$.
22. Divide 9·614 by ·0000019 and $\dfrac{2\frac{1}{2}}{5\frac{1}{2}}$ by ·0003, and multiply the sum of the quotients by ·0005.

Miscellaneous.

23. Express $\frac{1}{2}(6\frac{1}{3} + 2\frac{2}{3} - 3)$ as a decimal, and $·7 + \frac{3}{11}$ of $·825 + 4·13$ as a vulgar fraction.
24. Reduce $\left(\frac{3}{25} \text{ of } 2·48 - \frac{1}{100} \text{ of } ·02\right) \div 1000$ to a decimal.

25. Which is the greatest and which is the least of the expressions

$$(1)\ \frac{2}{3}+\frac{3}{4}; \qquad (2)\ 1\cdot41421; \qquad (3)\ \frac{1}{2}+\frac{1}{5}+\frac{5}{7}?$$

26. Find the sum, difference, product and two quotients of $10\cdot01$ and $\cdot0091$, and find the sum of the results.

27. Find within a thousandth the value of $3229\cdot1\dot{6} \times 1\cdot047\dot{5}$.

28. Arrange in order of magnitude:

$$(1)\ \frac{333}{106}; \qquad (2)\ 3+\cfrac{1}{7+\cfrac{1}{16}}; \qquad (3)\ 3\cdot1415926.$$

29. Find the G.C.M. of $1353\cdot6$ and $231\cdot48$; of $29\cdot75$ and $113\cdot9$; of $36\cdot795$ and $57\cdot98$; and of $376\cdot1034$ and 1081.

Make the number of decimal places in the two numbers the same (143); *find their* G.C.M. *as if they were integers, and cut off that number of decimal places in the result. See Art.* 135.

30. Find to 6 places of decimals the value of

$$(1)\ \frac{1}{5}+\frac{1}{5^2}+\frac{1}{5^3}+\frac{1}{5^4}+\dots$$

Express the first term as a decimal, and derive each term from the preceding by dividing by 5, *placing the results under one another.*

$$(2)\ \frac{1}{7}+\frac{1}{7^2}+\frac{1}{7^3}+\frac{1}{7^4}+\dots$$

$$(3)\ \frac{1}{10^2}\times\left\{1-\frac{3}{10^2}+\frac{3\cdot4}{1\cdot2}\cdot\frac{1}{10^4}-\frac{3\cdot4\cdot5}{1\cdot2\cdot3}\cdot\frac{1}{10^6}\right\}.$$

Express each term in the bracket separately as a decimal.

$$(4)\ 1+\frac{1}{1\cdot2}+\frac{1}{1\cdot2\cdot3}+\frac{1}{1\cdot2\cdot3\cdot4}+\frac{1}{1\cdot2\cdot3\cdot4\cdot5}+\dots$$

The second term is derived from the first by dividing by 2; *the third from the second by dividing by* 3, *&c.*

$$(5)\ \frac{1}{5}+\frac{1}{3}\cdot\frac{1}{5^3}+\frac{1}{5}\cdot\frac{1}{5^5}+\frac{1}{7}\cdot\frac{1}{5^7}+\frac{1}{9}\cdot\frac{1}{5^9}+\dots$$

First express as decimals $\frac{1}{5}, \frac{1}{5^3}, \frac{1}{5^5}, \dots$ *then write under one another* $\frac{1}{5}, \frac{1}{3}\cdot\frac{1}{5^3}, \frac{1}{5}\cdot\frac{1}{5^5}, \dots$ *and add.*

$$(6)\ 16\times\left\{\frac{1}{5}-\frac{1}{3}\cdot\frac{1}{5^3}+\frac{1}{5}\cdot\frac{1}{5^5}-\frac{1}{7}\cdot\frac{1}{5^7}+\dots\right\}-\frac{4}{239}.$$

First express as decimals and add the 1*st*, 3*rd*, 5*th*, ... *terms within the bracket, then the* 2*nd*, 4*th*, 6*th*, ... *terms; find the difference, and multiply by* 16; *and from the result subtract* $\frac{4}{239}$ *expressed as a decimal.*

CHAPTER VII.

EVOLUTION.

163. EVOLUTION is the operation by which we find any root of a given number.

The *square root, cube root, fourth root, fifth root,*... of a given number is the number whose *square, cube, fourth power, fifth power,*... is equal to the given number. Thus the square root of 25 is 5, because 5^2 is 25; the fourth root of 256 is 4, because 4^4 is 256.

The root of a number is denoted by writing $\surd$ (really *r*) before the number, and placing against it a small figure denoting which root is to be taken; thus the square root of 25 is denoted by $\sqrt[2]{25}$, or simply $\sqrt{25}$; the cube root of 64 by $\sqrt[3]{64}$; the fifth root of 128 by $\sqrt[5]{128}$.

SQUARE ROOT.

164. (1) In multiplying a number by itself we see that its square and the square of its units' figure have the same units' figure; thus 537^2 has the same units' figure as 7^2.

Now the squares of the simple numbers

1, 2, 3, 4, 5, 6, 7, 8, 9

are respectively

1, 4, 9, 16, 25, 36, 49, 64, 81,

and if a number ends with 0, its square also ends with 0; hence the squares of all numbers, integral or decimal, must end with either, 0, 1, 4, 5, 6, or 9; and therefore it follows that

A number ending with 2, 3, 7 *or* 8 *cannot be the square of any number, integral or decimal.*

(2) If a number ends with 1, 2, 3,... ciphers, its square must end with 2, 4, 6,... ciphers (41): also, if a number does not end with a cipher, its square does not end with a cipher; therefore

A whole number ending with an odd number of ciphers cannot be the square of a whole number.

(3) If a number has 1, 2, 3,... places of decimals, the last figure being significant, its square must have 2, 4, 6... places of decimals (148); therefore it follows that

A decimal whose last figure is significant, and which has an odd number of decimal places, cannot be the square of any number integral or decimal.

165. *When the square root of a whole number is not a whole number, neither is it a fraction whose numerator and denominator are whole numbers.*

For, if possible, let $\sqrt{53}$ be $7\frac{4}{9}$ or $\frac{67}{9}$, where $\frac{67}{9}$ is in its lowest terms; then by definition

$$\frac{67}{9} \times \frac{67}{9} \text{ or } \frac{67^2}{9^2} \text{ must be equal to } 53;$$

but 67 and 9 are prime to each other, therefore 67^2 and 9^2 are prime to each other (95), and therefore 67^2 is not divisible by 9^2, or $\frac{67^2}{9^2}$ is not a whole number.

166. A number which can neither be expressed (measured) by a whole number, nor by a fraction whose numerator and denominator are whole numbers, is called an *incommensurable or irrational* number; thus $\sqrt{2}$, $\sqrt{5}$, $\sqrt{53}$ are incommensurable numbers.

Again, numbers whose square roots can be expressed exactly either by a whole number or by a fraction are called *perfect squares:* thus 36 whose square root is 6, and $\frac{4}{9}$ whose square root is $\frac{2}{3}$, are perfect squares; whereas 53, and $\frac{4}{7}$ are not perfect squares.

167. The squares of

1, 2, 3, 4, 5, 6, 7, 8, 9, 10,

are respectively

1, 4, 9, 16, 25, 36, 49, 64, 81, 100,

which shews that the square of a number of 1 figure consists of either 1 figure or 2 figures.

If we affix a cipher (0) to each of the above numbers we must affix two ciphers (00) to their squares; hence the square of a number of 2 figures consists of either 3 or 4 figures.

If we affix 2 ciphers (00) to each of the above numbers, we must affix 4 ciphers (0000) to their squares; hence the square of a number of 3 figures consists of either 5 or 6 figures.

And, in like manner, every additional figure in the number makes *two* additional figures in the square; hence

The square of any number consists of twice as many figures, or twice as many less 1, *as there are in the given number.*

168. Hence conversely, if the square of a number be given, we may suppose its first two figures on the right to correspond with the first figure to the right in the number; the next two figures in the square with the second figure in the number, and so on; and the last two or last one figure in the square to correspond with the last figure in the number. Thus we may mark off the squares 529 and 459684 in this way, ,5,29 ,45,96,84,
which shews that the numbers of which these are the squares consist of 2 and 3 figures respectively. The parts into which these numbers are marked off are called *periods;* thus in 529 the periods are 5 and 29; and in 459684 they are 45, 96 and 84.

It is more usual, however, to place a dot over the units' figure, and every alternate figure in the square; hence there will be as many dots as there are periods, and each period will consist of the figure over which the dot is placed, and the figure to *its left*, if there be one: for example, in 97344 we point thus, $\dot{9}7\dot{3}4\dot{4}$; so that the first period is 9, the second 73, and the third 44.

169. The extraction of the square root of a number depends on the following proposition:

If from the square of a number we subtract the square of one part of it, the remainder is a product of two factors: one factor is twice that part increased by the other part, and the other factor is the other part.

Take 57, which is made up of the parts 50 and 7; now

$$\begin{aligned} 57^2 &= (50+7)\times(50+7) \\ &= 50\times 50+50\,.\,7+7\,.\,50+7\,.\,7 && (62) \\ &= 50^2+2\,.\,50\,.\,7+7\,.\,7 && (35) \\ &= 50^2+(2\,.\,50+7)\times 7; && (62) \end{aligned}$$

therefore $57^2-50^2=(2\,.\,50+7)\times 7.$

COR. When the part whose square is taken away is much greater than the other part, it follows from the above result that if we divide the remainder by twice the part whose square is taken away, the quotient will give a near approximation to the part left—and of course in excess.

170. *To extract the square root of a given number.*

Take 71289. Placing dots over the units' and every alternate figure, we see that there are 3 figures in the root, and that the first period is 7: and, since 7 lies between 4 and 9, the squares of 2 and 3, the root must lie between 200 and 300. Put down 200 as one part of the required root, and subtract its square from the given number, leaving 31289.

$$
\begin{array}{rrl}
 & \dot{7}1\dot{2}8\dot{9} & (\,200 \\
 & 40000 & \,\,60 \\
400 + 60 = 460) & 31289 & \,\,\,\,7 \\
60 & 27600 & \\
520 + 7 = 527) & 3689 & \\
 & 3689 &
\end{array}
$$

Divide the remainder 31289 by 2.200 or 400; the quotient is greater than 70 but less than 80; therefore the remaining part of the root is less than 80 (169). Let us try 70; but $(400+70)\times 70$ or 470.70 or 31900 is greater than 31289; therefore 70 is too great. Let us try 60; now $(400+60)\times 60 = 27600$, which is less than 31289; therefore the remaining part of the root lies between 60 and 70, and the required root lies between 260 and 270.

Subtract 460×60 or 27600 from 31289, leaving 3689: but we have now subtracted from the given number

$$200^2 + (2\,.\,200 + 60)\times 60 \text{ or } 260^2, \qquad (169)$$

we may therefore consider the root as consisting of two parts, one is 260, and the other still to be found is less than 10.

Divide the remainder 3689 by 2.260 or 520: the quotient is greater than 7 but less than 8; therefore the other part is less than 8. Let us try 7: now $(520+7)\times 7 = 3689$, and this subtracted from the remainder leaves 0; therefore 7 is the part required exactly, and therefore the root required is 267.

171. In the preceding example the given number was a *perfect square*, but the method is equally applicable if the number be not a perfect square; except that in the latter case the root can be found to a limited degree of accuracy only.

Find the square root of 5713.

```
                           5713·0000 ( 70
                           4900         5
          140+5=145 ) 813                ·5
                           725           ·08
      150+·5=150·5 ) 88·00             ·004
                           75·25
     151+·08=151·08 ) 12·7500
                           12·0864
151·16+·004=151·164 ) ·663600
                           ·604656
                           ·058944
```

Pointing the units' and each alternate figure, we find there are two periods, and that the first is 57; and since 57 lies between 49 and 64, the root lies between 70 and 80; therefore 70 gives the root correct within 10. Put down 70 as a part of the root, and subtract its square, leaving 813.

Divide 813 by 2 . 70 or 140: the quotient is greater than 5 but less than 6: therefore the remaining part of the root is less than 6 (169). Let us try 5: now $(140+5)\times 5=725$, which is less than 813; therefore the root lies between 75 and 76, and therefore 75 gives the root correctly within 1.

Subtract 725 and there remains 88; but we have now subtracted

$$70^2+(2\,.\,70+5)\times 5 \text{ or } 75^2, \qquad (169)$$

hence we may consider the square root of 5713 to consist of two parts, one of which is 75, and the other a part to be found, but which is less than 1.

Dividing the remainder 88 by 2 . 75 or 150: the quotient is greater than ·5 but less than ·6; therefore the remaining part of the root is less than ·6. Let us try ·5: now $(150+·5)\times ·5$ is 75·25, which is less than 88; therefore the root lies between 75·5 and 75·6, and therefore 75·5 gives the root correctly within ·1.

Proceeding in precisely the same way, the next step shews that 75·58 gives the root correctly within ·01, and the next that 75·584 gives it correctly within ·001; and continuing this process we can obtain the root to any degree of accuracy we please.

Let us now in these examples strike off all unnecessary ciphers, bring down the periods only when we actually require them, suppress the decimal point in all places except the root, and take the remainders and divisors for what they really are, and not for what they represent, and our work will stand thus:

71289 (267	5713·0000 (75·584
4	49
46) 312	145) 813
276	725
527) 3689	1505) 8800
3689	7525
	15108) 127500
	120864
	151164) 663600
	604656
	58944

172. We deduce then the following rule:

(1) *Place a dot over the units' figure, and over every alternate figure to its left, and to its right: if there be no units' figure, suppose a cipher placed there, and remember that each period consists of the figure over which the dot is placed and the figure to its* left.

(2) *Find the number whose square is immediately below the number in the first period: place that number in the root, and subtract its square from the first period.*

(3) *To the remainder bring down the next period, giving the first dividend; double the figure in the root, and see how often it is contained in the dividend when its last figure is omitted; set down the quotient as the second figure in the root and annex it to the trial divisor we have just used, giving the first divisor. Multiply this divisor by the second figure of the root, and if the product be not greater than the first dividend subtract it from the dividend, but if the product be greater, use a lower number for the root figure until it becomes less; subtract, and we thus get the second remainder.*

(4) *To this remainder bring down the next period: to the first divisor add its last figure, and see how often it is contained in the second dividend when its last figure is omitted; and proceed precisely as before.*

(5) *Continue the process till all the periods have been brought down: if there be no final remainder the given number is a perfect square, and its root is found; if there be a final remainder we have*

obtained the root not exactly but within a unit of its last decimal order, and the remainder shews by how much the square of the root obtained differs from the given number.

REMARK. If at any step the quotient figure is 0, set down 0 in the root, annex it to the trial divisor, bring down the next period and proceed as before.

Ex. Find the square root of ·08042896, and of 34·850.

(1) 0·08042896 (·2836	(2) 34·8500 (5·90338
4	25
48) 404	109) 985
384	981
563) 2028	11803) 40000
1689	35409
5666) 33996	118063) 459100
33996	354189
	1180668) 10491100
	9445344
	1045756

In Ex. (1) we put a cipher in the units' place, over which we placed the *first* point, and then over each alternate figure to the right; it is more usual simply *to suppose* the cipher to be put in the units' place.

In Ex. (2) we have found the square root, accurate within one hundred-thousandth, or ·00001; and the remainder shews that the square of the root obtained differs from the given number by ·0001045756.

173. When the number of figures to be found in the root is large, the work may be considerably contracted, as the following considerations and example will shew. When a certain number of figures have been found in the root, the divisor will contain at least the same number of figures, and the remainder either the same number or one less; and as the subsequent steps of the operation can alter only in a very slight degree the figures already obtained in the divisor and remainder, it follows that we may obtain by contracted division as many additional figures within one as we have already got: hence, when one more than half the required number of figures in the root has been obtained, we may cut off the last figure to the right in the divisor and proceed as in contracted division.

Example. Find to 12 places of decimals the square root of 3·141592653589.

Here we must have 13 figures in the root, therefore we find 7 in the usual way, and 6 by contraction.

```
               3·14,15,92,65,35,89 ( 1·772453850905
               1
         27 ) 214
              189
        347 ) 2515
              2429
       3542 ) 8692
              7084
      35444 ) 160865
              141776
     354485 ) 1908935
              1772425
    3544903 ) 13651089
              10634709
   35449068 ) 301638000
              283592544
  354490765 ) 1804545600
              1772453825
35449077009 ) 3209177750000
              3190416930 81
3544907701805 ) 18760569190000
                17724538509025
                 936030680975
```

```
               3·14,15,92,65,35,89 ( 1·772453
               1
         27 ) 214
              189
        347 ) 2515
              2429
       3542 ) 8692
              7084
      35444 ) 160865
              141776
     354485 ) 1908935
              1772425
    3544903 ) 13651089
              10634709
    3544906 ) 3016380 ( 850905
              2835925
               180455
               177245
                 3210
                 3190
                   20
                   18
                    2
```

174. In the preceding examples we have, for the sake of clearness, written down each product at full length, and then performed the subsequent subtraction; it will, however, both save time and be conducive to accuracy, if we combine the two operations (51). Thus, taking the last example, the work will appear as follows:

```
               3·14,15,92,65,35,89 ( 1·772453
27             214
347             2515
3542              8692
35444            160865
354485            1908935
3544903            13651089
3,5,4,4,9,0,6       3016380 (850905
                     180455
                       3210
                         20
                          2
```

FRACTIONS AND MIXED NUMBERS.

175. The square of a *fraction* is found by squaring its numr and denr: hence conversely the square root of a fraction is found by extracting the square root of its numr and denr.

(1) If the denr of the given fraction, or of the fractional part of the mixed number, be *a perfect square*, we apply the Rule directly, whether the numr be a perfect square or not, thus:

Ex. 1. $\sqrt{\frac{25}{64}} = \frac{\sqrt{25}}{\sqrt{64}} = \frac{5}{8}$.

Ex. 2. $\sqrt{8\frac{17}{64}} = \sqrt{\frac{529}{64}} = \frac{\sqrt{529}}{\sqrt{64}} = \frac{23}{8} = 2\frac{7}{8}$.

Ex. 3. $\sqrt{\frac{29}{64}} = \frac{\sqrt{29}}{\sqrt{64}} = \frac{5{\cdot}384164\ldots}{8} = {\cdot}673145\ldots$

Ex. 4. $\sqrt{33\frac{139}{144}} = \sqrt{\frac{4891}{144}} = \frac{69{\cdot}935684\ldots}{12} = 5{\cdot}827973\ldots$

(2) But if the denr of the given fraction or of the fractional part of the mixed number be *not a perfect square*, we reduce the fraction or the mixed number either

(1) to an equivalent fraction whose denr is a perfect square, and extract the square root of numr and denr, or

(2) to a decimal, and proceed in the ordinary way; thus:

$$\sqrt{\frac{8}{13}} = \sqrt{\frac{8\times 13}{13\times 13}} = \sqrt{\frac{104}{169}} = \frac{\sqrt{104}}{\sqrt{169}} = \frac{10{\cdot}198039\ldots}{13} = {\cdot}784464\ldots$$

or $= \sqrt{{\cdot}\dot{6}1538\dot{4}} = {\cdot}784464\ldots$

$$\sqrt{25\tfrac{8}{11}} = \sqrt{\frac{283}{11}} = \sqrt{\frac{283\times 11}{11\times 11}} = \sqrt{\frac{3113}{121}} = \frac{55{\cdot}794265\ldots}{11} = 5{\cdot}072205\ldots$$

or $= \sqrt{25{\cdot}\dot{7}\dot{2}} = 5{\cdot}072205\ldots$

EXERCISE 27.

Extract the square root of

1. 54756; 804609; 822649; 12809241; 97574884; 21224449.
2. 93·7024; ·128881; ·02819041; ·00822649; 3659·0401; ·49112064.
3. 5973303369; 32166944416; 7578747136; 5777216064; 6407522209.
4. 236·144689; 285970·396644; 400·54818769; ·00501361708761.
5. 41605·800625; 82606128544; 360117·609604; 938703069915 61.
6. 120888·68379025; 5783663477·05072081; 78702684186368O889.
7. [illegible]; 72002[illegible]; 32[illegible]; 564[illegible]; 3083[illegible]; 18·7; 3·361; 4738·027.

8. 24·2064, 3124·81 and 2·42064 × 312·481; ·004 × 15·625; $\frac{\cdot 00125}{\cdot 18}$.

9. Find within one ten-thousandth the square root of the following numbers, writing down in each case the difference between the square of the root obtained and the given number (172, 5):

20; ·1; 3·14159265; ·011; 175·250564; 12·566360.

10. Find to 7 places of decimals the square root of

·0068; $\frac{3}{5}$; $\frac{12}{13}$; $21\frac{5}{11}$; $27\frac{3}{14}$; $\frac{5\cdot 04}{\cdot 012}$; $\sqrt{\cdot 02} - \sqrt{\cdot 00\dot{2}}$.

11. Find to 11 places of decimals the square root of

·000190882; 9·79; 97·9; $\frac{16}{17}$; ·85; ·0683; ·3467; $\frac{17}{113}$.

Extract the square root of

12. 75·347; 3; and $\frac{5}{7}$; each to 14 places of decimals.

13. 57·57; ·00003532; and ·67; each to 17 places of decimals.

14. ·07; 1·77; and 50·420; each to 21 places of decimals.

CUBE ROOT.

176. In the same way as in square root (164, 165) we may shew that:

(1) *A number ending with ciphers, where the number of such ciphers is not a multiple of 3, cannot be the cube of a whole number.*

(2) *A number whose last figure is significant, where the number of decimal places is not a multiple of 3, cannot be the cube of any number, integral or decimal.*

(3) *If the cube root of a whole number be not a whole number, neither is it a fraction whose numerator and denominator are whole numbers.*

The cube root of such a number is said to be incommensurable.

(4) *The cube of any number consists of three times as many figures, or three times as many less 1 or less 2, as there are in the given number.*

(5) *If we divide a number into periods by placing dots over the units' figure and over every third figure to the left, the number of periods will give the number of figures in the root, and each period will consist of the figures over which the dot is placed and the two figures to its* left, *if they are so many.*

177. The extraction of the cube root of a number depends on the following proposition:

If from the cube of a number we subtract the cube of one part of it, the remainder is a product of two factors: one factor is the sum of 3 times the square of that part, 3 times the product of the

two parts, and the square of the other part; and the second factor is the other part.

Take 57, which is made up of the parts 50 and 7: its square is

$$50^2+2.50.7+7^2, \qquad (169)$$

and if we multiply each part of this sum separately by 50 and by 7, the sum will be equal to $57^2.57$ or 57^3.: hence

$$57^3=50^3+2.50^2.7+50.7^2+50^2.7+2.50.7^2+7^3$$
$$=50^3+3.50^2.7+3.50.7^2+7^3;$$

therefore $57^3-50^3=(3.50^2+3.50.7+7^2)\times 7.$ (62, 1)

COR. When the part whose cube is taken away is much greater than the other part, it will be seen from the above result that if we divide the remainder by 3 times the square of the part taken away, the quotient will give a near approximation to the part left—and of course in excess.

178. *To extract the cube root of a given number.*

(1) Take 94818816. Placing dots over the units' and every third figure, thus $9\dot{4}81\dot{8}81\dot{6}$, we see that there are 3 figures in the cube root, and that the first period is 94: and since 94 lies between 64 and 125, the cubes of 4 and 5, the cube root required must lie between 400 and 500. Put down 400 as one part of the root; and we must now find the other part, which is less than 100.

(2) We shall find it convenient to arrange our work in 3 columns, placing the given number in the third, thus:

I.	II.	III.	
400	160000	94818816 (	400
400	320000	64000000	50
800	480000	30818816	6
400	62500	27125000	
1200	542500	3693816	
50	65000	3693816	
1250	607500		
50	8136		
1300	615636		
50			
1350			
6			
1356			

Put 400 in col. i, multiply it by 400, giving 400^2, or 160000, which put in col. ii; multiply this by 400, giving 400^3 or 64000000, which place in col. iii and subtract, giving the remainder 30818816.

(3) Now add 400 to col. i, giving $2 \cdot 400$ or 800; multiply this by 400, giving $2 \cdot 400^2$ or 320000, which add to col. ii, giving $3 \cdot 400^2$ or 480000:—again, add 400 to col. i, giving $3 \cdot 400$ or 1200. Our three cols. now stand thus:

$$3 \cdot 400 \text{ or } 1200, \qquad 3 \cdot 400^2 \text{ or } 480000, \qquad 30818816.$$

Using the trial divisor $3 \cdot 400^2$ (177 Cor.), the quotient is greater than 60 but less than 70; therefore the other part of the root is *less* than 70. If we try 60, that is, if we multiply

$$3 \cdot 400^2 + 3 \cdot 400 \cdot 60 + 60^2$$

by 60, we shall find the result greater than the remainder. Let us then try 50: add 50 to col. i, giving $3 \cdot 400 + 50$ or 1250; multiply this by 50, giving $3 \cdot 400 \cdot 50 + 50^2$ or 62500, which add to col. ii, giving

$$3 \cdot 400^2 + 3 \cdot 400 \cdot 50 + 50^2 \text{ or } 542500;$$

multiply this by 50, giving

$$(3 \cdot 400^2 + 3 \cdot 400 \cdot 50 + 50^2) \times 50 \text{ or } 27125000,$$

which subtract from col. iii, giving the remainder 3693816.

But we have now subtracted

$$400^3 + (3 \cdot 400^2 + 3 \cdot 400 \cdot 50 + 50^2) \times 50 \text{ or } 450^3. \qquad (177)$$

We may therefore consider the root as made up of two parts, one of which is 450, and the other is to be found, and it is less than 10: hence we repeat the process of this paragraph (3).

(4) Add 50 to col. i, giving $3 \cdot 400 + 2 \cdot 50$ or 1300; multiply this by 50, giving $3 \cdot 400 \cdot 50 + 50^2$, which add to col. ii, giving

$$3 \cdot 400^2 + 6 \cdot 400 \cdot 50 + 3 \cdot 50^2,$$

$$\text{or } 3 \times (400^2 + 2 \cdot 400 \cdot 50 + 50^2) \text{ or } 3 \times 450^2; \qquad (177)$$

and again 50 to col. i, giving $3 \cdot 400 + 3 \cdot 50$ or $3 \cdot 450$ or 1350: hence our cols. stand thus:

$$3 \cdot 450 \text{ or } 1350, \qquad 3 \cdot 450^2 \text{ or } 607500, \qquad 3693816.$$

Using the trial divisor $3 \cdot 450^2$, the quotient is greater than 6 but less than 7: therefore the upper part of the root is less than 7. Let us try 6:—add 6 to col. i, giving $3 \cdot 450 + 6$ or 1356; multiply this by 6, giving $3 \cdot 450 \cdot 6 + 6^2$, which add to col. ii, giving

$$3 \cdot 450^2 + 3 \cdot 450 \cdot 6 + 6^2 \text{ or } 615636;$$

multiply this by 6, giving

$$(3 \cdot 450^2 + 3 \cdot 450 \cdot 6 + 6^2) \times 6 \text{ or } 3693816,$$

which subtract from col. iii, giving the remainder 0.

We have now subtracted from the given number

$$450^3+(3.450^2+3.450.6+6^2)\times 6 \text{ or } 456^3, \qquad (177)$$

and there is no remainder; hence the cube root of the given number is 456.

179. If the given number be not a perfect cube, the preceding method is equally applicable, but the root can only be found to a limited degree of accuracy. For example

Find the cube root of 34829.

I.	II.	III.	
30	900	34829·000	(30
30	1800	27000	2
60	2700	7829	·6
30	184	5768	·05
90	2884	2061	·007
2	188	1877·976	
92	3072	183·024000	
2	57·96	159·658625	
94	3129·96	23·365375000	
2	58·32	22·391272393	
96	3188·28	·974102607	
·6	4·8925		
96·6	3193·1725		
6	4·8950		
97·2	3198·0675		
·6	·685699		
97·8	3198·753199		
·05			
97·85			
·05			
97·90			
·05			
97·95			
·007			
97·957			

Point the units' and every third figure: the first period is 34, which lies between 27 and 64, the cubes of 3 and 4; hence the cube root of the given number lies between 30 and 40, and therefore 30 gives the cube root within 10.

Put 30 in col. i: multiply by 30, giving 900, which put in col. ii; multiply by 30, giving 27000, which put in col. iii and subtract, giving the remainder 7829. Complete cols. ii and i thus—add 30 to col. i, giving 60; multiply this by 30, giving 1800, which add to col. ii, giving 2700;

and now add 30 to col. i, giving 90:—therefore the cols. stand thus, 90, 2700, 7829.

Use the trial divisor 2700; the quotient is greater than 2 and less than 3: therefore the other part of the root must be less than 3. Let us try 2; the remainder will be 2061: therefore the root lies between 32 and 33, and therefore 32 gives the root within 1. Complete cols. ii and i, and the cols. will stand thus—96, 3072, 2061.

Use the trial divisor 3072: the quotient is greater than ·6 but less than ·7; therefore the other part of the root must be less than ·7. Let us try ·6: the remainder will be 183·024; therefore the root will lie between 32·6 and 32·7, and therefore 32·6 will give the root within ·1.

In like manner the next step shews that 32·65 gives the root within ·01: and the next that 32·657 gives the root within ·001: and continuing this process we can obtain the root to any degree of accuracy we please.

Let us now, in these examples, strike off all unnecessary ciphers, bring down the periods only when we actually require them, suppress the decimal point in all places except the root, and take the remainders and divisors for what they really are, and not for what they represent, and our work will stand thus:

I.	II.	III.	I.	II.	III.
4	16	94818816 (456	3	9	34829·000 (32·657
4	32	64	3	18	27
8	48	30818	6	27	7829
4	625	27125	3	184	5768
125	5425	3693816	92	2884	2061000
5	650	3693816	2	188	1877976
130	6075		94	3072	183024
5	8136		2	5796	159658625
1356	615636		966	312996	23365375
			6	5832	22391272393
			972	318828	974102607
			6	48925	
			9785	31931725	
			5	48950	
			9790	31980675	
			5	685699	
			97957	3198753199	

180. We deduce then the following Rule:—

(1) *Arrange for three columns, and head them respectively* i, ii, *and* iii; *in col.* iii *put the number whose cube root is to be extracted, and to its right provide a place for the root.*

(2) *Place a dot over the units' figure, and over every third figure to its left, and to its right if the number be a decimal: if there be no units' figure suppose a cipher placed there; and remember that each period consists of the figure over which the dot is placed and the two figures to its left, if there are so many.*

(3) *Find the number whose cube is immediately below the first period; place it in the root; place it also in col.* i; *multiply col.* i *by the root-figure, and place the product in col.* ii; *multiply col.* ii *by the root-figure and subtract the product from the first period in col.* iii.

(4) *To find the next figure in the root proceed thus:—complete the columns, that is, bring down the next period in col.* iii: *add the root-figure to col.* i; *multiply the sum by the root-figure and add the product to col.* ii; *add the root-figure to col.* i: *see how often the trial divisor (col.* ii) *is contained in col.* iii *when the last two figures are omitted; place the quotient in the root; bring it down to col.* i; *multiply col.* i *by the root-figure; put the product under col.* ii, *two places to the right, and add; multiply the sum by the root-figure, and place the product under col.* iii *and subtract, if the product be not too great:—if the product be too great, we must try the next lower figure in the root, or the next again, till we get a product small enough, and then subtract.*

(5) *Find the next and each succeeding figure in precisely the same way, and continue the process till all the periods have been brought down: if there be no final remainder, the given number is a perfect cube, and its cube root is found: if there be a final remainder, we have obtained the cube root within a certain degree of exactness only, determined by a unit of the last order in the root, and the remainder shews by how much the cube of the root obtained differs from the given number.*

REMARK 1. If at any step the root-figure is 0, bring down 0 to col i, and 00 to col. ii, and proceed as before.

REMARK 2. There can seldom be any doubt about the root-figure except at the *second* step; and here the numbers to be operated on are so small, that the process may be gone through *mentally* before the root-figure is actually put down.

181. When *one more than a third* of the figures required in the root have been obtained by the ordinary method, the rest may be found by contraction. For when a certain number of figures have been obtained in the root, there will be generally *twice* that number in the trial divisor and as many or as many less one in the remainder; and from this remainder as a dividend we can find as many root-figures as it contains figures, less one or less two at most. Instead therefore of bringing down new periods, we confine our work within the limits indicated by the remainder, and neglect those portions of col. ii and col. i which do not affect the figures we are seeking. The way in which this is done will be understood from the following example.

Ex. Find the cube root of ·78539816339745 to 12 places of decimals.

I.	II.	III.
9	81	·785398163397450 (·92263
9	162	729
18	243	56398
9	544	49688
272	24844	6710163
2	548	5089448
274	25392	1620715397
2	5524	1531147176
2762	2544714	89568221450
2	5528	76609659447
2764	2550252	12958562003 (507432201
2	165996	11768760950
27666	255191196	189801053
6	166032	178763634
27672	255357228	11037419
6	830349	10215066
276783	25536553149	822353
3	830358	766130
276786	25537383507	56223
3	13839	51075
276789	2553752190	5148
	13839	5107
	2553766029	41
	2	26
	25537662	15
	2	
	25537664	

The total number of figures in the root is to be 12, and since the integer next greater than $\frac{1}{3}$ of 12 is 5, we shall find the first 5 figures in the usual way, and the rest by contraction.

Arrange the 3 columns, and put the given number in col. iii; place a dot over the units' figure, and over every third figure to *its* right. The first period is 785, and therefore the first root-figure is 9. Put 9 in the root, prefixing the decimal point; put 9 also in col. i, multiply this 9 by 9, giving 81, which put in col. ii; multiply this 81 by 9, giving 729, which put under the first period in col. iii, and subtract, giving 56.

To find the second figure in the root, complete the columns: that is bring down the next period in col. iii, giving 56398; add 9 to col. i, giving 18; multiply this 18 by 9, giving 162, which add to col. ii, giving 243; add 9 to col. i, giving 27. See how often the trial divisor 243 is contained in 563, the quotient is 2: put 2 in the root, bring it down to col. i, giving 272; multiply this 272 by 2, and put the product 544 under col. ii, two places to the right, and add, giving 24844; multiply this by 2, and place the product 49688 under col. iii, and subtract, giving 6710.

In the same way find the third and each succeeding figure.

Suppose that we have now found five figures in the root, and completed col. i and ii, as if to find the 6th figure:—then the columns will be

276789   25537383507   12958562003

as they appear just *below* the broken line. As we do not intend to bring down another period in col. iii, we must cut off 1 figure in col. ii, and 2 figures in col. i, so that in multiplying up from one col. to the next, units of the same order may be under one another. Cut off therefore 7 in col. ii, and 89 in col. i. Use col. ii as a trial divisor, the quotient is 5; put 5 in the root, multiply col. i by 5, and add the product to col. ii; multiply col. ii by 5, and subtract the product from col. iii.

To find the next figure, complete the columns, which now means add the same number as before to col. ii. Cut off 9 from col. ii, and 67 from col. i. Use col. ii as a trial divisor, the quotient is 0; put 0 in the root, and the columns will remain unaltered. Again, cut off another figure in col. ii, namely 2, and the rest of the figures in col. i. Use col. ii as a trial divisor, the quotient is 7: and though there is no figure in col. i *to set down* from, yet from 7 times 2 plus 4 or from 18 we carry 1; add 1 to col. ii, multiply the sum by 2, and subtract the product from col. iii.

Again, add 1 to col. ii, completing it; cut off its last figure 4, and then the rest of the work is effected by contracted division only.

In this way the cube root of the given number is found to be ·92263507432201; which is correct to the last place of decimals.

182. In the preceding examples we have, for the sake of clearness, written down the products at full length, and then performed the subsequent addition or subtraction; it will however both save time and be conducive to accuracy, if we combine the two operations (51). Thus, taking the last example, the work will appear as follows:

I.	II.	III.
9	81	·783398163397456 (·92263
18	243	56398
272	24844	6710163
274	25392	1620715397
2762	2544724	89568221450
2764	2550252	11958561003 (507432101
27666	255191196	189801053
27672	255357228	11037419
276783	25536553149	822353
276786	25537383507	56223
276789	2553752190	5148
	255376602,9	41
	25537662	15
	25537664	

FRACTIONS AND MIXED NUMBERS.

183. The cube of a fraction is found by cubing its numerator and denominator; hence, conversely, the cube root of a fraction is found by extracting the cube root of its numr and denr.

(1) If the denominator of the given fraction, or of the fractional part of the given mixed number, be *a perfect cube*, we apply the Rule directly, whether the numr be a perfect cube or not; thus

Ex. 1. $\sqrt[3]{\frac{27}{64}}=\frac{\sqrt[3]{27}}{\sqrt[3]{64}}=\frac{3}{4}$.

Ex. 2. $\sqrt[3]{18\frac{26}{27}}=\sqrt[3]{\frac{512}{27}}=\frac{\sqrt[3]{512}}{\sqrt[3]{27}}=\frac{8}{3}=2\frac{2}{3}$.

Ex. 3. $\sqrt[3]{\frac{29}{64}}=\frac{\sqrt[3]{29}}{\sqrt[3]{64}}=\frac{3{\cdot}072317\ldots}{4}=\cdot768079\ldots$

Ex. 4. $\sqrt[3]{18\frac{500}{729}}=\sqrt[3]{\frac{13622}{729}}=\frac{\sqrt[3]{13622}}{\sqrt[3]{729}}=\frac{23{\cdot}882527\ldots}{9}=2{\cdot}653614\ldots$

(2) But if the denominator of the given fraction or of the fractional part of the given mixed number be *not a perfect cube*, we reduce the fraction or the mixed number either to

(1) an equivalent fraction whose denominator is a perfect cube, and extract the cube root of numerator and denominator: or to

(2) a decimal, and proceed in the ordinary way: thus

Ex. 5. $\sqrt[3]{\frac{5}{7}} = \sqrt[3]{\frac{5 \times 49}{7 \times 49}} = \frac{\sqrt[3]{245}}{\sqrt[3]{343}} = \frac{6\cdot2573248\ldots}{7} = \cdot8939035\ldots$

or $= \sqrt[3]{\cdot\dot{7}1428\dot{5}} = \cdot8939035\ldots$

Ex. 6. $\sqrt[3]{8\frac{5}{7}} = \sqrt[3]{\frac{61}{7}} = \sqrt[3]{\frac{2989}{343}} = \frac{14\cdot4048466\ldots}{7} = 2\cdot057\ 352\ldots$

or $\sqrt[3]{8\cdot\dot{7}1428\dot{5}} = 2\cdot0578352\ldots$

FOURTH ROOT, SIXTH ROOT, AND NINTH ROOT.

184. (1) Since $5^4 = (5^2)^2$ (66), we may find the fourth power of a number by squaring the number, and then squaring its square: hence, conversely, we may *extract the fourth root* of a number by extracting its square root, and then the square root of its square root.

(2) In like manner, since $5^6 = (5^2)^3$ and $= (5^3)^2$ (66), we may *extract the sixth root* of a number by extracting its cube root, and then the square root of its cube root; or by extracting its square root, and then the cube root of its square root.

(3) And since $5^8 = (5^4)^2 = \{(5^2)^2\}^2$ we may *extract the eighth root* of a number by extracting its square root, then the square root of its square root, and lastly the square root of that square root.

(4) Lastly, since $5^9 = (5^3)^3$, we may *extract the ninth root* of a number by extracting its cube root and then the cube root of its cube root.

EXERCISE 28.

Extract the cube root of

1. 912673; 12812904; 233·744896; ·007077888; ·000016730899.
2. ·014137569; 176464·0815290; ·081690010219; 355496768704.
3. ·056711623688; 2·222447625; 198767717056; 8452·26465300.

4. 7021212830 72; ·0977810365 43; 967·068262369; 513537536·5120.

5. 9316298194 1037; 7·986807258669; 179301192·791869.

6. 9713554096972 5504; 69653647767692748807928 9747.

7. $\frac{1728}{1331}$; $\frac{46656}{4913}$; 4[illegible]; 456[illegible]; 46[illegible]; 57[illegible]; 3845·296.

8. Find to 4 figures before contraction the cube root of

$$1034;\quad 5·912;\quad ·009968;\quad 5;\quad \frac{8}{13};\quad 7[illegible].$$

9. Find to 5 figures before contraction the cube root of

$$3·467;\quad 2·718281828 46;\quad \frac{3}{7};\quad ·07;\quad ·078759;\quad ·325142.$$

Extract the cube root of

10. ·78539; $18\frac{1}{7}$; ·001; and ·024, each to 14 places of decimals.

11. 2·187; ·003; 397·953; ·013; and 81·812703, each to 20 places.

12. $2+\sqrt{3}$; $2-\sqrt{3}$; $7+\sqrt{7}$, each to 13 places of decimals.

13. Extract the fourth root of

211309379856; 2·718281828 46; 435[illegible]; ·0008217; 62; 103[illegible].

14. Extract the sixth root of

$$260184053769595201;\quad 5368[illegible];\quad 75·347;\quad \frac{3}{7}.$$

15. Find the eighth root of 57[illegible]; ·003532; $\frac{5}{7}$.

16. Extract the ninth root of

$$·6898697810 56;\quad 3000;\quad \frac{8}{13};\quad 7+\sqrt{7};\quad 82[illegible].$$

17. Find the value of $\sqrt[3]{\frac{5·12}{33·75}}$; $\sqrt[3]{\frac{5·12}{·03375}}$; $\frac{\sqrt[3]{·512}+\sqrt[3]{·003375}}{\sqrt[3]{8}-\sqrt[3]{·001}}$.

18. Find the square root of 3, and then shew that the cube root of $26-15\sqrt{3}$ is equal to $2-\sqrt{3}$.

19. Divide the sum of the cube roots of 377149·515625 and 12771[illegible] by the square root of 5017[illegible].

20. In A.D. 1696, De Lagny, a Member of the Academy of Sciences, and afterwards a Fellow of the Royal Society, said that the most skilful computer could not in less than a month find within a unit the cube root of

$$696536483318640035073641037;$$

find the cube root (1) within a unit, and (2) to 15 places of decimals.

CHAPTER VIII.

RATIO AND PROPORTION.

185. RATIO is the relation which two quantities of the same kind bear to each other, with respect to the number of times that the first contains the other; thus, the ratio of 11 yards to 4 yards is determined by the number of times that 11 yards contains 4 yards, and therefore by the fraction $\frac{11}{4}$.

The ratio of 11 yards to 4 yards is written thus

$$11 \text{ yards} : 4 \text{ yards},$$

and is read 11 yards *to* 4 yards.

The two quantities must be of *the same kind* that the division may be possible, and the quotient will always be a number; thus, the ratio of 11 yards to 4 yards, of 11 hours to 4 hours, of 11 pence to 4 pence, are all determined by the number $\frac{11}{4}$: that is

$$11 \text{ yards} : 4 \text{ yards} = 11 \text{ hours} : 4 \text{ hours} = 11 \text{ pence} : 4 \text{ pence} = 11 : 4.$$

Hence in treating of ratios we usually consider the terms to be numbers: for at any time we can pass from quantities of the same kind to the numbers which measure them, and vice versa, whenever we find it necessary to do so.

186. The first term of a ratio is called the *Antecedent* and the second the *Consequent*. One ratio is said to be the *inverse* of another when the Antecedent and Consequent of the one are respectively the Consequent and Antecedent of the other:—thus the inverse ratio of 11 : 4 is the ratio of 4 : 11.

187. One ratio is *greater* or *less* than another according as the fraction determining the former is greater or less than the fraction determining the latter. Thus the ratio of 3 to 4 is greater than that of 5 to 7, because $\frac{3}{4}$ is greater than $\frac{5}{7}$.

188. As the value of a fraction is not altered by multiplying or dividing numerator and denominator by the same number, the value

of a ratio is not altered by multiplying or dividing both its terms by the same number: thus the ratio of 2 to 3 is the same as that of 4 to 6, of 12 to 18, of 20 to 30, &c.

189. The ratios that each of a set of numbers have to each other is expressed in the form of a continued ratio: thus the ratios that each of the numbers 3, 5, 7, 9 have to one another is written

$$3 : 5 : 7 : 9.$$

The values of these ratios will not be altered if we multiply or divide *all* the terms by the same number.

190. We *compound* ratios by multiplying the antecedents for a new antecedent, and the consequents for a new consequent: thus the ratio compounded of the ratios $2 : 3$, $5 : 7$ and $14 : 15$ is

$$2\times 5\times 14 : 3\times 7\times 15.$$

PROPORTION.

191. Four quantities are in proportion when the first has the same ratio to the second which the third has to the fourth. A proportion, therefore, is the equality of two ratios; thus, since $2 : 3$ and $10 : 15$ are equal ratios, we have the proportion

$$2 : 3 = 10 : 15,$$

which is read 2 to 3 equals 10 to 15.

This proportion is also written, as in Geometry, thus

$$2 : 3 :: 10 : 15,$$

and is then read 2 is to 3 as 10 is to 15.

Of this proportion 2 and 15 are called the *extremes*, and 3 and 10 the *means*: $2 : 3$ the *first* ratio, and $10 : 15$ the *second* ratio.

192. *In every proportion the product of the extremes is equal to the product of the means.*

Take the proportion $a : b = c : d$,

where a, b, c, d are numbers, either integral or fractional; then

$$\frac{a}{b} = \frac{c}{d},$$

therefore

$$\frac{a\times d}{b\times d} = \frac{b\times c}{b\times d} \qquad (139)$$

and since these fractions have a common denominator their numerators must be equal,

therefore $$a \times d = b \times c.$$

COR. Hence, $$d = \frac{b \times c}{a} \text{ and } a = \frac{b \times c}{d},$$

that is,—*Either extreme is equal to the product of the means divided by the other extreme.* And in like manner,—*Either mean is the product of the extremes divided by the other mean.*

REMARK. In any proportion we may replace the *numbers* of each ratio by quantities of which they are the measure, thus if

$$2 : 3 = 10 : 15,$$

we may have 2 men : 3 men = £10 : £15,

but we cannot apply the terms of this proportion to the results just obtained, for of course we cannot multiply a quantity by a quantity; but if the terms of one ratio be quantities and of the other numbers, we can apply them: thus if

$$2 : 3 = £10 : £15;$$

then $$£15 \times 2 = £10 \times 3; \text{ and } £15 = \frac{£10 \times 3}{2}.$$

193. *If four numbers are in proportion, they are in proportion when taken inversely.*

For example, if $a : b = c : d$, then inversely $b : a = d : c$.

Since $$a : b = c : d,$$

therefore $$b \times c = a \times d, \qquad (192)$$

and therefore $$\frac{b \times c}{a \times c} = \frac{a \times d}{a \times c},$$

or $$\frac{b}{a} = \frac{d}{c},$$

or $$b : a = d : c.$$

REMARK. This proposition holds whether the terms of both ratios are numbers, or the terms of one ratio be numbers and of the other quantities, or the terms of both ratios be quantities: thus if

2 men : 3 men = £10 : £15,

then 3 men : 2 men = £15 : £10.

193*. *If four numbers are in proportion, they are in proportion when taken alternately.*

For example, if $a : b = c : d$, then alternately $a : c = b : d$.

Since $\qquad \frac{a}{b} = \frac{c}{d}$,

therefore $\qquad a \times d = b \times c, \qquad (192)$

and therefore $\qquad \frac{a \times d}{c \times d} = \frac{b \times c}{c \times d}$,

or $\qquad \frac{a}{c} = \frac{b}{d}$,

or $\qquad a : c = b : d$.

REMARK. The result of this Article is applicable when the four terms of a given proportion are either all numbers or all quantities of the same kind: but not when the terms of each ratio are quantities of different kinds: for example, although

2 men : 3 men = £10 : £15,

we cannot have 2 men : £10 = 3 men : £15,

for each of the two ratios is impossible. But since

£2 : £3 = £10 : £15,

there is now no objection to our alternating the second and third terms, and therefore we have

£2 : £10 = £3 : £15.

194. *If we compound the corresponding ratios of two or more proportions the resulting ratios will form a proportion.*

For example, if $\qquad a : b = p : q,$
$\qquad c : d = q : r,$
$\qquad e : f = r : s,$

then $\qquad a \times c \times e : b \times d \times f = p \times q \times r : q \times r \times s$.

For $\qquad \frac{a}{b} = \frac{p}{q},\ \frac{c}{d} = \frac{q}{r}$ and $\frac{e}{f} = \frac{r}{s}$,

therefore $\qquad \frac{a}{b} \times \frac{c}{d} \times \frac{e}{f} = \frac{p}{q} \times \frac{q}{r} \times \frac{r}{s}$;

or $\qquad \frac{a \times c \times e}{b \times d \times f} = \frac{p \times q \times r}{q \times r \times s}, \qquad (129)$

or $\qquad a \times c \times e : b \times d \times f = p \times q \times r : q \times r \times s$.

COR. If in corresponding ratios the consequent of one be the antecedent of the next, the common terms may be made to disappear in the compound proportion: thus in the last proportion we may write

$$a \times c \times e : b \times d \times f = p : s. \qquad (188)$$

194*. When, in a proportion, the means are equal, each of them is called a *mean proportional* between the two extremes; thus, in the proportion

$$2 : 4 = 4 : 8,$$

4 is a mean proportional between 2 and 8.

And because $4 \times 4 = 2 \times 8$ or $4^2 = 2 \times 8$; therefore $4 = \sqrt{2 \times 8}$; (192) that is,—*A mean proportional between two numbers is the square root of their product.*

Ex. 1. Find a 4th proportional to $2\frac{2}{3}$, $3\frac{3}{4}$, and $4\frac{4}{5}$.

Now $\qquad 2\frac{2}{3} : 3\frac{3}{4} = 4\frac{4}{5}$: 4th proportional required;

$$\text{therefore 4th proportional} = \frac{3\frac{3}{4} \times 4\frac{4}{5}}{2\frac{2}{3}} = \frac{15}{4} \times \frac{3}{8} \times \frac{24}{5} = \frac{27}{4} = 6\tfrac{3}{4}.$$

Ex. 2. The ratio of A to B is $2 : 3$; and of B to C is $5 : 6$: find the ratio of A to C.

Now $$\frac{A}{B} = \frac{2}{3} \text{ and } \frac{B}{C} = \frac{5}{6};$$

$$\text{therefore } \frac{A}{B} \times \frac{B}{C} = \frac{2}{3} \times \frac{5}{6}, \text{ or } \frac{A}{C} = \frac{5}{9};$$

$$\text{or } A : C = 5 : 9.$$

Ex. 3. The ratio of A to B is $2 : 3$; of B to C is $5 : 6$; and of C to D is $7 : 8$: find the continued ratio of A, B, C and D.

$A : B = 2 : 3 = 70 : 105$, multiplying each term by 5×7;

$B : C = 5 : 6 = 105 : 126$;

$C : D = 7 : 8 = 126 : 144$;

therefore $A : B : C : D = 70 : 105 : 126 : 144$.

Ex. 4. A cask of 72 gallons consists of 11 parts brandy and 1 part water: how much water must be added that it may consist of 9 parts brandy and 1 water?

72 gallons is made up of 12 parts, therefore 1 part is 6 gallons and 11 parts is 66 gallons. But, while the brandy remains the same 66 gallons,

the water is to be increased so that the brandy is to the water as $9 : 1$;

$$\text{but } 9 : 1 = 66 : 7\tfrac{1}{3},$$

therefore $1\tfrac{1}{3}$ gallons of water must be added.

EXERCISE 29.

1. Simplify the ratios: $21 : 35$; $441 : 2401$; $\tfrac{3}{4} : \tfrac{5}{7}$; $3\tfrac{2}{3} : 5\tfrac{5}{6}$; $3\cdot16 : 7\cdot7\tfrac{3}{7}$; $1 : \tfrac{1}{2} : \tfrac{1}{3} : \tfrac{1}{4}$; $\tfrac{1}{2} : \tfrac{2}{3} : \tfrac{3}{4} : \tfrac{4}{5}$.

2. Express in its simplest form the ratio compounded of the ratios:

   $2 : 3$, $3 : 4$, $4 : 5$; $7 : 15$, $9 : 16$, $24 : 35$; $1\tfrac{2}{3} : 2\tfrac{1}{2}$; $2\tfrac{4}{17} : 7\tfrac{1}{5}$.

3. Of the following ratios which is greatest?

   $5 : 7$ or $7 : 9$; $8 : 15$ or $15 : 29$; $2\tfrac{5}{6} : 6\tfrac{4}{7}$ or $4\tfrac{1}{3} : 11\tfrac{2}{3}$.

4. Do the ratios $4\tfrac{1}{2} : 1\tfrac{3}{4}$ and $6\tfrac{1}{2} : 1\tfrac{3}{4}$ constitute a proportion?

5. Do the numbers $4\tfrac{1}{2}$, $\tfrac{1}{16}$, 13 and $\tfrac{1}{5}$ form a proportion? If not, find what the fourth number must be, so that a proportion may be formed.

6. What is the first term of the proportion of which the other three terms in order are 8, 9 and 12?

7. If one mean of a proportion is $5\tfrac{1}{3}$, and the two extremes are $2\tfrac{1}{4}$ and $7\tfrac{1}{2}$, what is the other mean?

8. Find a fourth proportional to

   $6\tfrac{2}{3}$, $4\tfrac{4}{7}$ and $13\tfrac{1}{3}$; $\cdot0004$, $1\cdot4$ and $\cdot02$; $\cdot1$, $\cdot01$ and $\cdot001$; $\cdot0625$, $3\cdot85$ and $12\cdot5$; 8351, $3\cdot69$ and $30\cdot57$, within $\cdot0001$.

9. Find a mean proportional to

$3\tfrac{1}{3}$ and $10\tfrac{2}{3}$; $\cdot1$ and $\cdot001$; $387\cdot908$ and $\cdot0187$ to 4 places of decimals.

10. Compare the rates of two locomotives, one of which travels $397\tfrac{3}{8}$ miles in $11\tfrac{5}{8}$ hours, and another which travels $263\tfrac{1}{12}$ miles in $8\tfrac{3}{8}$ hours.

11. If, when A makes a profit of £2, B makes £3; and when B makes a profit of £4, C makes £5; and when C makes a profit of £6, D makes £7: compare the profits of A, B, C and D.

12. One grocer to 19 lbs. of coffee adds 5 lbs. of chicory, and another to 27 lbs. of coffee adds 7 lbs. of chicory: compare the amount of coffee in the two mixtures.

13. A mixture is composed of 9 parts brandy and 1 water; 4 gallons of water are added, and the mixture contains 6 times as much brandy as water: how many gallons of brandy does it contain?

14. A barrel consists of 3 parts ale to 2 parts stout; how much of the mixture must be drawn off and replaced by stout, that the new mixture may be half and half?

15. A greyhound pursues a hare and takes 3 leaps for every 4 leaps of the hare, but 2 leaps of the hound are equal to 3 of the hare: compare the rates of hound and hare.

CHAPTER IX.

CONCRETE NUMBERS.

TABLES OF TIME, LENGTH, &c.

195. If one quantity contains another of the same kind an exact number of times, the first is said to be a *multiple* of the second, and the second a *submultiple* or *aliquot part* of the first.

196. Of quantities of the same kind we take an arbitrary but well-defined quantity of that kind as our *unit*, and finding how many times or parts of a time it is contained in each of them, we express them either as whole numbers or as fractions. But in this way very large quantities will be expressed by very high numbers, and very small quantities by fractions, which give by inspection little idea of their relative values; to obviate this inconvenience, we take such multiples and submultiples of the unit as will enable us to avoid very high numbers in one case, and troublesome fractions in the other. Thus of length, we take a yard as our unit, but to measure long lengths we use the mile, a high multiple of the yard, and to measure short lengths we use the inch, a small submultiple.

But in England, not only do these multiples and submultiples proceed, in any particular case, without uniformity, but the relation observed between them in any one case, is no guide in any other; thus the relation between the multiples and submultiples of the unit of *length*, is not that which obtains between those of *weight*, or of *time*, or of *capacity*, &c.

197. The principal quantities with which we shall be concerned are those of *length*, *surface*, *volume*, *weight*, *capacity*, *time*, and *money*. And as in England the unit of time is the only one derived from a fixed quantity in nature, we shall begin with the measures of time.

TIME.

198. The unit of time is the DAY, or, strictly speaking, the *mean solar day*. A *solar day* is the interval between two successive noons; but as these intervals are of unequal length, we take the mean or average of all the solar days in the year, and to this *mean solar day* we give in civil reckonings the name of DAY. The submultiples of the day are the *hour*, the *minute* and the *second*; and its multiples are the *week*, the *month*, and the *year*: their relations are set forth in the following table :

60 *seconds* (s.)	are	1 *minute* (m.).
60 *minutes*		1 *hour* (h.).
24 *hours*		1 DAY.
7 *days*		1 *week*.
4 *weeks*		1 *month*.
365 *days* or 366 *days* . .		1 *year*.

199. The solar year consists of 365·242242 mean solar days, or very nearly $365\frac{1}{4}$ days: hence to make the civil year correspond with the solar, we take three consecutive years of 365 days, and a fourth, called *leap-year*, of 366 days, those being leap-years of which the number is divisible by four; and this is called the Julian correction, after Julius Cæsar. But in this way we insert 100 days in 400 years, which is too much, for $·242242 \times 400$ is 96·8968 or 97 days nearly; to make the necessary correction, those years whose *centuries* are not divisible by four are not leap-years; thus 1700, 1800, 1900 are *not* leap-years, but 2000 is a leap-year; and this is called the Gregorian correction, after Pope Gregory XIII.

The civil year is divided into 12 *calendar* months, of which February contains 28, or in leap-year, 29 days: April, June, September, November, 30 days; and each of the rest 31 days.

LENGTH.

200. The unit of length is the YARD. In the Office of the Warden of the Standards at Westminster there is a solid bar of bronze 38 inches long and 1 inch square; near to each end a small cylindrical hole is sunk in which is inserted a gold plug; and the

distance between the centres of the gold plugs when the bronze is at a temperature of 62° F. is the imperial standard yard (41 and 42 Vict. c. 49, s. 10).

201. By the Act, 5 Geo. IV. c. 74, the imperial standard yard was declared to be the length of the pendulum, vibrating seconds of mean time in the latitude of London in vacuum at the level of the sea, in the proportion of 36 to 39·1393. In this way, it was enacted that a new standard yard should be constructed, if at any time it should be required; but when the old standard was rendered useless by the burning down of the Houses of Parliament, and its restoration rendered necessary, so much doubt was thrown by men of science as to how far the standard could be accurately restored by the above method, that the present standard was constructed from a comparison of copies that had been carefully made of the old standard (18 and 19 Vict. c. 72).

202. The multiples and submultiples of the yard, and their relation to each other, are set forth in the following table:—

12 *inches* (in.)	are 1 *foot* (ft.).
3 *feet*	1 YARD (yd.).
5½ *yards*	1 *rod* or *pole*.
4 *poles* or 22 *yards* . . .	1 *chain*.
10 *chains*	1 *furlong*.
8 *furlongs* or 1760 *yards* .	1 *mile*.

The following special measures are also used:—4 *inches* are 1 *hand* (used in measuring the height of horses); 6 *feet* are 1 *fathom*; and 100 *links* are 1 *chain* or 22 *yards*.

Linen and woollen drapers divide the YARD into *quarters*, and each quarter again into quarters called sixteenths or *nails*; thus 2¼ *inches* are 1 sixteenth or *nail*, and 4 *nails* or 9 inches are 1 *quarter* of a yard. Sometimes 5 quarters of a yard is called an *ell*.

SURFACE.

203. The unit of surface is a *square yard*; that is a square each of whose sides is a yard in length.

Take such a square and divide each of two adjacent sides into 3 equal parts, and through the points of division draw straight

lines parallel to the sides; we have then 3×3 or 9 squares, each of whose sides is a foot in length; that is 1 square yard is equal to 9 square feet. In like manner 1 square foot is equal to 12×12 or 144 square inches, &c.

The following is the Table of Surface:

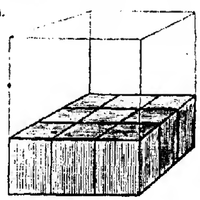
144 *square inches* . .	are 1 *sq. foot.*	
9 *sq. feet*	1 *sq. yard.*	
$30\frac{1}{4}$ *sq. yards* or $272\frac{1}{4}$ *sq. feet*	1 *sq. rod* or *sq. pole*, or	
	1 *perch*,	used in measuring land.
40 *perches*	1 *rood.*	
4 *roods*	1 *acre.*	
640 *acres* or 1760^2 *sq. yards*	1 *sq. mile.*	

204. Since 22 yards are 1 chain, 22×22 or 484 sq. yards are 1 sq. chain, and 10 sq. chains or 4840 sq. yards are 1 acre. Also since 100 links are 1 chain—

100^2 or 10,000 *sq. links* . . are 1 *sq. chain.*
and 10 *sq. chains* or 100,000 *sq. links* 1 *acre.*

VOLUME OF SOLIDITY.

205. The unit of volume is a *cubic yard*; that is a cube, each of whose sides is a yard in length.



Take such a cube, and divide each of three adjacent sides into 3 equal parts, and through each of the points of division draw planes parallel to the sides, then we have $3 \times 3 \times 3$ or 27 cubes, each of whose sides is 1 foot in length: that is 1 cubic yard is equal to 27 cubic feet. In like manner 1 cubic foot is equal to 12^3 or 1728 cubic inches, &c. Hence we have the following Table of Volume:—

1728 *cubic inches* . . .	are	1 *cu. foot.*
27 *cu. feet*		1 *cu. yard.*
1760^3 *cu. yards* . . .		1 *cu. mile.*

WEIGHT.

206. The unit of weight is the POUND; formerly called the Pound *Avoirdupois* to distinguish it from the Pound *Troy;* but as the Pound Troy has now no legal existence, the epithet Avoirdupois is unnecessary. In the office of the Warden of the Standards at Westminster there is deposited a cylinder of platinum marked P. S. 1844, 1 lb., and this weight is the imperial standard pound, and is the *only* unit or standard measure of weight from which all other weights, and all measures having reference to weight, are derived (41 & 42 Vict. c. 49, s. 13).

One sixteenth part of the Pound is an *ounce*, and one seven-thousandth part of the Pound is a *grain;* therefore 7000 grains is 16 ounces and $437\frac{1}{2}$ grains is an ounce. Also 480 grains is an *ounce troy* (41 & 42 Vict. c. 49, s. 14).

207. The multiples and submultiples of the Pound, and their relation to each other, are set forth in the following Table:—

16 *drams* or $437\frac{1}{2}$ *grs.*	are	1 *ounce* (oz.).
480 *grains* . .		1 *ounce troy* (oz. tr.).
16 *ounces* or 7000 *grs.*		1 POUND (lb.; *libra*).
14 *pounds* . .		1 *stone.*
28 *pounds* . . .		1 *quarter* (of a hundred-weight).
4 *quarters* or 112 *lbs.*		1 *hundred-weight* (cwt. = C, wt.).
20 *hundred-weight* .		1 *ton.*

208. All articles sold by weight are sold by imperial standard weight; except that

(1) Gold and silver, and articles made thereof, also platinum, diamonds, and other precious metals or stones, *may be* sold by the *ounce troy* or by any decimal parts of such ounce; and

(2) Drugs, when sold by *retail*, *may be* sold by a peculiar subdivision of the *troy ounce*, called Apothecaries' Weight (41 & 42 Vict. c. 49, s. 20).

209. As many questions in Arithmetical papers have reference to Troy weight, although with the exception of the Troy ounce it has ceased to exist, it is necessary for us to give the Table:—

24 *grains* . . .	are	1 *penny-weight* (dwt. = d, wt.).
20 *dwts.* or 480 *grains* .		1 *ounce troy* (oz. tr.).
12 *ounces tr.* or 5760 *grs.* .		1 *pound troy* (lb. tr.).

APOTHECARIES' WEIGHT.

20 *grains*	are	1 *scruple* (℈).
3 *scruples* or 60 *grs.* . .		1 *drachm* (ʒ).
8 *drachms* or 480 *grs.* .		1 *ounce troy* (℥).

CAPACITY.

210. The unit of capacity is the GALLON: and is the only standard measure of capacity from which all other measures of capacity, as well for liquids as for dry goods, are derived. The Gallon is defined to contain 10 lbs. weight of distilled water (41 & 42 Vict. c. 49, s. 15):—and as 1 cubic inch of distilled water weighs 252·458 grains, it follows that 10 lbs. or 70,000 grains of water, will fill 70,000 ÷ 252·458 or 277·274 cubic inches.

The GALLON its multiples and submultiples are used to measure liquids, and some dry goods, as grain, fruit, &c., according to the following Table:—

4 *gills* . . .	are	1 *pint.*	
2 *pints* . . .		1 *quart* (*or* quarter *of a gallon*).	
4 *quarts* . .		1 GALLON.	
2 *gallons* . . .		1 *peck.*	for *dry* goods only.
4 *pecks* or 8 *gallons*		1 *bushel.*	
8 *bushels* . . .		1 *quarter* (*of a ton*).	

211. *Apothecaries' Fluid Measure* is founded on the fact that

A pint of pure water, Weighs a pound and a quarter—

or 20 ounces. The following is the Table:—

60 *minims* (minima) .	are	1 *fluid drachm* (fl. dr.).
8 *fluid drachms* .		1 *fluid ounce* (fl. oz.).
20 *fluid ounces* . .		1 *pint* (O; *Octarius*).
8 *pints*		1 GALLON (C; *Congius*).

MONEY.

212. The unit of money is THE POUND or SOVEREIGN. The sovereign is made of standard gold, which is composed of 11 parts pure gold and 1 part alloy : 480 oz. troy of this metal are coined into 1869 sovereigns, so that a sovereign contains 123·274... grains of standard gold. The other *gold* coin is the half-sovereign.

The SHILLING is made of standard silver, which is composed of 37 parts pure silver and 3 parts alloy; 12 oz. troy of this metal is coined into 66 shillings, so that a shilling contains $87\frac{3}{11}$ grains of standard silver. The other silver coins are—the crown or five-shilling piece, the half-crown, the florin or two-shilling piece, the sixpence, the four-pence and the three-pence, which are respectively the half, the third and the quarter of a shilling. Though the four-pence is still in circulation, it has ceased to be coined for some time.

The PENNY is a bronze or copper coin made of metal which is composed of 95 parts copper, 4 tin, and 1 zinc; 1 lb. of this metal is coined into 48 pence, so that a penny weighs $\frac{1}{3}$ oz. The other copper coins are the half-penny, and the farthing or quarter-penny.

The following is the Table of Money:—

2 *farthings* . .	are	1 *half-penny*.
4 *farthings* or 2 *half-pence*		1 *penny*.
12 *pence*		1 *shilling*.
20 *shillings*		1 POUND or SOVEREIGN.

213. Pounds, shillings and pence are represented by £, *s*, and *d*; the initials of the Latin words *Libra*, *solidus*, and *denarius*. Farthings are written as fractions of a penny, and any lower sum is expressed as a fraction of a farthing, just as if the farthing were written as an integer; thus 2 shillings and 7 pence 3 farthings is written 2*s*. $7\frac{3}{4}$*d*.; and 2 shillings and 7 pence 3 farthings and $\frac{6}{18}$ of a farthing is written 2*s*. $7\frac{3}{4}\frac{6}{18}$.

The guinea is no longer in circulation; but we sometimes give the name to 21 shillings or £1. 1*s*.

214. A sovereign is of equal value with the metal composing it; for a person may take gold to the Mint, and on having it reduced

to standard gold (212) may have delivered to him an equal weight of gold coin, free of charge; whereas a shilling passes for more than its intrinsic value, and a penny for many times its intrinsic value. For this reason gold coin is a legal tender to any amount; but silver coin only up to 40 shillings, and copper coin only up to 12 pence.

PROPOSED DECIMAL COINAGE.

215. The following decimal system of coinage has been recommended by a Committee of the House of Commons, and has received the general assent of those arithmeticians and men of science who wish to see the introduction of a decimal system into England; there is little doubt, therefore, that should a change take place, this system will be adopted and sanctioned by Parliament.

In it, the *pound* or *sovereign* as at present defined will still be our *unit* of money: its submultiples will follow the decimal system and take the names of *florin*, *cent* and *mil*; so that

10 *mils* (*m.*) = 1 *cent* (*c.*).
10 *cents* = 1 *florin* (*f.*).
10 *florins* = 1 POUND or SOVEREIGN (£).

Hence £25. 7*f.* 8*c.* 9*m.* may be written £25·789 (140), and may be operated on as easily as the decimal 25·789.

The gold coins will remain as at present, the sovereign and the half-sovereign.

The silver coins will be the double florin, the florin, half-florin, 2 cents ($4\frac{4}{5}d.$) and 1 cent ($2\frac{2}{5}d.$).

The copper coins will be the 5 mils ($1\frac{1}{5}d.$), 2 mils ($\frac{24}{25}$ of $\frac{1}{2}d.$) and 1 mil ($\frac{24}{25}$ of $\frac{1}{4}d.$): so that the present half-penny and farthing may remain in circulation with their values depreciated 4 per cent.

216. The great drawback to the proposed system is that no sum of money of the present coinage, except a multiple of 6*d.*, can be paid *exactly* in the new coinage, for 1*d.* is $4\frac{1}{6}m.$; hence a fare, a toll, or a stamp of 1*d.*, must be paid for either by 4*m.* or 5*m.*, that is either by $\frac{1}{6}m.$ too little or by $\frac{5}{6}m.$ too much; or 2*d.* either by 8*m.* or 9*m.*, that is either by $\frac{1}{3}m.$ too little or $\frac{2}{3}m.$ too much, &c.; and therefore in the *transition* from the present to the proposed system there must be considerable inconvenience.

UNITED STATES.

217. The weights and measures of the United States differ from those of England in the following particulars only: (1) Their *cwt.* is really 100 lbs., and their *ton* 2000 lbs. (2) Their *unit of liquid measure* is our late *wine gallon*, and is about $\frac{5}{6}$ of our *standard gallon*. (3) Their *unit of dry measure* is our late *Winchester bushel*, and is about $\frac{32}{33}$ of our standard bushel. (4) Their money is altogether different from ours.

218. *Money.* The unit of money is the DOLLAR; and its submultiples follow the decimal system according to this Table:—

10 *mills* (*m.*)	= 1 *cent* (*ct.*).
10 *cents*	= 1 *dime* (*d.*).
10 *dimes*	= 1 DOLLAR (\$).
10 *dollars*	= 1 *eagle* (E).

Dimes are usually expressed as tens of cents, and mills are either neglected or expressed as fractions of a cent, so that accounts are kept in dollars and cents only: thus, 36·645 dollars is not read 3E. 6\$. 6*d.* 4*cts.* 5*m.* but 36\$. 64½*cts.*

The *gold* coins are composed of 9 parts pure gold and 1 part alloy; and are the eagle, weighing 258 grains, the half-eagle, &c.

The *silver* coins are composed of 9 parts pure silver and 1 part copper; and are the DOLLAR, weighing 412½ grains, the half-dollar, &c.

The *copper* or *bronze* coins are the cent and the half-cent.

The dollar is worth 4*s.* 2*d.* nearly, and therefore the cent ½*d.* nearly.

THE METRIC SYSTEM.

219. Formerly there existed in France the same want of uniformity in forming the multiples and submultiples of the units of weights and measures as exists at the present time in England; but soon after the Revolution of 1789 a Commission was nominated consisting of Borda, Condorcet, Monge, Lagrange and Laplace, for the purpose of preparing a new system of weights and measures. The system they recommended was established by the Legislature in 1801, and now prevails in almost its entirety, throughout the whole of France, under the name of *the metric system.* Since then

it has been introduced into Belgium, Holland and Switzerland; and to a greater or less extent into Italy, Germany, the United States and England.

In the formation of the multiples and submultiples the decimal system is followed exclusively; the Greek prefixes to any unit denoting multiples and the Latin prefixes denoting submultiples: thus, the metre being taken as the unit of length,

its multiples are			and its submultiples are		
deca-metre, denoting	10	metres	deci-metre, denoting	$\frac{1}{10}$	metre
hecto-metre „	100	„	centi-metre „	$\frac{1}{100}$	„
kilo-metre „	1000	„	milli-metre „	$\frac{1}{1000}$	„
myria-metre „	10,000	„			

The multiples of the myriametre and the submultiples of the millimetre have received no special name.

220. *Length.* The unit of length is the METRE:—it is also the *fundamental* unit, because from it every other unit of weight or measure is derived; and hence the name *metric* system. A metre is defined to be the ten-millionth part of the distance from the pole to the equator measured along the surface of the ocean; but is, in reality, the length of a rod of platinum, deposited in the Archives of the State, which is called the *standard metre*, and gives the legal length at the temperature of melting ice.

221. *Surface.* The unit of surface is the *square metre.* From Art. 195 we see that—

100 *sq. millimetres* = 1 *sq. centimetre.*
100 *sq. centimetres* = 1 *sq. decimetre.*
100 *sq. decimetres* = 1 *sq. metre.*
100 *sq. metres* = 1 *sq. decametre, &c.;*

hence 735·55069 sq. metres is 7 sq. decametres, 35 sq. metres, 55 sq. decimetres, 6 sq. centimetres, and 90 sq. millimetres.

In measuring land the square decametre is called the *are*, and the only multiple and submultiple are the *hectare* (or square hectometre) and *centiare* (or square metre): thus an estate may contain 67834·09 ares, or 678 hectares, 34 ares and 9 centiares.

The surface of a country is expressed in square kilometres.

222. *Volume.* The unit of volume is the *cubic metre.* From Art. 197 we see that

1000 *cu. millimetres* = 1 *cu. centimetre.*
1000 *cu. centimetres* = 1 *cu. decimetre.*
1000 *cu. decimetres* = 1 *cu. metre.*
1000 *cu. metres* = 1 *cu. decametre, &c.;*

but the multiples of the cubic metre are seldom used: hence 3456·034259963 cu. metres is 3456 cu. metres, 34 cu. decimetres, 259 cu. centimetres and 630 millimetres.

In measuring wood the cu. metre takes the name of *stère.*

223. *Capacity.* The unit of capacity is the *litre;* it is the capacity of a *cubic decimetre,* and therefore the capacity of the *kilolitre* is that of a *cubic metre.*

224. *Weight.* The unit of weight is the *gramme:* it is the weight *in vacuo* of a cubic centimetre of distilled water at 4° C. that is at its greatest density. Hence the *kilogramme* is the weight of a cubic decimetre of such water, that is of a *litre;* and 1000 kilogrammes, called a *millier* or *tonneau de mer,* is the weight of a cubic metre of such water, that is of a *kilolitre.*

But in reality a kilogramme is the weight of a cylinder of platinum whose height is equal to its diameter, deposited in the Archives of the State, and called the *legal standard.*

225. *Money.* The unit of money is the *franc:* it is a coin weighing 5 grammes, and is composed of 9 parts pure silver and 1 part copper.

The submultiples of the franc are the *decime* and the *centime,* which are respectively the tenth and the hundredth of the franc: so that

10 *centimes* = 1 *decime.*
10 *decimes* or 100 *centimes* = 1 *franc.*

Accounts however are kept in francs and centimes only, so that 76·85 francs is read 76 francs 85 centimes.

The *gold* coins are composed of 9 parts pure gold and 1 part alloy, and are the 20-franc piece, the 10-franc piece and 5-franc piece. The legal value of gold is 15½ times that of silver, and as

the franc weighs 5 grammes, the weight of the 20-franc piece $= 100$ grammes $\div 15\frac{1}{2} = 6.45161$ grammes.

The *silver* coins are composed of 9 parts pure silver and 1 part copper; and are those of 5 francs, 2 francs, 1 franc, 50 centimes and 20 centimes.

The *copper* or *bronze* coins are composed of 95 parts copper, 4 tin and 1 zinc; and are those of 10, 5, 2 and 1 centime.

226. We will now give the principal weights and measures of the metric system with their equivalents in the English system, and from them all the others may be easily derived.

MEASURES OF LENGTH.

Myriametre	= 10,000 metres	=	6·21382 miles.
Kilometre	= 1,000 ...	=	1093·63306 yards.
Metre	= 1 ...	=	1·09363 ...
			39·37079 inches.

Hence a *kilometre* is about $\frac{5}{8}$ mile and a *metre* about $1\frac{1}{11}$ of a yard.

LAND MEASURE.

Hectare	= 1 sq. hectometre	= 10,000 sq. metres	=	2·47114 acres.
Are	= 1 sq. decametre	= 100 ...	=	119·60331 sq. yards.
Centiare		= 1 ...	=	1·19603 ...

Hence a *hectare* is about $2\frac{1}{2}$ acres and an *are* about 4 perches.

MEASURES OF CAPACITY.

Kilolitre	*= 1000 litres*	*= 1 cu. metre*	=	*220·09668 gallons.*
Litre	= 1 ,,	= 1 cu. decimetre	=	·22010 ...
				1·76077 pints.

Hence a *litre* is about $\frac{7}{8}$ quart or $1\frac{3}{4}$ pints.

WEIGHTS.

Millier or Tonneau de mer	= 1,000,000 grammes	= 1 cu. metre of distilled water	=	19·68412 cwts.
			=	2204·6212 lbs. Av.
Quintal	= 100,000	= $\frac{1}{10}$ ...	=	220·46212 ...
Kilogramme	= 1000	= 1 cu. decim.	=	2·20462 ...
				15432·3487 grains.
Gramme	= 1	= 1 cu. centim.	=	15·43235 ...

Hence a *tonneau de mer* is a little less than a ton, and a *kilogramme* about $2\frac{1}{5}$ lb. Av.

CHAPTER X.

REDUCTION AND THE COMPOUND RULES.—INTEGERS.

REDUCTION.

227. THE unit of any quantity, its multiples and submultiples, are called its *denominations*: thus, of length, the mile, furlong, yard, foot, &c. are its various denominations.

The highest multiple and the lowest submultiple are called respectively the highest and lowest denomination of the quantity: thus, of length, the highest denomination is the mile, and the lowest the inch.

When a quantity is expressed in one denomination only it is called a *simple* quantity—as 7 yards; £5; 36 gallons.

When a quantity is expressed in several denominations it is called a *compound* quantity—as £7. 9*s*. $2\frac{3}{4}$*d*.; 15 days 17 h. 25 m. 37 s.

228. REDUCTION is the process by which we express (1) a simple or a compound quantity in terms of its lower denominations, or (2) a simple quantity in terms of its higher denominations.

229. I. *To express a quantity in terms of its lower denominations.* Descending Reduction.

For example, express £47. 14. $6\frac{3}{4}$ in farthings.

$$\begin{array}{r} \text{£.}\ \ s.\ \ d. \\ \underline{47\,.\,14\,.\,6\tfrac{3}{4}} \\ \underline{20} \\ 954 \\ \underline{12} \\ 11454 \\ \underline{4} \\ 45819 \text{ farthings.} \end{array} \qquad \begin{array}{l} \text{£.}\ \ s.\ \ d. \\ \underline{47\,.\,14\,.\,6\tfrac{3}{4}} \\ \underline{954} \times 12 \\ \underline{11454} \times 4 \\ 45819 \text{ farthings.} \end{array}$$

Since £1 is 20*s*., £47 is 47 × 20*s*. or 940*s*., and £47. 14*s*. is 940 + 14 or 954*s*.; that is, we multiply 47 by 20 and add 14. Again since 1*s*. is 12*d*., 954*s*. is 954 × 12*d*. or 11448*d*., and 954*s*. 6*d*. is 11448 + 6 or 11454*d*.; that is, we multiply 954 by 12 and add 6.

In like manner to reduce to farthings, we multiply 11454 by 4 and add 3, giving 45819*f*.; hence

$$\text{£}47.\ 14.\ 6\tfrac{3}{4} = 45819f.$$

And as a process similar to the above may be pursued in all cases of descending Reduction, we have the following Rule:

Multiply the number in the highest denomination by the number which tells how many of the next lower denomination makes one of the highest, and add in to the product the given number of that lower denomination (if any), and continue the process till we come to the denomination required.

230. II. *To express a simple quantity in terms of its higher denominations.* Ascending Reduction.

For example, reduce 45819 farthings to pounds.

$$\begin{array}{r} 4\,)\,\underline{45819\quad} \\ 12\,)\,\underline{11454\frac{3}{4}} \\ 20\,)\,\underline{95,4.\ 6\frac{3}{4}} \\ £47.\ 14.\ 6\frac{3}{4} \end{array}$$

Since 4*f*. is 1*d*. there are as many pence in 45819*f*. as 45819 contains 4; but 45819 contains 4, 11454 times and 3 over; therefore 45819*f*. is $11454\frac{3}{4}$*d*.

Again, since 12*d*. is 1*s*. there are as many shillings in 11454*d*. as 11454 contains 12: but 11454 contains 12, 954 times and 6 over; therefore 11454*d*. is 954*s*. 6*d*. and 45819*f*. is 954*s*. $6\frac{3}{4}$*d*.

Lastly, since 20*s*. is £1 there are as many pounds in 954*s*. as 954 contains 20: but 954 contains 20, 47 times and 14 over (53); therefore 954*s*. is £47. 14*s*. and 45819*f*. is £47. 14*s*. $6\frac{3}{4}$*d*.

And as a process similar to the above may be pursued in all cases of ascending Reduction, we have the following Rule:

Divide the number in the given denomination by the number which tells how many of this denomination make one of the next higher denomination, setting down the remainder as of the same denomination as its dividend: and continue this process till we come to the denomination required.

231. *Proof.* Reduction descending and ascending are inverse processes; if therefore we perform one process on a given quantity, and on the result the other process, we ought to get the original quantity. Thus, if by the descending process we find that £47. 14. $6\frac{3}{4}$ is 45819*f*., we ought by the ascending process to find that 45819*f*. is £47. 14. $6\frac{3}{4}$.

Hence every example of Reduction with its Proof is an example of both processes, and gives at the same time a guarantee for the correctness of the work.

Ex. 1. Reduce 365 days 5 h. 48 m. 51 s. to seconds.

da. h. m. s.		s.
365 5 48 51	*Proof.*	6,0) 3155693,1
1095, ×8 } 24		6,0) 52594,8 ... 51 s.
8765, ×60		24 { 8) 8765 ... 48 m.
525948, ×60		24 { 3) 1095 ... 5 h.
31556931 sec.		365 d. 5 h. 48 m. 51 s.

Here we first multiply the number of days by 24, or by 3 and by 8 in succession, adding in 5 to the *second* product; thus giving the number of hours.

Ex. 2. Reduce 993629 ounces to tons, &c.

oz.		tons. cwts. qr. lbs. oz.
16 { 4) 993629	*Proof.*	27 14 1 25 13
16 { 4) 248407 ... 1		954, ×4
28 { 4) 62101 ... 13 oz.		2217, ×7 } 28
28 { 7) 15525 ... 1		15519, ×4 } 28
4) 2217 ... 25 lbs.		62101, ×4 } 16
2,0) 55,4 ... 1 qr.		248404, ×4 } 16
27 tons 14 cwt. 1 qr. 25 lbs. 13 oz.		993629 oz.

232. There is a difficulty attending the application of the above Rule in reducing yards to poles and square yards to square poles, which we shall now explain.

To reduce yards to poles we have to divide by $5\frac{1}{2}$; but since $5\frac{1}{2}$ yds. is 11 half-yds. we multiply the yards by 2, and divide by 11, the remainder being half-yds.; and note that 1 half-yd. is $1\frac{1}{2}$ ft. or 1 ft. 6 in.

To reduce square yards to square poles we have to divide by $30\frac{1}{4}$; but since $30\frac{1}{4}$ sq. yds. is 121 qr.-sq. yds. we multiply the sq. yds. by 4 and divide by 121, the remainder being qr.-sq. yds.; and note that 1 qr.-sq. yd. is $2\frac{1}{4}$ sq. ft. or 2 sq. ft. 36 in.

Ex. 3. Reduce 606651 inches to miles.

```
12 ) 606651
  3 ) 50554...3 in.
      16851...1 ft.
              2
11 ) 33702
 4,0 ) 306,3...9 hf-yds. or 4 yds. 1 ft. 6 in.
   8 ) 76...23 p.                   1 ft. 3 in.
      9 m. 4 f. 23 p.       4 yds. 2 ft. 9 in.
```

The remainder in dividing by 11 is 9 half-yds., or $4\frac{1}{2}$ yds. or 4 yds. 1 ft. 6 in., and this added to the previous remainder, 1 ft. 3 in., gives a final remainder, 4 yds. 2 ft. 9 in.

Ex. 4. Reduce 37543473 sq. inches to acres.

```
              sq. in.
144 { 12 ) 37543473
    { 12 ) 3128622...9
        9 ) 260718...81 in.
             28968...6 ft.
                   4
121 { 11 ) 115872
    { 11 ) 10533...9
      4,0 ) 957...75 qr. yds. or 18 sq. yds. 6 ft. 108 in.
        4 ) 23 ...37 p.                      6 ft.  81 in.
            5 A.  3 r.  37 p.   19 yards     4 ft.  45 in.
```

The remainder in dividing by 121 is 75 qr.-yds., or $18\frac{3}{4}$ sq. yds. or 18 sq. yds. 6 ft. 108 in.; and to this we add the previous remainder, 6 ft. 81 in., giving the final remainder 19 sq. yds. 4 ft. 45 in.

233. There are some cases in Reduction where we cannot pass directly step by step from the given denominations to the one proposed. We must in such cases pass through an intermediate denomination common to both, and it will be advisable to keep such common denomination as high as possible.

Ex. 5. Reduce £25. 16s. $6\frac{1}{2}d.$ to francs of $9\frac{1}{2}d.$ each.

```
  £.  s.  d.
  25 . 16 . 6½
  516s.
  6198d.
19 ) 12397 ( 652 fr.
     99
      47
       9 = 4½d.
```

Here the common denomination is half-pence: but £25. 16. $6\frac{1}{2}$ is 12397 half-pence, and 1 franc is 19 half-pence; we therefore reduce 12397 half-pence to francs by dividing by 19, and the result is 652 francs with 9 half-pence or $4\frac{1}{2}d.$ over.

Ex. 6. Express 1 cwt. 1 qr. 25 lbs. in Troy weight.

```
       cwts.  qrs.  lbs.
        1      1     25
        5, × 28
      165, × 7000
    (8 ) 1155000 grs.
480 { 3 ) 144375
    (2,0,) 4812,5
       2406 oz. tr. 120 grs.
```

As Troy weight now consists simply of ounces troy of 480 grains, the common denomination is grains; we therefore reduce 1 cwt. 1 qr. 25 lbs. to grains, and then express the result in Troy ounces by Reduction ascending.

Exercise 30.

Reduce each of the following compound quantities to its lowest expressed denomination, giving the proof in each case:—

1. £343. 13. 5; £127. 16. 8¼; £47. 19. 11¾; £879. 18. 0¼.
2. 76 da. 19 h. 43 min. 57 s.; 43 weeks 5 h. 49 m. 57 s.
3. 7 miles 5 f. 32 p. 4 yds.; 25 fur. 39 p. 3 yds. 2 ft. 8 in.
4. 25 miles 6 fur. 17 p. 4 yds. 3 in.; 25 miles 459 yds. 31 in.
5. 17 acres 25 p. 25 yds.; 3 roods 17 p. 21 yds. 8 ft.
6. 8 acres 2 r. 34 p. 3 ft. 87 in.; 53 acres 21 p. 8 ft. 125 in.
7. 625 cu. yds. 19 ft. 1609 in.; 2767 cu. yds. 24 ft. 999 in.
8. 89 tons 17 cwt. 27 lbs. 15 oz.; 18 cwt. 73 lbs. 9 dr.
9. 165 oz. tr. 280 grs. 19 lbs. 8 oz. 14 dwts. 17 grs.
10. 8 tons 8 cwts. 98 lbs. 3045 grs.; 5 C. 7 O. 17 fl. oz. 5 fl. dr. 45 m.
11. 95 gall. 1 pt. 2 gills; 54 qrs. 7 bsh. 6 gall.
12. 75 yds. 3 qrs. 3 nl. 2 in.; 425 tons 19 cwt. 100 lbs. 15 oz. 200 grs.

Reduce each of the following simple quantities to its highest denomination, giving the proof in each case:—

13. 345679*d.*; 987653 half-pence; 130013 farthings.
14. 1,000,019 farthings; 1,000,000 min.; 4568657 sec.
15. 3055709 sec.; 25432245 sec.; 268543 in.
16. 1847638 ft.; 57383 yds.; 3126749 in.
17. 7865432 sq. in.; 12563257 sq. in.; 657345 sq. ft.
18. 895487 sq. yds.; 10,000,000 sq. in.; 25607809 sq. in.
19. 986877 cu. in.; 2099520 cu. in.; 87654 lbs.;
20. 378539 oz.; 1693539 drams; 20,000,000 grs.

21. 425095 grs. of gold; 882743 minims; 1879 pints.

22. 24357 gills; 4357 gallons of wheat; 97324 pints (dry).

23. Find the number of ounces troy in Three hundred millions three thousand eight hundred and forty grains of gold.

24. Find the number of miles, furlongs, &c. in Fifty millions five thousand six hundred and ninety inches.

25. Reduce 247 guineas 13*s.* $8\frac{1}{2}$*d.* to half-pence; $54\frac{1}{2}$ guineas to sixpences.

26. Reduce 5345 crowns to threepences; 23567 florins to half-crowns.

27. Reduce £25. 18. 9 to thalers of 3*s.* each; £443. 16. 8 to dollars of 4*s.* 2*d.* each; £137. 13. $6\frac{1}{4}$ to francs of $9\frac{1}{2}$*d.* each.

28. Find the number of square yards in a square mile; and then find how many perches there are in a square mile.

29. Reduce 2897 inches of *cloth* to yards, &c. Express 988 feet in cloth measure; and 34 ells 4 qrs. 3 nls. in yards, &c.

30. Express 2 cwts. 3 qrs. 17 lbs., 5 cwts. 18 lbs. 14 oz. in Tr. ounces.

31. A cash-box contains 89 sovereigns, 35 half-sovereigns, 19 half-crowns, 25 florins, 31 shillings and 15 sixpences: find the sum of all these coins in pence.

32. What will a penny a minute amount to in a year of 365 days 6 hrs.?

33. A child can paper 1 pin in 1 second; how many pins can it paper in a working week of $52\frac{1}{2}$ hours?

34. How many seconds are there from 6 A.M. Jan. 1 to 6 A.M. Feb. 1; from 8 A.M. 3 Mar. to 9 P.M. 22 May; from 10 A.M. 15 June to 7 A.M. 8 Dec.?

COMPOUND ADDITION.

234. COMPOUND ADDITION is the operation by which we find a single quantity which is equal to two or more quantities of the same kind put together.

This single quantity is called the *sum* of the given quantities.

235. We proceed upon precisely the same principles in the Compound that we do in the Simple Rules, and the only difference in the corresponding operations is this—that in the Simple Rules 10 units of one order always make 1 unit of the next higher order, whereas in the Compound Rules the Relation between two suc-

cessive orders is never uniform, but has to be sought from the Table of the quantity under consideration.

Making then the necessary alteration for this difference, the following is the Rule for Compound Addition:—

(1) *Write down quantities under one another, so that units of the same denomination may be in the same vertical column, and draw a line underneath.* (2) *Begin at the first column on the right, and find the sum of the numbers in that column; reduce the sum to the next higher denomination: set down the remainder under the column and carry the quotient to the first figure of the next column.* (3) *Having carried thus, find the sum of the second column: reduce, set down and carry as in the first column, and proceed in this way through all the columns:—*

PROOF. The proof given in Simple Addition (25) is equally applicable in Compound Addition.

Ex. 1. Find the sum of £125. 14. $9\frac{3}{4}$; £36. 9. $3\frac{1}{2}$; £267. 18. $7\frac{1}{4}$; £49. 13. 6; £157. 19. $11\frac{3}{4}$ and £87. 12. $5\frac{1}{2}$.

£.	s.	d.
125	14	$9\frac{3}{4}$
36	9	$3\frac{1}{2}$
267	18	$7\frac{1}{4}$
49	13	6
157	19	$11\frac{3}{4}$
87	12	$5\frac{1}{2}$
725	8	$7\frac{3}{4}$

Begin at the first or farthings column, and say 5, 6, 8, 11; now 11*f.* is $2\frac{3}{4}$*d.*; set down $\frac{3}{4}$*d.* and carry 2 to the first figure of the second column. For the second column say—7, 18, 24, 31, 34, 43; now 43*d.* is 3*s.* 7*d.*; set down 7 and carry 3 to the 12 of the next column. For the shillings column say—15, 34, 47, 65, 74, 88; now 88*s.* is £4. 8*s.*; set down 8 and carry 4 to the 7 of the next column: and now add the last column. Thus we have £725. 8. $7\frac{3}{4}$. N.B. We may if we please add up the shillings in two columns, but this is not advisable.

Proof. Begin at the top of each column and add downwards; we shall get at each step the same result as before.

236. In the addition of yards and sq. yards, and the subsequent reduction to poles and sq. poles, the same difficulty occurs as in

reduction (232): we must express the half-yard over as 1 ft. 6 in. and the quarter, half, and three-quarter sq. yard as 2 ft. 36 in., 4 ft. 72 in. and 6 ft. 108 in., and perform a second addition.

Ex. 2.

po.	yds.	ft.	in.
5	3	2	8
8	0	1	9
15	4	1	10
10	1	2	3
39	$4\frac{1}{2}$	2	6
		1	6
39	5	1	0

Ex. 3.

ac.	r.	po.	yds.	ft.	in.
2	3	15	20	3	31
7	2	18	32	6	15
1	1	25	31	8	25
3	0	34	27	7	100
15	0	15	$21\frac{1}{4}$	7	27
				2	36
15	0	15	22	0	63

In Ex. 2 the sum of the yards is 10, and 10 yds. is 1 po. $4\frac{1}{2}$ yds.; we therefore set down 4, and for the $\frac{1}{2}$ yd. we write down 1 ft. 6 in. for a subsequent addition.

In Ex. 3 the sum of the square yards is 112, and 112 sq. yds. is 3 sq. po. $21\frac{1}{4}$ sq. yds.; we therefore set down 21, and for the $\frac{1}{4}$ sq. yd. we write down 2 sq. ft. 36 sq. in. for a subsequent addition.

237. If the lowest denominations of the given compound quantities be mixed numbers, we add separately first the fractional and then the integral parts of such numbers (120, Rk. 3).

We will again remark that though farthings are written as fractions of a penny, and are put in the same column as pence, thus avoiding a fourth column, they are to be considered as whole numbers, just as much as pence or shillings.

Ex. 4. Find the sum of the two sets of quantities written down below:—

(1)

£.	s.	d.	
25 .	16 .	$7\frac{1}{2}\ \frac{2}{3}$	16
8 .	14 .	$2\frac{3}{4}\ \frac{5}{8}$	15
	17 .	$6\frac{0}{4}\frac{7}{12}$	14
31 .	2 .	$8\frac{1}{4}\ \frac{5}{6}$	20
19 .	19 .	$10\frac{1}{2}\ \frac{3}{4}$	18
86 .	10 .	$11\frac{3}{4}\frac{11}{24}$	83

(2)

oz.	dwts.	grs.	
5	16	$15\ \frac{2}{3}$	16
1	14	$23\ \frac{5}{8}$	15
	17	$0\frac{7}{12}$	14
2	4	$21\ \frac{5}{6}$	20
3	19	$8\ \frac{3}{4}$	18
14	15	$8\frac{11}{24}$	83

In (1) the lowest denomination consists of $2\frac{2}{3}f.$, $3\frac{5}{8}f.$, $\frac{7}{12}f.$, $1\frac{5}{6}f.$ and $2\frac{3}{4}f.$ The L.C.D. of the fractions is 24, which write down as a denominator in the *sum*, and at the side write down the numerators of the equivalent fractions; add these numerators giving 83, that is $\frac{83}{24}$ or $3\frac{11}{24}$; set down $\frac{11}{24}$ and carry 3 to the farthings, and proceed in the usual way.

EXERCISE 31.

	£	s.	d.
1.	7305	14	$8\frac{1}{2}$
	513	6	5
	8737	13	$4\frac{3}{4}$
	5928	11	$8\frac{1}{4}$
	67	5	$10\frac{3}{4}$
	536	17	$7\frac{1}{4}$

	£	s.	d.
2.	1287	14	$7\frac{1}{2}$
	3179	17	$4\frac{3}{4}$
	19	19	$11\frac{1}{4}$
	906	10	$3\frac{1}{4}$
	4594	13	8
	97	8	$9\frac{1}{2}$

	£	s.	d.
3.	14376	15	$10\frac{1}{4}$
	19058	18	$1\frac{1}{4}$
	25976	12	5
	10732	17	$8\frac{3}{4}$
	32147	12	6
	43275	9	$11\frac{3}{4}$

4. Add together £456. 14. $8\frac{3}{4}$, £9. 16. $4\frac{1}{2}$, £83. 18. $10\frac{1}{2}$, £17. 19. 7. £686. 7. $9\frac{1}{4}$, £8. 15. $6\frac{1}{2}$, £3548. 19. $9\frac{1}{2}$, and £95. 8. $8\frac{3}{4}$.

5. Add together £8. 19. $10\frac{3}{4}$, £1379. 17. $6\frac{1}{4}$, £897. 16. $9\frac{1}{2}$, £89. 18. 11, £4357. 8. $11\frac{1}{4}$, £52765. 15. $8\frac{3}{4}$, and £99. 19. $11\frac{3}{4}$.

	lbs.	oz.	grs.
6.	9	15	235
	18	11	96
	6	13	365
	13	9	192
	5	14	400

	cwts.	qrs.	lbs.	oz.
7.	14	2	25	11
	6	3	19	14
	3	0	21	12
	8	1	26	15
	10	3	18	9

	tons	cwts.	qrs.	lbs.	oz.
8.	25	18	2	17	12
	19	16	1	22	10
	8	17	3	26	14
	14	15	2	19	9
	37	19	3	27	15

	lbs.	oz.	dwts.	grs.
9.	13	9	14	21
	6	7	18	16
		11	16	19
	20	6	17	23
	9	10	15	20

	C.	O.	fl. oz.	fl. dr.	m.
10.	3	5	18	7	10
		7	13	1	45
	1	4	9	3	15
	2	0	19	5	20
		3	5	6	30

	gall.	qts.	pts.	gills
11.	57	3	1	3
	38	1	1	2
	45	2	0	3
	26	3	0	3
	18	2	1	2

	yds.	qrs.	nls.	in.
12.	5	1	2	2
	3	0	2	1
		3	1	2
	6	3	2	2
	1	0	0	1

	ells	qrs.	nls.	in.
13.	3	4	1	2
		2	0	2
	4	4	2	2
		3	3	1
	1	2	1	2

	qrs.	bsh.	pk.	gall.
14.	19	6	3	1
	38	7	1	1
	11	4	3	0
	4	7	3	1
	32	5	2	0

	days	h.	m.	s.
15.	18	18	35	47
	9	21	29	53
	8	17	54	36
	6	23	39	49
	13	16	59	28
	7	20	46	59

	mo.	wk.	da.	h.	m.
16.	18	3	6	17	43
	10	2	4	23	37
	6	0	5	43	55
	12	3	6	29	28
	8	1	3	35	48
	7	2	4	56	37

	yrs.	dys.	h.	m.	s.
17.	6	84	13	34	43
	7	129	21	45	28
	3	273	9	51	17
	9	65	16	28	36
	5	343	12	18	24
	8	95	22	55	49

	poles	yds.	ft.	in.
18.	25	4	2	9
	18	3	1	6
	9	1	0	8
	33	4	2	10
	3	5	2	7

	fur.	po.	yds.	ft.	in.
19.	3	15	3	1	4
	4	20	4	0	9
	2	8	5	2	6
		31	5	0	1
	6	18	2	2	10

	miles	f.	p.	yds.
20.	3	7	38	5
	9	3	23	2
		6	14	5
	8	5	17	4
	6	4	31	3

	per.	yds.	ft.	in.
21.	32	25	8	136
	16	18	3	67
	9	36	5	45
	20	15	8	129
	6	9	7	16

	rds.	per.	yds.	ft.	in.
22.	2	36	24	2	49
	1	27	9	3	90
		25	16	8	70
	3	18	29	5	115
		30	25	6	63

	cu. yds.	ft.	in.
23.	37	15	1084
	86	22	695
	9	13	1556
	24	8	924
	10	26	1688

Find the sum of

24. 16 cwt. 91 lb. 14 oz. 15 dr., 19 cwt. 68 lb. 11 oz. 12 dr., 9 cwt. 87 lb. 12 oz. 14 dr., 14 cwt. 103 lb. 13 oz. 10 dr., and 18 cwt. 95 lb. 10 oz. 13 dr

25. 9 lb. 12 oz. 230 grs., 3 lb. 10 oz. 125 grs., 5 lb. 13 oz. 289 grs., 8 lb. 14 oz. 365 grs., and 7 lb. 11 oz. 420 grs.

26. 27 miles 3 f. 150 yds. 2 ft., 19 miles 6 f. 73 yds. 1 ft., 10 miles 5 f. 95 yds., 8 miles 4 f. 219 yds. 2 ft., and 12 miles 7 f. 37 po. 1 ft.

27. 47 ac. 3 r. 27 per. 15 yds., 29 ac. 1 r. 36 p. 24 yds., 36 ac. 18 p. 19 yds., 9 ac. 2 r. 35 p. 20 yds., and 3 roods 9 p. 12 yds.

28. £18. 14. $4\frac{1}{2}\frac{3}{8}$, £9. 0. $10\frac{1}{4}\frac{1}{5}$, £3. 15. $7\frac{1}{4}\frac{3}{8}$, £11. 9. $3\frac{3}{4}\frac{1}{2}$, £8. 13. $6\frac{1}{4}\frac{2}{3}$, and £27. 17. $7\frac{1}{2}\frac{7}{12}$.

29. £2. 19. $3\frac{3}{4}\frac{1}{6}$, 18s. $10\frac{1}{4}\frac{5}{7}$, £5. 3. $7\frac{3}{4}\frac{8}{9}$, 7s. $6\frac{1}{2}\frac{3}{5}$, 15s. $0\frac{1}{2}\frac{7}{11}$ and £3. 6. $9\frac{1}{4}\frac{3}{5}$.

30. £25. 16. $3\frac{1}{4}\frac{5}{12}$, £16. 17. $10\frac{1}{2}\frac{7}{16}$, £7. 13. $6\frac{1}{4}\frac{5}{8}$, £12. 9. $11\frac{3}{4}\frac{5}{8}$, £22. 18. $9\frac{1}{2}\frac{8}{11}$, and £6. 10. $8\frac{3}{4}\frac{1}{3}$.

31. 13 cwt. 21 lb. $13\frac{5}{8}$ oz., 3 qrs. 18 lb. $9\frac{7}{16}$ oz., 25 lb. $15\frac{3}{4}$ oz. 1 qr. 13 lb. $3\frac{11}{16}$ oz., 2 qrs. 12 lb. $12\frac{5}{8}$ oz., and 4 cwt. 8 lb. $15\frac{3}{8}$ oz.

32. 25 poles 4 yds. 2 ft. $8\frac{3}{4}$ in., 17 po. 2 yds. $6\frac{3}{4}$ in., 2 po. 1 ft. $7\frac{7}{12}$ in. 15 po. 5 yds. 1 ft. $11\frac{8}{11}$ in., 6 po. 4 yds. 2 ft. $10\frac{5}{8}$ in., and 20 po. 3 yds. $9\frac{1}{2}$ in

COMPOUND SUBTRACTION.

238. COMPOUND SUBTRACTION is the operation by which we find what quantity is left when a smaller quantity is taken from a greater of the same kind.

For the terms used in Subtraction see Arts. 26 and 27.

239. Making the alteration spoken of in Art. 235 in the Rule for Simple Subtraction, we have the following Rule for Compound Subtraction :—

(1) *Write the Subtrahend under the Minuend, so that units of the same denomination may be under one another, and draw a line*

below. (2) *Begin with the lowest denomination and subtract each number in the Subtrahend from the one above it, and place the remainder immediately below; and if in any case the number in the Subtrahend be greater than the one above it, add to the latter the number which tells how many of its denomination makes one of the next higher, and then subtract, taking care to add* 1 *to the next number in the Subtrahend.* N.B. In borrowing we may sometimes find it more convenient to subtract the number in the Subtrahend from the number which tells how many of its denomination makes one of the next higher, and then *add* the number above it (124, Ex. 1).

Proof. The methods of proof are the same as in Simple Subtraction (31).

Ex. 1. Subtract £17. 8. 5¼ from £25. 14. 9¾.

£.	s.	d.
25 .	14 .	9¾
17 .	8 .	5¼
8 .	6 .	4½

Here every number in the Subtrahend is less than the number above it in the Minuend; we have therefore only to subtract and set down.

Ex. 2. Subtract £345. 17. 8¾ from £525. 14. 7½.

£.	s.	d.
525 .	14 .	7½
345 .	17 .	8¾
179 .	16 .	10¾

Begin with the farthings: as we cannot take 3 from 2 we borrow 4 (for 4*f.* is 1*d.*) giving 6, and 3 from 6 is 3; set down ¾*d.* and carry 1 to the pence. For the pence: 8 and 1 is 9, but as we cannot take 9 from 7 we borrow 12 (for 12*d.* is 1*s.*) giving 19, and 9 from 19 is 10; set down 10 and carry 1. For the shillings: 17 and 1 is 18, but as we cannot take 18 from 14 we borrow 20 (for 20*s.* is £1) giving 34, and 18 from 34 is 16; set down 16 and carry 1. For the rest proceed as in Simple Subtraction.

Or we may proceed thus: 3 from 4, 1, and 2 (added) 3; set down ¾*d.* and carry 1. 9 from 12, 3, and 7 (added) 10; set down 10 and carry 1. 18 from 20, 2, and 14 (added) 16; set down 16, &c.

Ex. 3. From 15 po. 3 yds. 2 ft. 3 in. take 8 po. 4 yds. 1 ft. 9 in. (See Art. 236.)

po.	yds.	ft.	in.
15	3	2	3
8	4	1	9
6	4½	0	6
		1	6
6	4	2	0

Here when we come to the yards we say 4 from 5½ is 1½, and 3 is 4½: set down 4, and for the half-yard write down 1 ft. 6 in. to be subsequently added.

Ex. 4. Find the difference between 35 sq. po. 14 yds. 6 ft. 81 in. and 13 sq. po. 25 yds. 7 ft. 108 in. (See Art. 236.)

sq. po.	yds.	ft.	in.
35	14	6	81
13	25	7	108
21	18¼	7	117
		2	36
21	19	1	9

When we come to the sq. yards, we say 26 from 30¼ is 4¼, and 14 is 18¼; set down 18, and for the quarter sq. yd. write down 2 ft. 36 in. to be subsequently added.

240. If the lowest denominations of the given compound quantities be mixed numbers, we subtract separately first the fractional and then the integral parts of such mixed numbers (125, 1).

Ex. 5. Subtract £32. 17. 9 3/7 from £87. 14. 2 4/5.

Ex. 6. From 15 cwts. 1 qr. 16 3/7 lbs. take 8 cwts. 3 qrs. 25 4/5 lbs.

£.	s.	d.		cwts.	qrs.	lbs.	
87	14	2 4/5	28	15	1	16 3/7	15
32	17	9 3/7	15	8	3	25 4/5	28
54	16	4 13/35		6	1	18 22/35	

Here the L.C.D. of the fractions is 35, which write down as a denominator in the remainder, and at the side write down the numerators of the equivalent fractions, 28 and 15. In Ex. (5) we say 15 from 28 is 13 and set down 13/35, and proceed in the usual way. In Ex. (6) as we cannot take 28 from 15 we borrow 35, for 35/35 is 1, and say 28 from 35 is 7, and 15 is 22 (124, Ex. 1): set down 22/35, and carry 1 to the whole number 25 in the lbs., and proceed as usual.

Exercise 32.

Find the difference between

1. £3548. 9. 6 and £95. 14. 8¾; £536. 8. 7¼ and £89. 13. 9¾; £20 and 3s. 10½d.

2. £837. 14. 2½ and £358. 18. 6¾; £86. 15. 9¼ and £9. 18. 11½; £2 and 2½d.

3. 73 guineas 9s. 5½d. and 47 guineas 16s. 8¾d.

4. From 1000 guineas take the sum of £131. 9. 2¼ and £457. 8. 7½.

5. What is the final remainder in taking £139. 17. 8½ as many times in succession as possible from £457. 13. 0¾?

6. From the sum of £25. 16. 9½ and £19. 12. 7¾ take the difference between £76. 8. 6½ and £37. 13. 8¾.

	lbs.	oz.	drs.
7.	13	9	10
	6	13	14

	cwt.	qrs.	lbs.	oz.
8.	14	2	15	8
	7	2	17	11

	tuns	cwt.	qr.	lbs.	oz.
9.	25	13	1	5	6
	12	17	3	9	12

	lbs.	oz.	dwts.	grs.
10.	20	6	9	12
	15	9	12	19

	C.	O.	fl. oz.	fl. dr.	m.
11.	6	3	12	1	15
	2	6	17	5	40

	gall.	qts.	pt.	gill.
12.	57	2	1	2
	26	3	1	3

	yds.	qrs.	nl.	in.
13.	9	1	1	$\frac{1}{2}$
	3	3	3	2

	ells	qrs.	nl.	in.
14.	5	2	1	1
	2	4	1	2

	lds.	qrs.	bsh.	pk.	gall.
15.	7	3	5	2	0
	3	4	7	3	1

	days	h.	m.	s.
16.	18	18	35	47
	13	22	41	54

	mo.	wk.	da.	h.	m.
17.	10	2	3	14	28
	6	4	4	19	37

	yrs.	da.	h.	m.	s.
18.	7	129	13	26	17
	3	273	18	34	29

	po.	yds.	ft.	in.
19.	9	1	0	8
	4	3	2	10

	fur.	po.	yds.	ft.	in.
20.	5	15	3	1	4
	2	26	4	2	9

	miles	f.	p.	yds.
21.	29	3	23	2
	7	6	31	4

	per.	yds.	ft.	in.
22.	32	18	3	67
	20	25	8	86

	roods	per.	yds.	ft.	in.
23.	2	16	9	5	49
		23	18	7	90

	cu. yds.	ft.	in.
24.	87	8	924
	35	23	1688

Find the difference between

25. 5 poles and 3 yds. 3 in.; 47 acres 1 r. 18 per. 12 yds. and 24 ac. 2 r. 31 per. 17 yds.

26. 19 cwt. 68 lbs. 10 oz. 7 dr. and 12 cwt. 103 lbs. 14 oz. 10 dr.

27. 9 lbs. 9 oz. 125 grs. and 4 lbs. 14 oz. 347 grs.

28. 60 oz. Troy and 20,000 grs.; 8 lbs. and 50,000 grains.

29. £37. 14. $5\frac{1}{4}\frac{2}{5}$ and £24. 9. $8\frac{3}{4}\frac{1}{6}$; £9. 6. $3\frac{1}{3}\frac{5}{14}$ and 18*s*. $8\frac{1}{2}\frac{1}{3}\frac{7}{8}$.

30. £3. 17. $7\frac{2}{4}\frac{6}{11}$ and £1. 19. $10\frac{1}{4}\frac{1}{18}$; 3 cwt. 1 qr. $7\frac{7}{11}$ lbs. and $24\frac{5}{8}$ lbs.

31. 5 fur. 15 po. and 4 po. 3 yds. $8\frac{7}{13}$ in.

32. 11 oz. 15 dwts. $16\frac{8}{21}$ grs. and 18 dwts. $21\frac{11}{24}$ grs.

33. What is left out of 1000 guineas after paying £200. 19. $9\frac{1}{2}$ to one person, £457. 13. $8\frac{3}{4}$ to another, and £73. 16. $2\frac{1}{4}$ to a third?

34. A man receives £46. 8. $7\frac{1}{2}$, £235. 16. $8\frac{3}{4}$, 45 guineas and £127. 14. 6; and then pays £53. 12. $2\frac{1}{2}$, £87. 9. $4\frac{1}{4}$ and £197. 15. $8\frac{3}{4}$; how much will he have remaining over?

COMPOUND MULTIPLICATION.

241. COMPOUND MULTIPLICATION is the operation by which we find the sum of a compound quantity called the Multiplicand, repeated as many times as there are units in a given number called the Multiplier. The sum found is called the *Product.*

Hence the Multiplier is an *abstract* number,—a number of times; and the Product is a *concrete* number of the same kind as the Multiplicand.

242. CASE I. *When the Multiplier is not greater than* 12.

Here, while proceeding on the same principle as in Simple Multiplication, we must make such modification as the circumstance mentioned in Art. 235 requires; and we have then the following Rule:—

(1) *Write the Multiplier under the lowest denomination of the Multiplicand and draw a line below.* (2) *Beginning with the lowest denomination multiply each of them in succession by the Multiplier, and reduce, set down, and carry precisely as in Compound Addition.*

Ex. 1. Multiply £325. 13. 6¾ by 7.

£	s.	d.
325 .	13 .	6¾
		7
2279 .	14 .	11¼

Here we say 7 times 3 is 21, and 21*f.* is 5¼*d.*; set down ¼*d.* and carry 5. 7 times 6 is 42, and 5 carried is 47, and 47*d.* is 3*s.* 11*d.*; set down 11 and carry 3. 7 times 13 is 91 and 3 is 94, and 94*s.* is £4. 14*s.*; set down 14, and carry 4. And now proceed as in Simple Multiplication.

243. CASE II. *When the Multiplier is the product of two or more numbers each not greater than* 12, *multiply by each of those numbers in succession, and the last result will be the Product required* (36).

Ex. 2. Multiply 2 tons 8 cwts. 3 qrs. 13 lbs. by 96.

Since 96 is 8 . 12 or 12 . 8 we may first multiply by 8 and the result by 12; or we may first multiply by 12 and the result by 8; thus

tons	cwts.	qrs.	lbs.
2	8	3	13
			8
19	10	3	20
			12
234	11	0	16

tons	cwts	qrs.	lbs.
2	8	3	13
			12
29	6	1	16
			8
234	11	0	16

244. CASE III. *When the Multiplier exceeds or falls short of a product within the table by a number not greater than* 12, *multiply by such product* (Case II.) *and then by this number and add or subtract for the required Product* (62, 63).

Ex. 3. Multiply £17. 8. $5\frac{1}{4}$ by 139.

Since 139 is 132 + 7, or 144 − 5, we may multiply by 132 and then by 7, and add, or we may multiply by 144 and then by 5, and subtract.

£.	s.	d.		
17	8	$5\frac{1}{4}$		
		12		
209	1	3		
		11		
2299	13	9	product by	132
121	19	$0\frac{3}{4}$	......	7
2421	12	$9\frac{3}{4}$	......	139

£.	s.	d.		
17	8	$5\frac{1}{4}$		
		12		
209	1	3		
		12		
2508	15	0	product by	144
87	2	$2\frac{1}{4}$	......	5
2421	12	$9\frac{3}{4}$	......	139

245. CASE IV. *When the Multiplier is a number beyond the range of the table.*

For example suppose the Multiplier to be 2479. Let us multiply the given quantity by 10 3 times in succession; the first result will give the product by 10, the second by 100, and the third by 1000; now let us multiply the given quantity by 9, the first result by 7, the second by 4, and the third by 2; in this way we have found the products of the given quantity by 9, by 70, by 400, and by 2000, and the sum of these partial products will give the Product by 2479 (41).

We have then the following Rule:—

Multiply by 10 *as many times in succession as there are figures in the Multiplier less* 1; *then multiply the Multiplicand by the units' figure of the Multiplier, the first product by the tens' figure, the second product by the hundreds' figure. The sum of these partial products will give the Product required.*

Ex. 4. Multiply £3. 15. $7\frac{3}{4}$ by 2479.

£.	s.	d.		£.	s.	d.		
3	15	$7\frac{3}{4}$	× 9 =	34	0	$9\frac{3}{4}$	product by	9
		10						
37	16	$5\frac{1}{2}$	× 7 =	264	15	$2\frac{1}{2}$	......	70
		10						
378	4	7	× 4 =	1512	18	4	......	400
		10						
3782	5	10	× 2 =	7564	11	8	......	2000
				9376	6	$0\frac{1}{4}$	......	2479

When the Multiplier is a large number, as in this Example, and we are told to proceed by Compound Multiplication, the following is the simplest method.

£.	s.	d.
3	15	$7\frac{3}{4}$
		2479
9376	6	$0\frac{1}{4}$

```
 4 ) 7437
      1859 ... ¾d.
     17353
12 ) 19212
      1601 ... 0d.
     12395
     2479
20 ) 38786
      1939 ... 6s.
      7437
     £9376
```

We first multiply 3*f.* by 2479 giving 7437*f.* or 1859*d.* and $\frac{3}{4}$*d.*, set down $\frac{3}{4}$*d.* and carry 1859*d.* to the pence: then multiply 7*d.* by 2479 and add in 1859, giving 19212*d.* or 1601*s.* and 0*d.*, set down 0*d.* and carry 1601 to the shillings: then multiply 15*s.* by 2479 and add in 1601 giving 38786*s.*, or £1939. 6*s.*, set down 6*s.* and carry 1939 to the £'s: lastly multiply £3 by 2479, and add in £1939, giving £9376. Thus the product is £9376. 6. $0\frac{1}{4}$.

Whichever method be employed, the other may be adopted as a *Proof*.

246. If the lowest denomination of the Multiplicand be a mixed number, we multiply separately first the fractional and then the integral part of such mixed number (131, Rk.).

Ex. 5. Multiply £345. 17. $2\frac{3}{4}\frac{7}{8}$ by 8; 5 yds. 2 ft. $7\frac{5}{8}$ in. by 12.

(1)

£.	s.	d.
345	17	$2\frac{3}{4}\frac{7}{8}$
		8
2766	17	$11\frac{1}{2}\frac{7}{8}$

(2)

yds.	ft.	in.
5	2	$7\frac{5}{8}$
		12
70	1	$7\frac{1}{2}$

In (1) we say 8 times $\frac{7}{9}$ is $\frac{56}{9}$ or $6\frac{2}{9}$; set down $\frac{2}{9}$ and carry 6: 8 times 3*f.* is 24*f.*, and 6*f.* is 30*f.* or 7½*d.*; set down ½*d.* and carry 7, &c.

In (2) we say 12 times $\frac{5}{8}$ is $\frac{60}{8}$ or $7\frac{4}{8}$ or 7½*d.*; set down ½ and carry 7, &c.

247. If the weight, value,... of one unit be given, we can by Compound Multiplication find the weight, value,... of any number of units of the same kind; for example:—If the value of 1 oz. of gold is £3. 15. 7¾ the value of 2479 oz. is found by multiplying £3. 15. 7¾ by 2479. (See Ex. 4.)

EXERCISE 33.

Multiply

1. £19. 18. 7¼ by 8; £3. 9. 7¾ by 12; £87. 8. 11¾ by 10.
2. £37. 19. 9½ by 9; £374. 12. 10¼ by 7; £549. 13. 7¾ by 11.
3. £65. 18. 1½ by 63; £873. 6. 5¼ by 72; £489. 11. 10¾ by 88.
4. £907. 17. 3¾ by 108; £17. 18. 8¾ by 121; £28. 13. 7½ by 144.
5. £57. 13. 7¾ by 39; £43. 15. 9¾ by 53; £9. 10. 11½ by 86.
6. £2. 15. 6¼ by 94; £75. 14. 0¾ by 127; £87. 0. 11¾ by 151.
7. £14. 3. 8¼ by 347; £356. 16. 4¾ by 193.
8. £27. 14. 3½ by 856; £40. 13. 7½ by 1085.
9. £9. 15. 10¾ by 4508; £16. 12. 9¼ by 7249.
10. 18*s.* 7¾*d.* by 9384; £3. 18. 11½ by 57089.
11. £567. 13. 8¾ by 8736 and by 98736.
12. 3 lbs. 8 oz. 18 dwts. 8 grs. by 35 and by 96.
13. 27 gallons 3 qts. 1 pt. 3 gills by 36 and by 236.
14. 4 qrs. 7 bush. 3 pk. 1 gall. by 12 and by 144.
15. 17 weeks 4 d. 13 h. 27 min. 36 sec. by 9 and by 79.
16. 8 miles 5 fur. 7 ch. 12 yds. by 132; 7 ac. 3 r. 25 p. by 157.
17. 23 cu. yds. 6 ft. 459 in. by 8 and by 72.
18. 24 lbs. 9 oz. 135 grs. by 7 and by 70.
19. 9 tons 15 cwts. 1 qr. 24 lbs. by 11 and by 459.
20. 5 fur. 30 po. 4 yds. 1 ft. by 7 and by 84.
21. 25 miles 6 fur. 23 po. 3 yds. 2 ft. 8 in. by 56 and by 83.
22. 2 roods 27 p. 15 yds. 8 ft. by 6 and by 10.
23. 37 ac. 3 r. 19 po. 28 yds. 4 ft. 103 in. by 8 and by 75.
24. 3 yds. 3 qrs. 3 nl. 2 in. by 7 and by 59.
25. £15. 7. $8\frac{1}{2}\frac{1}{16}$ by 9; £36. 15. $10\frac{3}{4}\frac{11}{16}$ by 12.

26. £3. 13. $3\frac{1}{4}\frac{18}{21}$ by 56; £2. 18. $7\frac{3}{4}\frac{13}{17}$ by 132.

27. 13*s*. $8\frac{1}{2}\frac{88}{163}$ by 120; £3. 17. $8\frac{1}{2}\frac{11}{16}$ by 43.

28. £4. 13. $7\frac{1}{4}\frac{11}{17}$ by 93; £2. 16. $10\frac{3}{4}\frac{13}{18}$ by 139.

29. 80 oz. tr. 15 dwts. $19\frac{8}{13}$ grs. by 42.

30. 14 cwts. 3 qrs. 25 lbs. $13\frac{8}{21}$ oz. by 96.

31. 3 furlongs 34 po. 4 yds. 1 ft. $8\frac{1}{3}\frac{1}{6}$ in. by 99.

Find the value of

32. 207 quarters of wheat at 56*s*. 8*d*. a quarter.

33. 78 lbs. 5 oz. of quinine at 8*s*. $8\frac{3}{4}$*d*. an ounce.

34. 3 cwts. 2 qrs. 10 lbs. of tea at 2*s*. $4\frac{3}{4}$*d*. per lb.

35. 3569 oz. tr. of gold at £3. 17. $10\frac{1}{2}$ per ounce.

36. 3 roods 24 po. 27 yds. of building land at £3. 16. $6\frac{3}{4}$ per sq. yard.

COMPOUND DIVISION.

248. COMPOUND DIVISION is the operation by which—

(1) We find how many times one compound quantity contains another compound quantity of the same kind.

(2) We break up a compound quantity into as many equal parts as there are ones in a given number, and thus find the value of one of these parts.

In the first case the Divisor is a compound quantity of the same kind as the Dividend, and the Quotient telling *how many times* is an *abstract* number. In the second case the Divisor is an abstract number, and the Quotient telling *the value of each part* is a compound quantity of the same kind as the Dividend. Thus if 2*s*. $9\frac{1}{2}$*d*. $\times 56 =$ £7. 16. 4, it follows that £7. 16. 4 contains 2*s*. $9\frac{1}{2}$*d*. 56 times, where the Divisor is *concrete* and the Quotient *abstract*; it also follows that £7. 16. 4 divided into 56 equal parts gives 2*s*. $9\frac{1}{2}$*d*. in each part, where the Divisor is *abstract*, and the Quotient *concrete* and of the same kind as the Dividend.

249. CASE I. *When the Divisor is a compound quantity of the same kind as the Dividend.*

If the Divisor and Dividend were simple quantities (227) of the same kind we should proceed as in Simple Division: thus £10 contains £2, 10 cwts. contains 2 cwts., 10 gallons contains 2 gallons,

as often as 10 contains 2. And if the Dividend and Divisor are compound quantities we can reduce them to simple quantities of the same denomination, and then proceed as before.

We have then the following Rule:—

Reduce Dividend and Divisor to the same denomination, and then proceed as in Simple Division. N.B. Do not carry the reduction lower than is necessary.

Ex. 1. How many times does £35. 3. 9 contain £2. 14. 1?

£.	s.	d.		£.	s.	d.		
2	14	1		35	3	9		649) 8445 (13
54				703				1955
649				8445				8

Reducing each sum to pence, we have then to find how often 8445*d.* contains 649*d.* and the Quotient is 13, and 8*d.* over; and the *complete* Quotient is $13\frac{8}{649}$. (133.)

250. If the value, weight,... of one unit be given, we can by this case of Compound Division find the number of units of value, weight,... in any other given quantity of the same kind; for example: If £2. 14. 1 be the price of 1 cwt. of sugar, we find what number of cwts. £35. 3. 9 is the price of, by dividing £35. 3. 9 by £2. 14. 1.

251. CASE II. *When the Divisor is an abstract number.*

If we regard the various denominations of the Dividend as so many successive orders whose relationship to one another, instead of being uniform, is irregular and to be found from the tables, we may proceed in Compound precisely as in Simple Division; for example:—Divide £25. 15. 8 by 7, that is divide £25. 15. 8 into 7 equal parts (248, 2). Now £25 divided by 7 gives £3 with £4 over; put then £3 in each of the 7 parts and there still remains to be divided £4. 15. 8. But £4. 15*s.* is 95*s.*, and 95*s.* divided by 7 gives 13*s.* and 4*s.* over; put then 13*s.* further into each of the 7 parts making £3. 13*s.*, and there still remains to be divided 4*s.* 8*d.* Again, 4*s.* 8*d.* is 56*d.*, and 56*d.* divided by 7 gives 8*d.* exactly; put then 8*d.* further into each of the seven parts making £3. 13. 8,

	£.	s.	d.
7)	25 .	15 .	8
	3 .	13 .	8

and the division is completed; therefore £25. 15. 8 divided by 7 gives £3. 13. 8.

We have then the following Rule:—

(1) *Make the same arrangements for Dividend, Divisor and Quotient as in Simple Division.* (2) *Divide the highest denomination by the Divisor; reduce the remainder to the next lower denomination, add in the number of that denomination in the Dividend and divide the result by the Divisor: and so proceed step by step through all the denominations.*

Ex. 1. Divide £3685. 4. $9\frac{1}{2}$ by 678.

```
      £.   s.  d.   £.  s.  d.
678 ) 3685 . 4 . 9½ ( 5 . 8 . 8½
      3390
       295
        20
      5904 ( 8
      5424
       480
        12
      5769 ( 8
      5424
       345
         4
      1382 ( 2
      1356
       26f. = 6½d.
```

More briefly thus:—

```
      £.   s.  d.   £.  s.  d.
678 ) 3685 . 4 . 9½ ( 5 . 8 . 8½
      295
      5904 ( 8
       480
      5769 ( 8
       345
      1382 ( 2
      26f. = 6½d.
```

∴ the Quotient is £5. 8. $8\frac{1}{2}$ and $6\frac{1}{2}d.$ over.

Again, since 26*f.* divided by 678 is $\frac{26}{678}f.$ or $\frac{13}{339}f.$, complete Quotient = £5. 8. $8\frac{1}{2}\frac{13}{339}$.

252. *When the Divisor is the product of two or more factors, divide by each of them in succession* (52), *and find the remainder as in Simple Division.*

Ex. 2. Divide £3458. 17. $9\frac{1}{2}$ by 72:—(1) giving the Quotient and Remainder, and (2) the complete Quotient.

```
        £.     s.   d.
72 { 9 ) 3458 . 17 . 9½
   { 8 )  384 .  6 . 5 0/4 ...2f.
           48 .  0 . 9½ ...38f.
```

The final remdr. is $4 \times 9 + 2$ or 38*f.*
∴ Quotient is £48. 0. $9\frac{1}{2}$, and $9\frac{1}{2}d.$ over.

```
        £.     s.   d.
72 { 9 ) 3458 . 17 . 9½
   { 8 )  384 .  6 . 5 0/4 2/9
           48 .  0 . 9½ 19/36
```

The second remdr. from the farthings is $4\frac{2}{9}$ or $\frac{38}{9}$; and $\frac{38}{9} \div 8$ is $\frac{38}{72}$ or $\frac{19}{36}$,
∴ complete Quotient = £48. 0. $9\frac{1}{2}\frac{19}{36}$.

Ex. 3. Divide 53 miles 3 fur. 23 po. 4 yds. 2 ft. by 35.

	miles	fur.	po.	yds.	ft.	in.
7)	53	3	23	4	2	
5)	7	5	3	2	0	8...4 in.
	1	4	8	3	2	2...32 in.

yds.	ft.	in.
16	1	6
2	0	8
18	2	2

In dividing by 5 the remainder from the poles is 3 poles, or $16\frac{1}{2}$ yards, or 16 yds. 1 ft. 6 in., which added to 2 yds. 0 ft. 8 in. in the Dividend gives 18 yds. 2 ft. 2 in., to be further divided by 5. The second partial remainder is 4 in.; therefore the full remainder is $4 \times 7 + 4$, or 32 in. or 2 ft. 8 in.

The complete Quotient = 1 mile 4 fur. 8 po. 3 yds. 2 ft. $2\frac{32}{35}$ in.

253. *When the Divisor is* 10, 100, 1000,... *cut off* 1, 2, 3... *figures to the right in each succeeding Dividend; the figures to the left will at each step give the Quotient and the figures to the right the remainder* (53).

Ex. 4. Divide £85432. 19. $11\frac{1}{2}$ by 1000.

£. s. d.
1000) 85,432 . 19 . $11\frac{1}{2}$
20
8,659
12
7,919
4
3,678

£. s. d.
85,432 . 19 . $11\frac{1}{2}$
8,659
7,919
3,678 $\qquad \frac{678}{1000} = \frac{339}{500}$

The Quotient is £85. 8. $7\frac{3}{4}$, and 678*f.* or 14*s.* $1\frac{1}{2}$*d.* over, and the complete Quotient is £85. 8. $7\frac{3}{4}\frac{339}{500}$.

Ex. 5. Divide £85432. 19. $11\frac{1}{2}$ by 900.

£. s. d.
900 { 9) 85432 . 19 . $11\frac{1}{2}$
100) 94,92 . 11 . $1\frac{1}{4}$...1*f.*
18,51
6,13
,53

£. s. d.
9) 85432 . 19 . $11\frac{1}{2}$
100) 94,92 . 11 . $1\frac{1}{4}\frac{1}{9}$
18,51
6,13
,$53\frac{1}{9}$ $\qquad \frac{53\frac{1}{9}}{100} = \frac{478}{900} = \frac{239}{450}$

The final remdr. is $53 \times 9 + 1$ or 478*f.* or 9*s.* $11\frac{1}{2}$. ∴ the Quotient is £94. 18. 6 and 9*s.* $11\frac{1}{2}$*d.* over; and the complete Quotient £94. 18. $6\frac{3}{4}\frac{239}{450}$.

Ex. 6. Divide £227. 9. $4\frac{1}{2}\frac{3}{8}$ by 42.

Ex. 7. Divide 58 gallons 2 qts. $1\frac{5}{8}$ pts. by 5.

£. s. d.
6) 227 . 9 . $4\frac{1}{2}\frac{3}{8}$
7) 37 . 18 . $2\frac{1}{2}\frac{7}{8}$
5 . 8 . $3\frac{3}{4}\frac{1}{16}$

	gallons	qt.	pt.
5)	58	2	$1\frac{5}{8}$
	11	2	$1\frac{37}{40}$

In Ex. 6 the first remainder from the farthings is $5\frac{3}{4}$ or $\frac{23}{4}$, and $\frac{23}{4}\div 6$ is $\frac{23}{24}$; the second remainder is $1\frac{7}{8}$ or $\frac{15}{8}$, and $\frac{15}{8}\div 7$ is $\frac{15}{56}$.

In Ex. 7 the remainder from the pints is $4\frac{5}{8}$ or $\frac{37}{8}$, and $\frac{37}{8}\div 5$ is $\frac{37}{40}$.

254. If the value, weight, length,... of any number of units be given, we can by this case of Compound Division find the value, weight, length,... of one unit of the same kind; for example:—If the area of a field containing 32 equal allotments be 19 acres 3 r. 20 p., the area of one allotment is found in dividing 19 acres 3 r. 20 p. by 32.

EXERCISE 34.

Divide

1. £286. 3. 2 by £1. 11. 1¼; £727. 1. 7¼ by £15. 9. 4¾.
2. £144. 13. 11½ by 9s. 11¾d.; £1376. 11. 10¾ by 10s. 2½d.
3. £9961. 7. 6½ by £35. 16. 7¾; £501803. 15. 8¾ by £76. 13. 4¾.
4. How many francs of 9½d. each are contained in £87. 16. 8½?
5. How many dollars of 4s. 1½d. each must be given in exchange for £235. 10. 9?
6. How many bars of gold each weighing 5 oz. 13 dwts. 21 grs. can be made out of a bar weighing 88 lbs. 8 oz. 14 dwts. 15 grs.?
7. How many bars of iron each weighing 11 lbs. 10 oz. 11 drs. must be taken to make up a weight of 4 tons 8 cwts. 3 lbs. 6 oz. 15 drs.?
8. How many pieces of ribbon, each 5¾ yards long, can be cut off a length of 100 yards; and what length will remain over?
9. How many jars, each containing 2 gall. 3 qts. 1 pt. 3 gills, can be filled out of a cask containing 285 gallons?
10. How many portions of time each equal to 1 day 7 h. 45 min. 56 sec. are contained in 346 days 18 h. 34 min. 32 sec.?
11. How many lengths each equal to 9 poles 3 yds. 1 ft. 3 in. will make up 1 mile 6 fur. 26 p. 4 yds. 2 ft. 9 in.?
12. How many allotments each equal to 2 roods 5 per. 13 yds. 6 ft. 108 in. can be formed out of 158 acres 2 r. 20 per.?

Find the Quotient and Remainder in dividing—

13. £614. 2. 6¼ by 7; £592. 1. 7¾ by 9; £8159. 7. 11½ by 12.
14. £6455. 5. 10¼ by 11; £579. 18. 0 by 45; £1328. 13. 6 by 56.
15. £3839. 18. 7¾ by 96; £4718. 14. 8 by 132; £87. 12. 3½ by 10.
16. £876. 2. 11 by 100; £9658. 17. 3¼ by 100; £32486. 9. 7 by 1000.
17. £9797. 5. 6 by 900; £2075721. 8. 6 by 800; £12862. 1. 6 by 19.
18. £462. 12. 9¼ by 83; £878. 2. 3 by 23; £6263. 7. 0 by 167.
19. £7549. 17. 6 by 859; £77573. 18. 9½ by 4578.

Find the *complete* Quotient in dividing—

20. £37. 12. $2\frac{1}{2}$ by 11; £59. 15. $5\frac{1}{4}\,\frac{1}{8}$ by 9.

21. £23. 4. $10\frac{3}{4}\,\frac{4}{7}$ by 12; £167. 15. $2\frac{1}{2}\,\frac{1}{7}$ by 45.

22. £150. 12. $9\frac{1}{2}\,\frac{5}{8}$ by 56; £997. 18. $10\frac{1}{4}\,\frac{8}{13}$ by 53.

23. £246. 13. $8\frac{1}{4}\,\frac{12}{17}$ by 257; £2357. 16. $10\frac{1}{2}\,\frac{14}{15}$ by 100.

24. £887. 9. $4\frac{1}{4}\,\frac{7}{8}$ by 60; £1692. 3. $1\frac{1}{2}\,\frac{6}{23}$ by 2300.

Divide—giving in the first example the Remainder, and in the second the complete Quotient—

25. 878 weeks 4 d. 15 h. 37 min. 36 s. by 9 and by 56.

26. 679 miles 7 fur. 125 yds. 2 ft. 6 in. by 11 and by 120.

27. 4285 cu. yards 6 ft. 1689 in. by 23 and by 85.

28. 487 tons 13 cwts. 1 qr. 25 lbs. 14 oz. by 9 and by 397.

29. 3964 lbs. 9 oz. 15 dwts. 18 grs. by 12 and by 97.

30. 443 miles 5 f. 8 ch. 13 yds. by 10 and by 95.

31. 409 tons 3 cwts. 26 lbs. 8 drs. by 11 and by 96.

32. 5863 gallons 3 qts. 1 pt. 3 gills by 8 and by 75.

33. 6564 loads 1 qr. 4 bsh. 2 pk. 1 gall. by 5 and by 67.

34. 325 miles 7 fur. 25 po. 3 yds. 2 ft. 3 in. by 7 and by 11.

35. 478 miles 6 fur. 19 po. 2 yds. 1 ft. 10 in. by 96 and by 4397.

36. 854 acres 3 r. 27 p. 8 yds. 8 ft. 45 in. by 9 and by 246.

37. 485 yds. 3 qrs. 3 nl. 2 in. by 7 and by 11.

38. Divide 789 lbs. 12 oz. $14\frac{3}{4}$ drs. by 67; 657 gall. 2 qts. 1 pt. $2\frac{8}{11}$ gills by 43.

39. If 47 bushels of wheat cost £17. 11. $6\frac{1}{4}$, what is the price of 1 bushel, and of 1 peck?

40. If 67 pieces of cloth measure 2335 yds. 2 qrs. 7 in., what is the length of 1 piece?

41. If 28 lbs. 9 oz. of gold be worth £1343. 6. $10\frac{1}{2}$, what is the worth of 1 ounce?

42. If the area of 145 allotments be 415 acres 10 perches, what is the area of 1 allotment?

43. A chest of tea weighing 1 cwt. 1 qr. 15 lbs. cost £22. 10. $9\frac{1}{2}$, what is the cost of 1 lb.?

44. If a man's net income be £1785. 12. 6, how much may he spend on an average per day and per week to the nearest farthing, so as not to run into debt? Reckon 52 weeks, and 365 days to the year.

255. SOME APPLICATIONS OF THE PRECEDING RULES.

I. METHOD OF REDUCTION TO THE UNIT.

If the value, weight, length,... of any number of units be given, we can by Compound Division find that of one unit of the same kind (254); and the value, weight, length,... of one unit being found we can by Compound Multiplication find that of any number of units of the same kind (247). The solution which combines these two processes is called *The Method of Reduction to the Unit.*

Ex. 1. If 16 yards of cloth cost £9. 16. 8, how much will 25 yards cost?

Since the cost of 16 yards is 16 times the cost of 1 yard, the cost of 1 yard is found by *dividing* £9. 16. 8 by 16, and is therefore $\frac{£9.\ 16.\ 8}{16}$; and the cost of 25 yards is found by *multiplying* the cost of 1 yard by 25 and is therefore $\frac{£9.\ 16.\ 8}{16} \times 25$ or $\frac{£9.\ 16.\ 8 \times 25}{16}$.

The process may be arranged thus:—

Since 16 yards cost £9. 16. 8

1 yard costs $\frac{£9.\ 16.\ 8}{16}$

and 25 yards cost $\frac{£9.\ 16.\ 8}{16} \times 25$ or $\frac{£9.\ 16.\ 8 \times 25}{16}$.

		£.	s.	d.
		9	16	8
				5
		49	3	4
				5
16	4)	245	16	8
	4)	61	9	2
		15	7	3½

		£.	s.	d.
16	4)	9	16	8
	4)	2	9	2
			12	3½
				5
		3	1	5½
				5
		15	7	3½

∴ cost of 25 yards = £15. 7. 3½.

REMARK. The *order* of the operations is indifferent, but as a general rule the Multiplication should precede the Division.

Ex. 2. The cost of 2 qrs. 21 lbs. of sugar is £1. 10. 5¾: what is the cost of 4 cwt. 3 qrs. 18 lbs. at the same rate?

2 qrs. = 56 lbs.	4 cwts. = 448 lbs.
21	3 qrs. 18 lbs. = 102
77	550

Since cost of 77 lbs. $= £1.\ 10.\ 5\tfrac{3}{4}$

therefore .... 1 lb. $= \dfrac{£1.\ 10.\ 5\frac{3}{4}}{77}$

and ....550 lbs. $= \dfrac{£1.\ 10.\ 5\frac{3}{4} \times \overset{50}{\cancel{550}}}{\underset{7}{\cancel{77}}}$.

£.	s.	d.
1	10	5¾
		10
15	4	9½
		5
7) 76	3	11½
10	17	8½

£.	s.	d.
7) 1	10	5¾
	4	4¼
		10
2	3	6½
		5
10	17	8½

∴ cost of 4 cwt. 3 qrs. 18 lbs. = £10. 17. 8½.

II. AVERAGES.

The *Average* or *Mean* of any number of given quantities of the same kind, is that quantity which when put in place in each of the given quantities makes their sum the same. Hence to find the Average of any number of quantities we divide the sum of them by their number.

Ex. The receipts at a railway station are as follows: Jan. £245. 17. 10; Feb. £201. 18. 9; March, £285. 14. 7; April, £305. 2. 2; May, £346. 7. 8; and June, £400. 15. 3: find the average receipts per month.

£.	s.	d.
245	17	10
201	18	9
285	14	7
305	2	2
346	7	8
400	15	3
6) 1785	16	3
197	12	8½

The sum of the receipts for the 6 months is found to be £1785. 16. 3: hence the average month's receipts, got by supposing the receipts of every month to be equal, is found by dividing this sum by 6, and is £197. 12. 8½.

III. Revolution of Wheels.

A wheel in making one revolution passes over a length of ground precisely equal to its circumference: hence if we multiply the circumference by the number of revolutions made, we shall find the length of ground passed over; and conversely, if we divide the length of ground passed over by the circumference we shall find the number of revolutions, or by the number of revolutions we shall find the circumference.

Ex. How many revolutions will a carriage wheel 3 yds. $2\frac{1}{2}$ feet in circumference, make in a journey of 7 miles 3 fur. 35 poles?

yds.	ft.
3	$2\frac{1}{2}$
3	
11	
2	
23 half-feet.	

fur.	p.
59	35
40	
2395	
$5\frac{1}{2}$	
11975	
$1197\frac{1}{2}$	
$13172\frac{1}{2}$ yds.	
3	
$39517\frac{1}{2}$	
2	

23) 79035 (3436
100
83
145
7 half-feet = $3\frac{1}{2}$ feet.

∴ No. of revolutions is 3736 and $3\frac{1}{2}$ ft. over.

IV.

Ex. How many sovereigns, half-sovereigns, crowns, florins, shillings, sixpences, and threepences, and of each an equal number, are there in £67. 18. 8?

s.	*d.*
20	0
10	0
5	0
2	0
1	0
	6
	3
38	9
12	
465*d.*	

£	*s.*	*d.*
67	18	8
20		
1358		
12		

465) 16304 (35
2354
29 = 2*s.* 5*d.*

Every collection of one of each of these coins amounts to 38*s.* 9*d.*; hence there will be as many coins of each kind as £67. 18. 8 contains 38*s.* 9*d.*;—that is, there will be 35 coins of each kind, and a remdr 2*s.* 5*d.*

Ex. 1. Find the nearest sum of money to £197. 11. 6 that can be divided by 23 without remainder.

```
    £.   s.  d. £.  s.  d.
23) 197 . 11 . 6 ( 8 . 11 . 9½
     13
     271 ( 11
      18
     222 ( 9
      15
      60 ( ½
      14f.
```

From the work it appears that if the given sum be diminished by 14f. or 3½d. there will be no remainder, or if it be increased by 9f. or 2¼d., so as to make the last partial dividend 69, there will be no remainder; hence the *nearest* sum required is—

£197. 11. 6 + 2¼d. or £197. 11. 8¼.

Ex. 2. If £197. 11. 6 be given for 23 pieces of cloth, find to the nearest penny the price given for each piece.

From the last Ex. it appears that £8. 11. 9 a piece would give 15d. too little, and £8. 11. 10 would give 8d. too much: hence to the nearest penny the price would be £8. 11. 10.

VI. Barter and Exchange.

Ex. 1. How many pounds of tea at 3s. 2½d. a lb. must a grocer give in exchange for 35 yards of cloth at 12s. 5d. a yard?

We must first find what 35 yards at 12s. 5d. a yard amount to; and the number of times 3s. 2½d. is contained in this amount will give the number of pounds of tea.

```
s.  d.                s.  d.
3 . 2½                12 . 5
12                    12
38                    149
 2                      2
77                    298
                       35
                     1490
                     894
          ( 7 ) 10430
       77 (11 )  1490
                  135 and 35 half-pence over.
```

Therefore he will give 135 lbs. and 1s. 5½d.

Ex. 2. How many francs of $9\frac{1}{2}d.$ each will be given in exchange for 475 thalers at $2s.\ 11\frac{1}{2}d.$ each?

```
 d.          d.
 9½          35½
 2            2
 --
19           71
            475
            475
           3325
       ----------
       19 ) 33725 ( 1775
            147
            142
             95
              0
```

The value of 475 thalers is 33725 half-pence, and 1 franc is 19 half-pence; therefore there will be as many francs as 33725 half-pence contains 19 half-pence: or the number of francs is 1775.

VII. MIXTURES—ALLIGATION.

Ex. 1. A tea-merchant mixes 25 lbs. of tea at $1s.\ 9d.$ a lb., 40 lbs. at $2s.\ 5d.$, and 27 lbs. at $3s.\ 2d.$: at what rate per lb. must he sell the mixture?

```
            s.  d.   s.   d.
25 lbs. at  1 . 9 = 43 . 9
40 lbs. at  2 . 5 = 96 . 8
27 lbs. at  3 . 2 = 85 . 6
                  ----------
92            92 ) 225 . 11 ( 2 . 5¼
                   184
                    41
                    12
                   503
                   460
                    43
                     4
                   172
                    92
                    80f.
```

It appears from the work that he mixes 92 lbs., and that its cost is $225s.\ 11d.$, and that the cost of 1 lb. is $2s.\ 5\frac{1}{4}d.$, with a remainder of 80*f.* or $20d.$; hence by selling the whole at $2s.\ 5\frac{1}{4}d.$ a lb. he would lose $20d.$, but if he sells so as only *not to lose* he must sell at $2s.\ 5\frac{1}{2}d.$ a lb., and then he would gain 12*f.* or $3d.$

Ex. 2. At what rate per lb. must he sell the mixture, so as to gain £2. 6. 3 upon the transaction?

```
      s.    d.
     225 . 11
      46 .  3   s.   d.
    -----------
92 ) 272 .  2 ( 2 . 11½
     88
    1058 ( 11
     46
     184 ( ½
      0
```

The tea cost him $225s.\ 11d.$ and his gain is to be $46s.\ 3d.$, therefore he must sell it for the *sum* of $225s.\ 11d.$ and $46s.\ 3d.$, or for $272s.\ 2d.$; and dividing this sum by 92 we get $2s.\ 11\frac{1}{2}d.$, the selling price of 1 lb.

Ex. 3. How many lbs. of tea-dust (which cost him nothing) must be put in the mixture, to enable him to sell the tea at 2*s*. 5*d*. per lb. and gain at the same time 8*s*. 6*d*. on the transaction?

```
     s.   d.
    225 . 11
      8 .  6
    --------
    234 .  5
     12
   -------
29 ) 2813 ( 97
     261
     ---
      203
      203
```

Including his gain of 8*s*. 6*d*. he must receive for the mixture 234*s*. 5*d*.; and dividing this sum by 2*s*. 5*d*., it appears that he must sell 97 lbs.; that is, to the 92 lbs. of tea he must add 5 lbs. of dust; that is

No. of lbs. of tea-dust $= 97 - 92 = 5$.

VIII. Weekly and Daily Expenditure.

A man has a yearly income of £486. 15 and he sets aside £63 for charity, insurance and other purposes; what is the greatest sum he can spend per week, without getting into debt?

```
         £.    s.   d.
        486 . 15 . 0
         63 .  0 . 0
        ------------
52 { 4 ) 423 . 15 . 0
   {13 ) 105 . 18 . 9
        ------------
          8 .  2 . 11¾ ... 4f
```

The sum he may spend in one year or 52 weeks is £423. 15; and dividing this sum by 52, we see that he may spend £8. 2. 11¾ every week, and have 1*d*. over at the end of the year. If he spends £8. 3 per week, he will run into debt.

IX. Division of Money.

Ex. Divide £16. 5. 6 among A, B and C so that A may have £1. 2. 6 more than B, and B 16*s*. 9*d*. more than C.

```
£.  s.  d.                              £.   s.   d.
0 . 16 . 9 to be given to B            16 .  5 .  6
0 . 16 . 9 }                            2 . 16 .  0
1 .  2 . 6 } . . . . . A.          3 ) 13 .  9 .  6
----------                             --------------
2 . 16 . 0                              4 .  9 . 10 C's share.
                                     ∴ 5 .  6 .  7 B's . . .
                                   and 6 .  9 .  1 A's . . .
```

Here B has 16*s*. 9*d*. more than C, and A has £1. 2. 6 + 16*s*. 9*d*. more than C: if we take away these sums, to be subsequently given to B and A respectively, their shares will be equal to that of C:—dividing then the remainder £13. 9. 6 by 3 we get C's share.

x. Men, Women and Boys.

Ex. Divide £15. 12. 6 among 7 men, 9 women, and 11 boys, so that each man may receive three times as much as a boy, and each woman twice as much as a boy.

The 7 men will receive as much as 7×3 or 21 boys, and the 9 women as much as 9×2 or 18 boys; therefore 7 men 9 women and 11 boys will receive as much as $21 + 18 + 11$ or 50 boys; but they receive £15. 12. 6; therefore 1 boy will receive £15. 12. $6 \div 50$ or 6*s.* 3*d.*; and a woman will receive 6*s.* 3*d.* $\times 2$ or 12*s.* 6*d.*; and a man 6*s.* 3*d.* $\times 3$ or 18*s.* 9*d.*

```
7 men   = 21 boys                £.   s.   d.
9 women = 18 .  .         50 { 5 ) 15 . 12 . 6
11 boys = 11 .  .            { 10 ) 3 .  2 . 6
          --                            6 . 3 boy's share.
          50                           12 . 6 woman's .
                                       18 . 9 man's .  .
```

Exercise 35.

I.

1. Find the average of the following scores at cricket:—29, 67, 35, 18, 0, 43, 79, 137, 51, 38, 0 and 91.

2. Reduce 5792685 inches to miles, &c., and prove the result.

3. If 9 yards of cloth cost £7. 0. 0¾, what will 32 yards cost?

4. Divide £189. 5. 7½ among 3 men, so that one of them may have 15 guineas more than either of the other two.

5. A dealer buys a chest of tea weighing 185 lbs. at the rate of 4 lbs. for 7*s.* 6*d.*;—he finds 6 lbs. of it to be worthless, but sells the remainder at an advance of 2½*d.* a lb. on the cost price: find his gain.

6. 1869 sovereigns weigh 480 oz. Troy; find the weight of 1001784 sovereigns in standard weight, the lowest denominations being lbs., oz. and grs.

7. Find the least sum of money that must be subtracted from £663. 14. 8 to make the remainder divisible by 37.

8. A person has an income of £670. 13. 8, and for the first 7 months he spends on an average £58. 16. 9¾ a month: how much must he spend during each of the remaining 6 months, so as not to run into debt?

9. How many lbs. of tea at 3*s.* 2½*d.* a lb. must be given in exchange for 46 yards of silk at 8*s.* 0¾*d.* a yard?

10. A grocer mixes 40 lbs. of tea at 2*s.* 4¾*d.* a lb., 48 lbs. at 2*s.* 8¼*d.* a lb., and 64 lbs. at 3*s.* 2¾*d.* a lb.; find the value of 1 lb. of the mixture.

II.

11. How many Napoleons of 15*s.* 9½*d.* each can be obtained for 5658 thalers of 2*s.* 11½*d.* each?

12. From 15 miles 3 f. 20 p. take 3 po. 4 yds. 1 ft. 3 in.

13. The driving wheel of a locomotive is 5 yds. 2 ft. 9 in. in circumference, and makes on an average 3 revolutions a second: find the rate of the train per hour.

14. How many grains are there in 1 oz.? How many in 1 oz. Troy? Express 11 oz. tr. 168 grs. in imperial weight; the denominations being oz. and grains.

15. If a cannon-ball travel at the rate of 1250 feet per second, how long would it be in passing from the earth to the moon, a distance of 240000 miles?

16. A man buys 25 sheep for £36 and 30 more for £46; what will he gain or lose by selling them at £1. 10. 6 a-piece?

17. A merchant bought thirty-five pieces of cloth measuring on an average 19 yards each at 3*s.* 10½*d.* a yard, and sold them at 5*s.* 7*d.* a yard: what profit did he make?

18. A purse and the money it contains are worth £1. 18. 6, and the money is 10 times the value of the purse: how much does the purse contain?

19. A grocer mixes 19 lbs. of tea at 1*s.* 10¾*d.* a lb., 26 lbs. at 2*s.* 3½*d.* a lb. and 27 lbs. at 1*s.* 6¾*d.* a lb.; at how much per lb. must he sell the mixture so as to gain £2. 3. 4 on his outlay?

20. Divide £20. 2. 6 into two sums of money, one of which contains as many half-crowns as the other contains shillings.

III.

21. The daily receipts of a grocer for the week are as follows:—Monday, £4. 15. 3¾; Tuesday, £5. 13. 0½; Wednesday, £7. 16. 10½; Thursday (being a holiday), nothing; Friday, £3. 18. 11, and Saturday, £15. 19. 7¾; find his average daily receipts (1) excluding Thursday, and (2) including Thursday.

22. Which is heavier, 1 lb. of gold or 1 lb. of sugar? 1 oz. of gold or 1 oz. of sugar; and why? Reduce 513769 grains of gold and 15346907 oz. of sugar to their higher denominations.

23. To march at quick step is to take 108 paces of 2 ft. 8 in. per minute—what rate is this per hour? How long would it take a body of soldiers to march 36 miles at quick step?

24. If 1 cwt. of sugar cost £2. 11. 4, what will be the cost of 2 qrs. 13 lbs.?

25. What annual income would enable a person to spend 8*s*. 9*d*. a day and save £7. 16. 10½ every calendar month?

26. Divide £2451. 10*s*. among A, B, and C, so that A may have £178. 13. 6 more than B, and C £325. 14. 7½ less than B.

27. Find the salary of a person who pays £12. 7. 11 income-tax, when the tax is 7*d*. in the pound.

28. A brigade in close column occupies 1 rood 8 p. 1 yd. 6 ft. of ground, and each man occupies 3 sq. ft. 9 in.; of how many men does the brigade consist?

29. Find the value of 1 ton 2 cwt. 1 qr. 8 lbs. of copper coin, when 3 pennies weigh 1 oz.

30. There are three quantities, 6 hours, 20 minutes, and £36; multiply one of these by the quotient of the other two, and state fully the result of the operation.

IV.

31. If 17 chests of tea can be bought for £301. 10. 9: how much must be given for 56 chests?

32. Reduce 1847638 inches to miles, &c., and 59496275 sq. in. to acres, &c., and prove the correctness of the result in each case.

33. Divide £11. 15. 4½ among A, B and C, so that A may receive twice as much as B, and B twice as much as C.

34. If the cost of 3050 things be £17340. 10. 5, find the cost of 3099 things at the same rate. *Find the value of 49 things.*

35. If butter be bought at 5½ guineas the cwt., and retailed at 1*s*. 4½*d*. the lb., how much will be gained on every cwt.?

36. What weight of gold coin is equal to 3 cwts. of copper coin?

37. 120 tons of coal are purchased for £87. 16. 9; find to the nearest farthing the price at which they must be retailed per ton, so that no loss may be incurred; and at that price what profit will accrue?

38. If a person has an income of £535. 17. 6 a year, and he spends daily £1. 3. 10½, how much will he have over at the end of the year?

39. How many dollars of 4*s.* $1\frac{3}{4}$*d.* each must be given in exchange for 4950 thalers of 2*s.* $11\frac{1}{2}$*d.* each?

40. A spirit merchant mixes 26 gallons at 12*s.* 3*d.* a gallon with 39 gallons at 13*s.* 4*d.* a gallon: how many gallons of water must he add to the mixture so as to sell it at 10*s.* 9*d.* a gallon?

V.

41. Deduct £2. 13. $8\frac{3}{4}$ from £56. 5*s.*, and divide the resulting sum equally among 29 persons to the nearest farthing; how much will each person receive, and how much will remain over?

42. Divide £9. 17. $8\frac{1}{2}$ between two persons so that one may have four times as much as the other.

43. What weight of sugar at $4\frac{1}{2}$*d.* a lb. must be given in exchange for a chest of tea weighing 84 lbs. at 3*s.* $1\frac{1}{2}$*d.* a lb.?

44. A loaded truck weighs 10 tons 14 cwt. 2 qrs.: the goods are five times the weight of the empty truck, find the weight of the goods.

45. A pipe of wine containing 116 gallons is bought for £112: how much water must be added to it to allow of its being sold at 17*s.* 6*d.* a gallon?

46. Find the weight of copper coin required to pay a debt of £1000, when 3 pennies weigh 1 oz.

47. At the end of a week £54. 3*s.* is paid in wages to an equal number of men, women and boys; a man earns 4*s.* 6*d.*, a woman 3*s.* 3*d.* and a boy 1*s.* 9*d.* a day: how many of each class are there?

48. Sound travels at the rate of 1142 ft. per second; what is the distance of the thunder-cloud, when the thunder succeeds the lightning at an interval of 9 seconds?

49. The average price of a quarter of wheat for 19 years was 56*s.* 8*d.* a quarter; for the first five years the average price was 61*s.* $3\frac{1}{2}$*d.* a quarter, for the next 4 years 58*s.* $0\frac{1}{2}$*d.*, for the next 7 years, 53*s.* $5\frac{3}{4}$*d.*: find the average of the last 3 years.

50. There are 6 presses at work striking off sovereigns, half-sovereigns, florins, shillings, sixpences, and fourpences respectively, and each at the rate of 2500 per hour: find the value of the money struck off in 13 days of 9 hours each.

VI.

51. The mean height of 6 mountains is 10357 feet; find what the height of the seventh mountain must be, in order that the mean height of the seven mountains may be 10643 ft.

52. If 47 yards of cloth cost £34. 10. $3\frac{3}{4}$, find the cost of 99 yards.

53. The forewheel and hindwheel of a carriage are respectively 2 yds. 2 ft. 6 in. and 4 yds. 1 ft. 4 in. in circumference: how many more revolutions will the forewheel make than the hindwheel in going over a distance of 19 miles 2 fur. 120 yds.?

54. How many grains are there in 1 lb. Av.? how many in 1 lb. Troy? Express 21560 lbs. Troy in Av. weight: the lowest denominations being oz. and grains.

55. Divide £250. 19. 9 among A, B and C, so that B may receive 3 times, and C 5 times, as much as A.

56. A person after paying an income-tax of 5*d.* in the pound has remaining £856. 15. 5; find his full income.

57. If a waggon carries away each time 1 cu. yd. 5 ft. 1440 in. of earth, how many loads are carried away in excavating 2347 cu. yds.: and how much is carried in the last load?

58. A man exchanges 45 sheep at £1. 5. 9 each and 37 pigs at £3. 13. 6 each for 13 oxen at $16\frac{1}{2}$ guineas each, the difference being paid or received in money; how much does he pay or receive?

59. Tithes of the value of £448. 10*s.* are commuted for an equal number of bushels of wheat, barley, and oats; how many bushels of each kind will be received when wheat is sold at 7*s.* 2*d.* a bushel, barley at 4*s.* 9*d.*, and oats at 3*s.* 5*d.*?

60. Find the distance between two towns when £30. 18. 8 is paid for the fare of 17 first class passengers at $2\frac{1}{2}$*d.* a mile, of 26 second class at $1\frac{3}{4}$*d.* a mile, and of 40 third class at 1*d.* a mile.

VII.

61. If a person spend £135. 8. $8\frac{3}{4}$ in 23 weeks, how much will he spend in a year?

62. A has £100. 11. $4\frac{1}{2}$, and B has 64391 farthings: if A receives 34567 farthings, and B receives £58. 16. $7\frac{3}{4}$, which will then have the most, and by how much?

63. Reduce 294322493 sq. in. to acres, &c. and prove the result.

64. Divide £119. 16. 3 among 36 persons in such a way that 17 of them may each receive 18*s.* 9*d.* more than each of the rest.

65. Reduce 35 tons 19 cwts. 99 lbs. 12 oz. 135 grs. to grains. *First, reduce* 12 *oz.* 135 *grs.* to grains.

66. The construction of a mile of a certain railway costs a million pounds: at this rate what is the cost of construction, computed to the nearest penny, of every inch of its length?

67. A grocer buys 4 cwt. of sugar at $6d.$ per lb., and 8 cwt. at $4\frac{1}{2}d.$ per lb. He sells 6 cwt. at $5\frac{1}{2}d.$ per lb., at what rate per lb. must he sell the remainder so as neither to gain nor lose?

68. A merchant laid out £69. $6s.$ in spirits which he bought at $12s.$ $10d.$ a gallon; he retailed it at $16s.$ $6d.$ a gallon, making a profit of £11. $11s.$: how many gallons must he have lost by leakage?

69. £75 is paid in wages at the end of the week to a certain number of men, twice as many women, and three times as many children; each man earns $4s.$ $2d.$ a day, each woman $2s.$ $9d.$, and each child $2s.$ $4d.$; how many children are there?

70. There are five presses striking off at the same rate florins, shillings, sixpences, fourpences and threepences, and the value of the money coined in a day of 8 hours is £2672. 2. 8: how many coins does a press strike off in one hour?

VIII.

71. If a person has a yearly income of £350, and he spends at the rate of £78. 10. $8\frac{2}{3}$ in 85 days, how much will he be able to lay by at the end of the year?

72. Find the circumference of the wheel of a locomotive which makes on an average 4 revolutions in a second, and which performs a journey of 76 miles in 1 hour 36 min.

73. Find the greatest weight that is contained exactly in 3 tons 5 lbs. and in 20 tons 3 cwts. 2 qrs.

74. If the value of the United States dollar be $4s.$ $2d.$, how many dollars must be given for £500? Find also the least number of pounds that contains an exact number of dollars.

75. Divide £15. $6s.$ among 12 men, 17 women and 26 children in such a way that a man shall receive 3 times as much as a child, and a woman twice as much as a child: what does a woman receive?

76. A gives B 112 gallons of brandy at $32s.$ $6d.$ a gallon, and receives in return £40. 12. 6 and 780 yards of cloth: what is the price of the cloth per yard?

77. What is the least sum of money that can be paid in francs of $10d.$ each, in half-crowns, in thalers of $2s.$ $11d.$ each, and in dollars of $4s.$ $2d.$ each?

78. Employ short division in dividing 195477 by 7920; write down the final remainder, and compare the process by which 195477 in. may be reduced to furlongs, yds., ft. and in.

79. Divide £115. 2. 6 among 20 women and 25 men, so that each woman may receive 15*s.* more than each man: how much will each woman receive?

80. How long would a column of men 2145 feet in length take to march through a street 3 furlongs long at the rate of 75 paces a minute, each pace being $2\frac{3}{4}$ feet?

IX.

81. If 25 ounces of gold is worth £97. 6. $10\frac{1}{2}$, what will be the worth of 15 bars each weighing 5 lbs. 3 oz.?

82. By the payment of 2*s.* 1*d.* in London a banker will give credit at Calcutta for 1 rupee: how many rupees may be received in Calcutta for the payment of £5025. 6. 8 in London?

83. In how many days of 8 hours each will a person be able to count 1,000,000 sovereigns at the rate of 80 per minute? How many will remain to be counted on the morning of the 26th day?

84. A merchant buys 84 gallons of whiskey at 16*s.* 9*d.* a gallon, and sells it at 16*s.* 6*d.* a gallon, making a profit of 10 guineas: how many gallons of water did he add to the whiskey?

85. On the reduction of the income-tax from 9*d.* in the pound to 4*d.*, a person saves £29. 15. 10; find his gross income.

86. A man bought 150 apples at 2 a penny, and 150 more at 3 a penny, and mixed them and sold the whole at 5 for 2*d.*; how much did he lose, and where did the loss occur?

87. The total stock of gold coin and bullion in the Bank of England on a certain day being of the value of £16548126, and the weight of it 354160 lbs., determine the value of an ounce of gold.

88. The weekly wages at a mill amount to £186. 4*s.* In the mill a certain number of women are employed at 2*s.* 10*d.* a day, five times as many men at 5*s.* 6*d.* a day, and 6 times as many boys at 2*s.* 4*d.* a day: how many men are employed?

89. 1 lb. Troy of standard silver is coined into 66 shillings: what would be the error in weight if 264 crowns were considered as each weighing 1 oz. Av.? what in value?

90. A body of military one furlong in length is about to pass through a defile 3 miles 44 yds. long at quick step, which is 108 steps of 2 ft. 8 in. each per minute; what time must elapse before the last man clears the defile?

REDUCTION AND THE COMPOUND RULES—FRACTIONS.

REDUCTION.

256. While "times" denotes the multiplication of a quantity by an integer, "of" denotes its multiplication by a fraction, and either "times" or "of" its multiplication by a mixed number:—thus we say 3 *times* 7 gallons, $\frac{3}{5}$ *of* 7 gallons, and either $4\frac{3}{5}$ times 7 gallons or $4\frac{3}{5}$ of 7 gallons, but each expression denotes the multiplication of 7 gallons by 3, by $\frac{3}{5}$, and by $4\frac{3}{5}$ respectively.

3 times 7 gallons is then 7 gallons $\times$ 3, and it is often written 3×7 gallons; but since the Multiplier must be a number, we must either read the expression as 3 times 7 gallons, or we must suppose Multiplicand and Multiplier to be interchanged. Also 3×7 gallons is often used for (3×7) gallons, but unless we especially wish to shew that the operation *has been performed* although desirous of exhibiting the factors, no harm will arise from neglecting the bracket.

257. In Reduction we have to consider the two following cases:—

(i) To reduce a *fraction* of one denomination to a lower denomination; and conversely,

(ii) To reduce a quantity of one denomination to a *fraction* of a higher denomination.

I.

For example, reduce £$\frac{11}{64}$ to shillings, and $\frac{7}{16}$ *s.* to pence.

Now $\qquad$ £$\frac{11}{64} = \frac{11}{64}$ of 20*s.* $= \left(\frac{11}{64} \times 20\right)$ *s.* $= \frac{11 \times 20}{64}$ *s.*

and $\qquad$ $\frac{7}{16}$ *s.* $= \frac{7}{16}$ of 12*d.* $= \left(\frac{7}{16} \times 12\right)$ *d.* $= \frac{7 \times 12}{16}$ *d.*

hence to reduce a fraction of a pound to shillings we multiply the fraction by 20, and to reduce a fraction of a shilling to pence we multiply the fraction by 12. And as a like method is applicable to fractions of other denominations we have this Rule—

Multiply the fraction of the given denomination by the number which tells how many of the lower denomination make one of the given denomination.

Ex. 1. $\frac{8}{27}$ of a day $= \frac{8 \times 24}{27}$ hours $= \frac{64}{9}$ hrs. $= 7\frac{1}{9}$ hrs.

and $= \frac{8 \times 24 \times 60}{27}$ min. $= \frac{1280}{3}$ min. $= 426\frac{2}{3}$ min.

and $= \frac{8 \times 24 \times 60 \times 60}{27}$ sec. $= 25600$ sec.

II.

257*. Reduce $5\frac{1}{4}d.$ to the fraction of a shilling, and $3\frac{7}{18}s.$ to the fraction of a pound.

Now $5\frac{1}{4}d. = 5\frac{1}{4}$ of $1d. = 5\frac{1}{4}$ of $\frac{1}{12}s. = \left(5\frac{1}{4} \times \frac{1}{12}\right)s. = \frac{5\frac{1}{4}}{12}s.$

and $3\frac{7}{18}s. = 3\frac{7}{18}$ of $1s. = 3\frac{7}{18}$ of £. $\frac{1}{20} = \left(3\frac{7}{18} \times \frac{1}{20}\right)$£. $= \frac{3\frac{7}{18}}{20}$ £:

hence to reduce any number of pence to the fraction of a shilling we divide the number by 12, and to reduce any number of shillings to the fraction of a pound we divide by 20. And as a like method is applicable to other denominations we have this Rule—

Divide the number of the given denomination by the number which tells how many of that denomination make one of the higher denomination.

Ex. 2. $18\frac{3}{4}$ grs. $= \frac{18\frac{3}{4}}{24}$ dwts. $= \frac{75}{96}$ dwts. $= \frac{25}{32}$ dwts.;

and $= \frac{18\frac{3}{4}}{24 \times 20}$ oz. $= \frac{75}{96 \times 20}$ oz. $= \frac{5}{128}$ oz.;

and $= \frac{18\frac{3}{4}}{24 \times 20 \times 12}$ lb. $= \frac{75}{96 \times 20 \times 12}$ lb. $= \frac{5}{1536}$ lb.

258. Sometimes in reducing a fraction of one denomination to a fraction of another denomination we have to employ both the descending and the ascending process: for example,

Reduce $\frac{13}{14}$ of a guinea to the fraction of £1:

here the denomination *common* to guineas and pounds is shillings: we

therefore reduce the given quantity to shillings, and the result to the fraction of a pound: thus

$$\frac{13}{14} \text{ of a guinea} = \frac{13 \times 21}{14} s. = \frac{39}{2} s. = £\frac{39}{2 \times 20} = £\frac{39}{40}.$$

259. The preceding cases enable us

(i) To reduce a fraction of one denomination to a compound quantity of lower denomination: and

(ii) To reduce a compound quantity to a fraction of a higher denomination.

I.

Ex. 1. Find the value of $\frac{37}{45}$ of a lb. Troy.

$$\frac{37}{45} \text{ lb. Troy} = \frac{37 \times 12}{45} \text{ oz.} = \frac{148}{15} \text{ oz.} = 9\tfrac{13}{15} \text{ oz.}$$

$$\frac{13}{15} \text{ oz.} = \frac{13 \times 20}{15} \text{ dwts.} = \frac{52}{3} \text{ dwts.} = 17\tfrac{1}{3} \text{ dwts.}$$

$$\frac{1}{3} \text{ dwt.} = \frac{1 \times 24}{3} \text{ grs.} = 8 \text{ grs.}$$

$$\therefore \frac{37}{45} \text{ lb. Troy} = 9 \text{ oz. } 17 \text{ dwts. } 8 \text{ grs.}$$

260. If the denominator of the fraction and the several multipliers by which we pass from the given denomination to the lower denominations have *no* common factor, we shall find it easier to proceed by Compound Division than by Reduction: for example,

Find the value of $£\frac{5}{7}$.

Since $£\frac{5}{7} = \frac{£5}{7}$ (103) we have simply to divide £5 by 7, thus

$$\begin{array}{r|l} & £. \quad s. \quad d. \\ 7 & 5 \,.\, 0 \,.\, 0 \\ \hline & \quad 14 \,.\, 3\tfrac{3}{7} \end{array}$$

or, without any *arrangement*, we may write off at once

$$£\frac{5}{7} = 14s.\ 3\tfrac{3}{7}d.$$

In like manner, $\frac{8}{9}$ cwt. = 3 qrs. 15 lbs. 8 oz. $14\frac{2}{9}$ drs.

261. And if they *have* a common factor, we shall often find it convenient to combine the processes of Reduction and Division: for example,

Find the value of $\frac{37}{44}$ of a pole.

$$\frac{37}{44}\text{ pole} = \frac{37}{44} \times \frac{11}{2}\text{ yds.} = \frac{37}{8}\text{ yds.}$$

$$= 4\text{ yds. } 1\text{ ft. } 10\tfrac{1}{2}\text{ in.}$$

where the result $\frac{37}{8}$ yds. is got by Reduction, and the final result by Division.

II.

262. Ex. 2. Reduce 5 cwts. 3 qrs. 24 lbs. to the fraction of a ton.

$$24\text{ lbs.} = \frac{24}{28}\text{ qr.} = \frac{6}{7}\text{ qr.}$$

$$\therefore 3\text{ qrs. } 24\text{ lbs.} = 3\tfrac{6}{7}\text{ qrs} = \frac{3\frac{6}{7}}{4}\text{ cwt.} = \frac{27}{28}\text{ cwt.},$$

and $$5\text{ cwts. } 3\text{ qrs. } 24\text{ lbs.} = 5\tfrac{27}{28}\text{ cwts.} = \frac{5\frac{27}{28}}{20}\text{ ton} = \frac{167}{560}\text{ ton.}$$

Ex. 3. Express £37. 16. 6¾ in pounds only.

$$6\tfrac{3}{4}d. = \frac{6\frac{3}{4}}{12}s. = \frac{27}{48}s. = \frac{9}{16}s.;$$

$$\therefore 16s.\ 6\tfrac{3}{4}d. = 16\tfrac{9}{16}s. = \frac{16\frac{9}{16}}{20}£. = \frac{265}{320}£. = \frac{53}{64}£.;$$

and $$\therefore £37.\ 16.\ 6\tfrac{3}{4} = £37\tfrac{53}{64}.$$

263. Sometimes instead of proceeding step by step through all the intervening denominations from the lowest to the one required, as in the two preceding Examples, we reduce the given quantity to its lowest denomination and reach the result in a single step.

Ex. 2*. Reduce 5 cwts. 3 qrs. 24 lbs. to the fraction of a ton.

cwts.	qrs.	lbs.
5	3	24
23, × 4		
92, × 7		
668		

$\therefore$ 5 cwts. 3 qrs. 24 lbs. = 668 lbs.

$$= \frac{668}{28 \times 4 \times 20}\text{ ton}$$

$$= \frac{167}{560}\text{ ton.}$$

EXERCISE 36.

Reduce

1. $10\frac{1}{2}d.$ to the fraction of a shilling; $15\frac{3}{8}s.$ to the fraction of a pound.
2. $\frac{3}{5}$ of a farthing to the fraction of $1s.$; $\frac{5}{8}d.$ to the fraction of £1.
3. $235\frac{5}{7}d.$ to shillings, and to the fraction of a pound.
4. $\frac{3}{8}$ dwt. to the fraction of 1 lb. Troy; $\frac{5}{9}$ min. to the fraction of a week.
5. $12\frac{1}{3}$ oz. to the fraction of a cwt.; $18454\frac{1}{3}$ seconds to hours.
6. $1067\frac{6}{17}$ cu. inches to the fraction of a cu. yard.
7. $4\frac{2}{5}$ feet to the fraction of a furlong; $\frac{5}{7}$ sq. yds. to that of an acre.
8. $492\frac{1}{5}$ hours to the fraction of a year of $365\frac{1}{4}$ days.

Reduce

9. $\frac{4}{7}s.$ to farthings; £$\frac{3}{32}$ to shillings; £$\frac{5}{162}$ to pence.
10. £$\frac{1}{1320}$ to the fraction of a penny; $\frac{1}{250}s.$ to the fraction of a farthing; $\frac{1}{1344}$ of a guinea to the fraction of a farthing.
11. $\frac{9}{700}$ cwt. to the fraction of 1 lb.; $\frac{1}{4}$ ton to qrs.; $\frac{1}{25}$ oz. to grs.
12. $\frac{2}{15}$ lb. Troy to dwts.; $\frac{1}{14}$ gallon to the fraction of a pint.
13. $\frac{4}{1055}$ of a mile to poles; $3\frac{4}{1056}$ miles to yards.
14. $\frac{2}{121}$ of an acre to sq. yards; $5\frac{4}{121}$ acres to perches.

Reduce

15. $\frac{3}{175}$ lb. to the fraction of 1 lb. Troy; $\frac{1}{5}$ nail to that of a foot.
16. $\frac{2}{25}$ of a yard to the fraction of an ell; $\frac{5}{9}$ link to the fraction of a ft.
17. $12\frac{1}{2}$ oz. Av. to the fr. of 1 lb. Tr.; 35 lbs. $8\frac{2}{3}$ oz. Tr. to that of a cwt.

Reduce

18. $16s.\ 9\frac{3}{4}d.$, $6s.\ 11\frac{3}{4}d.$, $14s.\ 4\frac{4}{7}d.$ to the fractions of £1.
19. £4. 9. $2\frac{1}{4}$, £3. 19. $8\frac{1}{4}$ and £5. 16. $11\frac{3}{4}$ to pounds only.
20. 4 oz. 15 dwts. 15 grs. to the fraction of a pound Troy; and then to the fraction of a lb.
21. 15 cwts. 3 qrs. 17 lbs. 8 oz. to cwts. and to the fraction of a ton.
22. 5 bushels 3 pks. 1 gall. to the fraction of a quarter.
23. 3 qrs. 27 lbs. 9 oz. $12\frac{2}{3}$ dr. to the fraction of a cwt.
24. 3 furlongs 29 po. 4 yds. 1 ft. 9 in. to the fraction of a mile.

Reduce

25. 8 perches 27 yds. $6\frac{3}{16}$ ft. to perches, and to the fraction of an acre.

26. 72 days 6 hrs. 56 m. 15 s. to the fraction of a year of $365\frac{1}{4}$ days.

Find the value in lower integral denominations of

27. $\frac{13}{48}s.$, $4\frac{12}{13}d.$, £$\frac{3}{11}$, £$\frac{1}{250}$; $13\frac{8}{9}s.$, £$8\frac{827}{960}$, £$\frac{7}{11}$, £$5\frac{377}{480}$.

28. $\frac{5}{6}$ lb. Troy; $\frac{4}{7}$ lb.; $\frac{15}{16}$ oz. Troy; $2\frac{3}{16}$ lbs. Troy; $\frac{4}{7}$ cwt.

29. $\frac{1735}{1792}$ of a ton; $\frac{7}{11}$ of an acre; $5\frac{7}{8}$ furlongs; $\frac{803}{1056}$ of a mile.

30. $\frac{17}{45}$ of a day; $\frac{29}{165}$ of an acre; $2\frac{4}{5}$ of $\frac{4}{15}$ of a cwt.; $3\frac{41}{180}$ ells.

31. $\dfrac{3\frac{1}{8}}{12\frac{1}{17}}$ of a year of 365 days; $\dfrac{3\frac{1\frac{3}{4}}{4}}{20}$ of a ton; $\dfrac{6\frac{11\frac{3}{8}}{12}}{14}$ of a week.

32. $1\frac{4}{5}$ of $\dfrac{1}{1+\dfrac{1}{9+\dfrac{3}{4}}}$ of a gallon; $\dfrac{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}}{\frac{1}{2\frac{1}{2}}+\frac{1}{3\frac{1}{3}}+\frac{1}{4\frac{1}{4}}}$ of $\frac{4}{7}$ of a perch.

Find the value of

33. £$7\frac{3}{4}$ + $7\frac{7}{18}s.$ + $6\frac{3}{4}d.$; £$3\frac{5}{8}$ + $7\frac{1}{3}s.$ + $4\frac{5}{8}d.$

34. $\frac{3}{7}$ cwt. + $8\frac{3}{4}$ lbs. + $3\frac{9}{10}$ oz.; $\frac{3}{8}$ of a ton + $\frac{5}{8}$ of a cwt. + $\frac{3}{4}$ of a lb.

35. $3\frac{7}{11}$ miles − $7\frac{7}{12}$ fur. + $35\frac{11}{12}$ po.; $\frac{5}{32}$ sq. mile + $\frac{7}{10}$ acre + $\frac{5}{8}$ of a rood.

36. $\frac{4}{5}$ of a week + $\frac{3}{4}$ of a day + $\frac{4}{7}$ of an hour + $\frac{3}{4}$ of a minute.

MULTIPLICATION, DIVISION, &c.

264. (i) To multiply a quantity by a fraction—*multiply by the numerator and divide the product by the denominator* (128).

(ii) To multiply a quantity by a mixed number—(1) *multiply separately by the integer and by the fraction and add* (131 Rk.); or (2) *reduce the mixed number to a fraction and proceed by* (1).

(iii To divide a quantity by a fraction—*multiply by the denominator and divide the product by the numerator* (134 Cor.).

(iv) To divide a quantity by a mixed number—*reduce the mixed number to a fraction, and proceed by* (iii).

(v) To take a fractional part of a quantity—*multiply the quantity by the fraction* (128); that is, *proceed by* (i) *or* (ii).

Ex. 1. Multiply £25. 14. $9\frac{3}{4}$ by $\frac{7}{11}$.

Ex. 2. Divide 15 tons 13 cwts. 2 qrs. 20 lbs. by $\frac{3}{7}$.

£.	s.	d.
25	14	$9\frac{3}{4}$
		7
11) 180	3	$8\frac{1}{4}$
16	7	$7\frac{1}{4}\frac{2}{11}$

tons.	cwts.	qrs.	lbs.
15	13	2	20
			7
3) 109	15	3	0
36	11	3	$18\frac{2}{3}$

Ex. 3. Multiply £45. 12. $6\frac{3}{4}$ by $12\frac{9}{25}$.

Ex. 4. Divide 345 lbs. 9 oz. 16 dwts. 20 grs. by $13\frac{5}{7}$.

£.	s.	d.	
45	12	$6\frac{3}{4}$	
		9	
5) 410	13	$0\frac{3}{4}$	
5) 82	2	$7\frac{1}{4}\frac{2}{5}$	
16	8	$6\frac{1}{4}\frac{2}{25}$	product by $\frac{9}{25}$
547	10	9	12
563	19	$3\frac{1}{4}\frac{2}{25}$	...............$12\frac{9}{25}$

$13\frac{5}{7} = \frac{96}{7}$

lbs.	oz.	dwts.	grs.
345	9	16	20
			7
12) 2420	8	17	20
8) 201	8	14	$19\frac{1}{3}$
25	2	11	$20\frac{1}{3}\frac{1}{4}$

Ex. 5. Find the difference between $\frac{7}{8}$ of £35. 14. $7\frac{1}{2}$ and $2\frac{5}{6}$ of £5. 14. $9\frac{3}{4}$.

£.	s.	d.	
35	14	$7\frac{1}{2}$	
		7	
8) 250	2	$4\frac{1}{2}$	
31	5	$3\frac{1}{2}\frac{1}{8}$	
16	5	$3\frac{1}{2}\frac{1}{4}$	
14	19	$11\frac{3}{4}\frac{3}{4}$	Difference reqd.

£.	s.	d.	
5	14	$9\frac{3}{4}$	
		5	
6) 28	14	$0\frac{3}{4}$	
4	15	$8\frac{1}{4}\frac{1}{2}$	product by $\frac{5}{6}$
11	9	$7\frac{1}{2}$	2
16	5	$3\frac{1}{2}\frac{1}{4}$	

(vi) When the quantity is simple or can be easily reduced to a simple quantity, the arrangement above is usually dispensed with; we cancel, multiply, and proceed by reduction (259) or by compound division (260).

Ex. 6. $\frac{5}{7}$ of 2 guineas $=\frac{5}{7}$ of 42*s.* $=\frac{5\times 42}{7}$*s.* $=30$*s.* $=$ £1. 10*s.*

Ex. 7. $3\frac{3}{4}$ of half-a-crown $=\frac{15}{4}\times\frac{5}{2}$*s.* $=\frac{75}{8}$*s.* $=9$*s.* $4\frac{1}{2}$*d.* (260)

Ex. 8. $\frac{8}{21}$ of £4. 10*s.* $=\frac{8}{21}\times 90$*s.* $=\frac{240}{7}$*s.* $=34\frac{2}{7}$*s.* $=$ £1. 14. $3\frac{3}{7}$.

Ex. 9. Find the value of $\frac{3}{4}$ of a guinea $+\frac{3}{8}$ of a crown $+\frac{3}{5}$ of 7*s.* 6*d.* $-\frac{3}{4}$ of 2*d.*

$$\begin{array}{llr@{\ .\ }r@{\ .\ }l}
 & & \text{£} & s. & d. \\
\frac{3}{4}\text{ of a guinea} = \frac{3}{4}\text{ of }21s. & = \frac{63}{4}s. = & 0 & 15 & 9 \\
\frac{3}{8}\text{ of a crown} = \frac{3}{8}\text{ of }5s. & = \frac{15}{8}s. = & & 1 & 10\frac{1}{2} \\
\frac{3}{5}\text{ of }7s.\ 6d. = \frac{3}{5}\text{ of }\frac{15}{2}s. & = \frac{9}{2}s. = & & 4 & 6 \\
\hline
 & & 1 & 2 & 1\frac{1}{2} \\
\frac{3}{4}\text{ of }2d. & = \frac{3}{2}d. = & & & 1\frac{1}{2} \\
\hline
 & & 1 & 2 & 0\ \text{Value reqd.}
\end{array}$$

Ex. 10. $\frac{2}{7}$ of £15 $+3\frac{3}{7}$ of £1 $+\frac{1}{3}$ of $\frac{5}{7}$ of $\frac{3}{5}$ of £1 $+\frac{2}{3}$ of $\frac{3}{7}$ of 1*s.*

$$\begin{aligned}
&= \text{£}\tfrac{2}{7}\times 15 + \text{£}\tfrac{24}{7} + \text{£}\tfrac{1}{3}\times\tfrac{5}{7}\times\tfrac{3}{5} + \tfrac{2}{3}\times\tfrac{3}{7}s. \\
&= \text{£}\tfrac{30}{7} + \text{£}\tfrac{24}{7} + \text{£}\tfrac{1}{7} + \tfrac{2}{7}s. \\
&= \text{£}\tfrac{55}{7} + \tfrac{2}{7}s. = \text{£}7.\ 17\tfrac{1}{7}s. + \tfrac{2}{7}s. \\
&= \text{£}7.\ 17\tfrac{3}{7}s. \\
&= \text{£}7.\ 17.\ 5\tfrac{1}{7}. \qquad\qquad (260)
\end{aligned}$$

Ex. 11. From $\frac{7}{18}$ of a mile take $\frac{5}{7}$ of $3\frac{4}{5}$ poles.

$$\begin{array}{ll r@{\ .\ }r@{\ .\ }r@{\ .\ }r@{\ .\ }l}
 & & \text{fur.} & \text{po.} & \text{yds.} & \text{ft.} & \text{in.} \\
\frac{7}{18}\text{ mile} = \frac{7}{18}\times 8\text{ fur.} & = \frac{28}{9}\text{ fur.} = & 3 & 4 & 2 & 1 & 4 \\
\frac{5}{7}\text{ of }3\frac{4}{5}\text{ poles} = \frac{5}{7}\times\frac{19}{5}\text{po.} & = \frac{19}{7}\text{po.} = & & 2 & 3 & 2 & 9\frac{3}{7} \\
\hline
 & & 3 & 1 & 3\frac{1}{2} & 1 & 6\frac{4}{7} \\
 & & & & & 1 & 6\ \ldots\ldots(236) \\
\hline
\therefore\ \text{Difference} & = & 3 & 1 & 4 & 0 & 0\frac{4}{7}
\end{array}$$

EXERCISE 37.

Find the value of

1. £3. 16. $8\frac{3}{4} \times \frac{5}{7}$; £6. 18. $7\frac{3}{4}\frac{3}{4} \times \frac{8}{15}$.

2. £34. 12. $5\frac{3}{4} \times 11\frac{7}{11}$; £2507. 19. $8\frac{3}{4}\frac{3}{5} \times 30\frac{5}{8}$.

3. 7 lbs. 9 oz. 15 dwts. $22\frac{8}{11}$ grs. $\times 4\frac{3}{5}$; 2 qrs. 3 lbs. $11\frac{5}{7}$ oz. $\times 12\frac{7}{8}$.

4. 19 hrs. 43 m. $56\frac{8}{13}$ s. $\times 12\frac{7}{10}$; 10 ac. 3 r. 37 p. $15\frac{8}{9}$ yds. $\times 10\frac{6}{7}$.

5. £15. 8. $4\frac{3}{4} \div \frac{3}{5}$; £18. 16. $7\frac{1}{2}\frac{5}{7} \div 3\frac{2}{3}$.

6. £34. 16. $9\frac{1}{2} \div 9\frac{5}{8}$; £8. 18. $5\frac{3}{4}\frac{5}{9} \div 1\frac{6}{7}$.

7. £3976. 18. $7\frac{3}{4} \div 23\frac{11}{16}$; £879. 16. $7\frac{3}{4}\frac{14}{17} \div 8\frac{13}{15}$.

8. 4 lbs. 9 oz. 14 dwts. $15\frac{5}{7}$ grs. $\div \frac{5}{8}$; 8 days 15 h. 48 m. $57\frac{13}{63}$ s. $\div 4\frac{7}{8}$.

9. 13 cwts. 3 qrs. 26 lbs. $15\frac{7}{8}$ oz. $\div 3\frac{4}{7}$; 6 m. 7 fur. 15 p. $0\frac{6}{13}$ yds. $\div 3\frac{5}{12}$.

10. $\frac{7}{11}$ of £5. 19. $3\frac{1}{2}$; $\frac{11}{12}$ of £3. 14. $10\frac{3}{4}\frac{1}{3}$.

11. $3\frac{3}{8}$ of £8. 13. $8\frac{3}{4}\frac{5}{8}$; $29\frac{3}{5}$ of £125. 17. $6\frac{3}{4}\frac{3}{16}$.

12. $\frac{19}{67}$ of 789 lbs. 12 oz. $14\frac{3}{8}$ drs.; $\frac{4}{9}$ of 657 gal. 2 qts. 1 pt. $3\frac{6}{11}$ gills.

13. £10. 12. $4\frac{3}{4} \times 1\frac{7}{18}$ − £5. 18. $6\frac{3}{4} \div 10\frac{4}{15}$.

14. $7\frac{15}{17}$ of £7. 15. $3\frac{1}{4} - 1\frac{1}{11}$ of £9. 5. $10\frac{2}{3} - 3\frac{7}{8}$ of £3. 12. $11\frac{1}{3}$.

15. 5 yds. 2 ft. $5\frac{3}{5}$ in. $\times 7\frac{3}{8}$ − 9 yds. 2 ft. $7\frac{3}{8}$ in. − 15 yds. 1 ft. $9\frac{3}{5}$ in. $\div 3\frac{5}{8}$.

16. Take $\frac{4}{15}$ of £4. 10. 9 from $\frac{8}{9}$ of £6. 6. 9.

17. Find the difference between $\frac{4}{5}$ of 1*s.* 6*d.* and $\frac{2}{3}$ of 2*s.* 6*d.*

18. By how much is $1\frac{3}{5}$ of £2. 7. $11\frac{3}{4}$ greater than $2\frac{2}{3}$ of £1. 3. $2\frac{2}{5}$?

19. Find the difference between a thirty-fifth part of 95 guineas and a twenty-third part of £65. 18. 8.

20. Find the value of 500 times the difference between an eighty-fourth part of $2\frac{1}{3}$ cwts. and a thirtieth part of 1 cwt. 0 qr. 3 lbs.

Find the value of

21. $\frac{4}{5}$ of $\frac{4}{11}$ of 16*s.* 6*d.*; $\frac{1}{4}$ of $\frac{8}{15}$ of half-a-crown; $\frac{9}{35}$ of $1\frac{1}{4}$ guineas.

22. $\frac{8}{15}$ of £$5\frac{2}{3}$; $\frac{5}{12}$ of half-a-guinea; $\frac{2}{3}$ of $\frac{5}{6}$ of 13*s.* 4*d.*

23. $\dfrac{3\frac{1}{3}}{1\frac{2}{3}}$ of £4. 14. 6; $\dfrac{1}{5}$ of $\dfrac{4}{7}$ of $\dfrac{8}{19}$ of $2\frac{3}{8}$ of 247 guineas.

Find the value of

24. $\dfrac{3\frac{1}{4}}{7\frac{1}{2}}$ of 10 ft. $6\frac{1}{2}$ in.; $\dfrac{5}{6}$ of $\dfrac{\frac{3}{7}}{4\frac{2}{7}}$ of $3\frac{3}{4}$ sq. yds.; $4\frac{4}{11}$ of 2 qrs. 3 nls. $1\frac{3}{8}$ in.

25. $2\frac{2}{11}$ of 3 cwt. 3 qrs. 20 lbs.; $4\frac{1}{11}$ of $\dfrac{8}{1\frac{3}{4}}$ of 5 miles 3 f. 37 p. $4\frac{1}{8}$ yds.

26. $\dfrac{7\frac{3}{4} - 3\frac{1}{8}}{18\frac{1}{3} \div \frac{5}{7}}$ of 3 ac. 1 r. 35 p.; $\dfrac{5\frac{9}{11}}{3\frac{4}{7} + \frac{3}{8}}$ of $\left(3\frac{4}{7} - \dfrac{2}{5}\right)$ of 5 cwts. 2 qrs. 10 lbs. $7\frac{3}{4}$ oz.

27. $\dfrac{3\frac{5}{6}}{7\frac{5}{8}}$ of $\dfrac{4}{7\frac{1}{2}}$ of $4\frac{5}{7}$ cu. feet; $\dfrac{3\frac{1}{2}}{3\frac{1}{3}}$ of $(3\frac{4}{7} + 1\frac{3}{8})$ of 5 days $17\frac{3}{8}$ hrs.

28. $\dfrac{4\frac{2}{5}}{7\frac{4}{5}}$ of $(8\frac{1}{7} - 3\frac{3}{5})$ of 5 lbs. $9\frac{2}{25}$ oz. Troy.

Find the sum of

29. $\dfrac{3}{5}$ of 6*s*. 8*d*., $\dfrac{5}{7}$ of £2. 3. 9, and $\dfrac{9}{11}$ of £4. 14. 5.

30. $\dfrac{3}{4}$ of $\dfrac{5}{8}$ of £1, $\dfrac{2}{3}$ of $\dfrac{5}{9}$ of 2*s*. 6*d*., and $\dfrac{3}{4}$ of $10\frac{1}{2}$*d*.

31. $\dfrac{2}{9}$ of £1. 1*s*., $\dfrac{1}{2}$ of $\dfrac{3}{4}$ of £1, $\dfrac{1}{6}$ of $\dfrac{3}{4}$ of a crown, and $\dfrac{1}{3}$ of $\dfrac{5}{8}$ of 1*s*.

32. £$\dfrac{5}{13}$, $\dfrac{3}{16}$ of 6*s*. 8*d*., $\dfrac{1}{40}$ of a crown, and $\dfrac{9}{13}$ of a penny.

33. $\dfrac{3}{7}$ of £15, $\dfrac{5}{8}$ of $\dfrac{1}{2\frac{1}{3}}$ of £1. 12*s*., and $\dfrac{4}{7}$ of 3*d*.

34. $\dfrac{15\frac{3}{8}}{7\frac{1}{2}}$ of £1, $\dfrac{1}{3}$ of £140. 10. 6, and $2\frac{3}{11}$ of half-a-guinea.

Find the value of

35. $2\frac{1}{7}$ of £1. 9*s*. + $\frac{9}{14}$ of 18*s*. 2*d*. − $\frac{3}{11}$ of $4\frac{1}{2}$*d*.

36. $(\frac{3}{7} + \frac{1}{14})$ of a guinea − $\frac{1}{13}$ of £1 − $\frac{2}{3}$ of half-a-crown.

37. $\dfrac{7}{8}$ of £5. 10. 6 − $\dfrac{4}{7}$ of 2 guineas + $\dfrac{2}{3\frac{1}{3}}$ of $\dfrac{4}{\frac{3}{5} - \frac{2}{7}}$ of £$\dfrac{5}{9}$.

38. $\dfrac{1\frac{5}{11}}{1\frac{3}{5}}$ of £8. 8. $5\frac{1}{4}$ − $\dfrac{3\frac{7}{11}}{4\frac{2}{7}}$ of $\dfrac{10\frac{1}{2}}{7\frac{1}{2}}$ of £2. 0. 6.

39. $(2\frac{1}{4} \div 2\frac{1}{3})$ of £10. 2. $0\frac{3}{4}$ + $\left(\dfrac{5}{6}\right)^2$ of £13. 11. $7\frac{1}{2}$.

40. $7\frac{3}{5}$ of a year of $365\frac{1}{4}$ days + $3\frac{8}{13}$ of $\frac{1}{3}$ of a week + $\frac{2}{3}$ of $5\frac{3}{4}$ hours.

41. From $\frac{4}{1117}$ cwt. take $\frac{4}{15}$ of 2 lbs. 8 oz. 10 drs.; from $6\frac{1}{13}$ days take the sum of $\frac{2}{11}$ of a week and $\frac{1}{3}$ of a minute.

Compare the values of

42. $\frac{1}{16}$ of £1, $\frac{1}{20}$ of a guinea, and $\frac{8}{35}$ of a crown.

43. $\frac{1}{21}$ of £1, $\frac{1}{22}$ of £1. 1*s.*, and $\frac{1}{4}$ of 3*s.* 9½*d.*

44. $\frac{7}{1760}$ of a mile, $1\frac{3}{7}$ poles, and $\frac{8}{25}$ of a chain.

265. *To find what fraction one concrete quantity is of any other of the same kind.*

For example, express £1. 18. 9 as a fraction of £2. 6. 3. Now £1. 18. 9 is 465*d.* and £2. 6. 3 is 555*d.*; hence if we divide £2. 6. 3 into 555 equal parts and take 465 of these parts we shall get £1. 18. 9; and therefore £1. 18. 9 is $\frac{465}{555}$ of £2. 6. 3.

Or, since 1*d.* = $\frac{1}{555}$ of 555*d.*; 465*d.* = $\frac{465}{555}$ of 555*d.*

and therefore £1. 18. 9 = $\frac{465}{555}$ of £2. 6. 3.

Or again; £1. 18. 9 = 465*d.* = $\frac{465 \times 555}{555}$*d.* = $\frac{465}{555} \times 555$*d.* (110)

= $\frac{465}{555}$ of £2. 6. 3.

$\therefore$ in each case Fraction required $= \dfrac{465}{555} = \dfrac{31}{37}$.

But perhaps the simplest as well as the most general way of finding what fraction one quantity is of another of the same kind is by finding how many times or parts of a time the first quantity contains the second, or, in other words, by finding the quotient of the first divided by the second (103). Thus taking the preceding example

$$\text{Fraction required} = \frac{£1.\ 18.\ 9}{£2.\ \ 6.\ 3} = \frac{465d.}{555d.} = \frac{465}{555} = \frac{31}{37}. \qquad (249)$$

Hence to find what fraction one concrete quantity is of any other of the same kind we have this Rule—

Express the two quantities in the same denomination* (249), *and for the required fraction put the first in the numerator and the second in the denominator.

Ex. 1. Reduce 13*s*. $6\frac{3}{4}$*d*. to the fraction of £1. 12. $4\frac{1}{2}$.

$$\begin{array}{ll} s. & d. \\ 13 & .\ 6\frac{3}{4} \\ 162 & \\ 651\,f. & \end{array} \qquad \begin{array}{ll} s. & d. \\ 32 & .\ 4\frac{1}{2} \\ 388 & \\ 1554\,f. & \end{array}$$

$\therefore$ Fraction req. $= \frac{651}{1554} = \frac{31}{74}$.

13*s*. $6\frac{3}{4}$*d*. $= 13\frac{27}{48}$*s*. $= 13\frac{9}{16}$*s*.

£1. 12. $4\frac{1}{2} = 32\frac{9}{24}$*s*. $= 32\frac{3}{8}$*s*.;

$\therefore$ Fraction required $= \dfrac{13\frac{9}{16}s.}{32\frac{3}{8}s.} = \dfrac{217}{518}$

$= \dfrac{31}{47}$.

Ex. 2. What part of $\frac{2}{3}$ of $\frac{6}{7}$ of 3 guineas is $\frac{3}{4}$ of $\frac{8}{9}$ of 15*s*. 9*d*.?

$$\text{Fraction required} = \frac{\frac{3}{4}\text{ of }\frac{8}{9}\text{ of }15\frac{3}{4}s.}{\frac{2}{3}\text{ of }\frac{6}{7}\text{ of }63s.} = \frac{3}{4}\times\frac{8}{9}\times\frac{63}{4}\times\frac{3}{2}\times\frac{7}{6}\times\frac{1}{63}$$

$$= \frac{7}{24}.$$

Ex. 3. What fraction of $5\frac{8}{9}-3\frac{1}{4})$ of 3 tons 16 cwts. 3 qrs. $22\frac{2}{5}$ lbs. is $7\frac{3}{5}$ of $\dfrac{3\frac{3}{4}}{3\frac{17}{19}}$ of 5 cwts. 3 qrs. $3\frac{1}{2}$ lbs.?

$$\begin{array}{lll} \text{cwts} & \text{qrs.} & \text{lbs} \\ 5 & .\ 3 & .\ 3\frac{1}{2} \\ 4 & & \\ \hline 23 & & \\ 28 & & \\ \hline 647\frac{1}{2}\text{ lbs.} & & \end{array} \qquad \begin{array}{lll} \text{cwts.} & \text{qrs.} & \text{lbs.} \\ 76 & .\ 3 & .\ 22\frac{2}{5} \\ 4 & & \\ \hline 307 & & \\ 28 & & \\ \hline 8618\frac{2}{5}\text{ lbs.} & & \end{array}$$

$$\therefore \text{Fraction required} = \frac{7\frac{3}{5}\text{ of }\frac{3\frac{3}{4}}{3\frac{17}{19}}\text{ of }647\frac{1}{2}}{(5\frac{8}{9}-3\frac{1}{4})\text{ of }8618\frac{2}{5}} = \frac{\frac{38}{5}\times\frac{15}{4}\times\frac{19}{74}\times\frac{1295}{2}}{(2\frac{8}{9}-\frac{1}{4})\times\frac{43092}{5}}$$

$$= \frac{38}{5}\times\frac{15}{4}\times\frac{19}{74}\times\frac{1295}{2}\times\frac{36}{95}\times\frac{5}{43092}$$

$$= \frac{5}{24}.$$

Ex. 4. What fraction of 2 lbs. 10 oz. Av. must be added to 1 lb. 8 oz. Troy to give 3 lbs. 7 oz. 10 dwts.?

We see that this question reduces itself to finding— What fraction of 2 lbs. 10 oz. Av. is 1 lb. 11 oz. 10 dwts.?

$$\begin{array}{rrr} \text{lbs.} & \text{oz.} & \text{dwts.} \\ 3 & .\ 7 & .\ 10 \\ 1 & .\ 8 & .\ 0 \\ \hline 1 & .\ 11 & .\ 10 \end{array}$$

Now $\qquad 11$ oz. 10 dwts. $= 11\frac{1}{2}$ oz. Tr. $= \dfrac{11\frac{1}{2}}{12}$ lb. Tr. $= \dfrac{23}{24}$ lb. Tr.;

$\therefore$ 1 lb. 11 oz. 10 dwts. $= 1\frac{23}{24}$ lb. Tr. $= 1\frac{23}{24} \times 5760$ grains,

and $\qquad 2$ lbs. 10 oz. Av. $= 2\frac{10}{16}$ lbs. Av. $= 2\frac{5}{8} \times 7000$ grains;

$$\therefore \text{Fraction required} = \frac{1\frac{23}{24} \times 5760}{2\frac{5}{8} \times 7000} = \frac{47}{24} \times \frac{8}{21} \times \frac{5760}{7000}$$

$$= \frac{2256}{3675}.$$

EXERCISE 38.

1. What fraction is 9*s.* 11*d.* of 13*s.* 5*d.*; and 14*s.* $10\frac{3}{4}$*d.* of £3. 2. $6\frac{3}{4}$?

2. What fraction of £2. 19. $0\frac{1}{4}$ is £1. 10. $0\frac{3}{4}$?

3. What fraction of £7. 7. 11 is £3. 4. $0\frac{3}{4}$?

4. Express 13*s.* $10\frac{1}{2}\frac{7}{8}$*d.* as a fraction of £2. 9. 7.

5. Express 17*s.* $2\frac{9}{4}\frac{8}{11}$*d.* as a fraction of £1. 1*s.*

6. What fraction is 27 lbs. 12 oz. 15 drs. of 3 cwts. 3 qrs. 21 lbs.?

7. What part of 13 cwts. 2 qrs. 21 lbs. is 11 cwts. 1 qr. 14 lbs. 15 oz.?

8. What fraction of 5 ac. 2 r. 17 p. is 2 ac. 31 p.?

9. Reduce 3 qrs. $3\frac{3}{8}$ nls. to the fraction of $2\frac{2}{13}$ ells.

10. Reduce 3 qts. 1 pt. $2\frac{2}{3}$ gills to the fraction of 5 gall. 2 qts. 1 pt.

11. What fraction of 8 lbs. $12\frac{4}{5}$ oz. is 3 lbs. 9 oz. $62\frac{1}{2}$ grs.?

12. Reduce 3 qrs. 3 lbs. 1 oz. $12\frac{3}{8}$ drs. to the fraction of 4 cwts. 1 qr. 14 lbs.

13. Express 2 fur. 29 p. 2 ft. 10 in. as a fraction of 1 mile 5 f. $16\frac{1}{2}$ po.

14. What part of $4\frac{3}{8}$ of £1 is $3\frac{3}{8}$ of a guinea?

15. What part of $4\frac{3}{8}$ of an oz. Tr. is $2\frac{4}{5}$ of an oz.?

16. Express the difference between £$7\frac{4}{5}$ and $\frac{4}{7}$ of £7 as a fraction of £10. 6. 8.

17. Reduce $\frac{5}{8}$ of 1 yd. 3 qrs. $3\frac{3}{8}$ nls. to the fraction of an ell.

18. What fraction of 3 lbs. 3 oz. 6 dwts. 15 grs. is 1 lb. 5 oz. 12 dr.?

19. What fraction is 28 sq. yds. 7 ft. 49 in. of $8\frac{8}{11}$ of $5\frac{3}{8}$ perches?

20. What fraction is $19\frac{41}{44}$ of 4 cu. yds. 18 ft. 1127 in. of $\frac{7}{18}$ of 200 cu. yds.?

21. Express $\frac{7}{18}s. - \frac{7}{16}d.$ as a fraction of $3s.\ 2\frac{1}{2}d.$

22. What part of six guineas and a half is £4. 18. $11\frac{3}{5}\frac{1}{2}$?

23. If 25 francs 25 c. be equal to £1, what fraction of 1s. is 1 franc?

24. If 160 dollars be equal to £33, what fraction is $\frac{3}{8}$ of a dollar of $\frac{11}{14}$ of a crown?

25. What fraction of $\frac{12}{5}$ of a crown is $\frac{5}{12}$ of a guinea, and what ratio does their difference bear to their sum?

26. Reduce $\frac{1}{2}\frac{3}{4}\frac{1}{8}$ of £1 to the fraction of a Portuguese gold corôa of 5000 reis, when 1000 reis $= 55d.$

27. Express $\frac{4}{5}$ of £1 + $\frac{3}{5}$ of a guinea − $\frac{4}{5}$ of 1s. as a fraction of $3\frac{3}{4}$ guineas.

28. What fraction of a year of $365\frac{1}{4}$ days is 27 days 16 h. 29 m. 4 s.?

29. What part of £7. 0. $8\frac{3}{4}$ is $\dfrac{3\frac{5}{8}}{7\frac{1}{3}}$ of $\dfrac{4}{7\frac{1}{4}}$ of £3. 7. $5\frac{1}{4}$?

30. Reduce $\dfrac{3\frac{1}{3}}{1\frac{1}{12}}$ of $\left\{\dfrac{19}{120} \text{ of } £1 - \dfrac{17}{48} \text{ of } 1s.\right\}$ to the fraction of 27s.

31. What weight is the same fraction of 15 cwts. 2 qrs. 13 lbs. that £1. 11. $10\frac{1}{2}$ is of £3. 10. $1\frac{1}{2}$?

32. Express $\dfrac{7\frac{3}{4} - 3\frac{1}{3}}{18\frac{1}{3} \div \frac{1}{4}}$ of £33. 14. $5\frac{3}{4}$ as a fraction of £157. 17. $8\frac{1}{2}$.

33. What fraction of a ton added to $\frac{4}{5}$ of 2 cwts. will make it equal to 1 cwt. 2 qrs. 11 lbs.?

34. Reduce the difference between $\frac{3}{5}$ oz. and $\frac{9}{16}$ oz Troy to the fraction of $\frac{3}{4}$ lb.

35. Reduce $7\frac{1}{3}\frac{4}{5}$ of 10 oz. 18 dwts. 11 grs. to the fraction of 8 lbs. $8\frac{7}{12}$ oz. Av.

36. A uniform bar of steel weighs 3 qrs. 15 lbs. $13\frac{3}{4}$ oz.; what part must be cut off to weigh 58 lbs. $4\frac{1}{3}$ oz.?

37. What fraction of £2. 19. 6 must be added to $\dfrac{3\frac{1}{2}}{4\frac{1}{3}}$ of $(3\frac{2}{3} + 1\frac{5}{6})$ of $13s.\ 1\frac{1}{2}d.$ to make the sum equal to £3. 5s.?

38. What fraction of $\dfrac{7\frac{1}{3}}{4\frac{2}{3}}$ of £30. 13. $2\frac{3}{4}$ is $(8\frac{1}{4} - 3\frac{2}{3})$ of £5. 9. $11\frac{1}{4}$?

39. Find a sum of money that shall be the same part of £14. 7. $9\frac{2}{3}$ that 4 oz. 7 dwts. 5 grs. is of 8 oz. 10 dwts. 15 grs.

266. SOME APPLICATIONS OF THE PRECEDING RULES.

1. METHOD OF REDUCTION TO THE UNIT.

(1) If 6 cwts. of sugar cost £33. 15. 0

$$1 \text{ cwt.} \;.\;.\;.\; \text{costs } \frac{£33.\ 15.\ 0}{6}. \qquad (254)$$

(2) If $\frac{3}{5}$ cwt. of sugar cost £3. 7. 6

$$\tfrac{1}{5} \text{ cwt.} \;.\;.\;.\;.\;.\; \frac{£3.\ 7.\ 6}{3}$$

$$\text{and } \tfrac{5}{5} \text{ cwt.} \;.\;.\;.\;.\;.\; \frac{£3.\ 7.\ 6 \times 5}{3}$$

$$\text{or } 1 \text{ cwt.} \;.\;.\;.\;.\;.\; \frac{£3.\ 7.\ 6}{\frac{3}{5}}. \qquad (134)$$

(3) If $6\frac{3}{5}$ cwts. of sugar cost £37. 2. 6,

$$\text{or if } \tfrac{33}{5} \;.\;.\;.\;.\;.\;.\; £37.\ 2.\ 6,$$

$$\text{then } 1 \text{ cwt.} \;.\;.\;.\;.\;.\; \frac{£37.\ 2.\ 6}{\frac{33}{5}} \text{ or } \frac{£37.\ 2.\ 6}{6\frac{3}{5}}.$$

Hence whether the *number* of cwts. be integral or fractional or mixed, the value of 1 cwt. may always be got by dividing the given sum by this number. And generally

If the value, weight, length......of *any number* of things be given, the value, weight, length......of 1 of them may be always found by division.

Ex. If $6\frac{3}{4}$ lbs. of silver be worth £24. 9. $4\frac{1}{2}$, what is the value of 3 lbs. 9 oz. 12 dwts.?

$$£24.\ 9.\ 4\tfrac{1}{2} = £24.\ 9\tfrac{3}{8}s. = £24\,\frac{9\frac{3}{8}}{20} = £24\tfrac{15}{32}.$$

$$3 \text{ lbs. } 9 \text{ oz. } 12 \text{ dwts.} = 3 \text{ lbs. } 9\tfrac{3}{5} \text{ oz.} = 3\,\frac{9\frac{3}{5}}{12} \text{ lbs.} = 3\tfrac{4}{5} \text{ lbs.}$$

Since $6\frac{3}{4}$ lbs. is worth £$24\frac{15}{32}$,

1 lb. . . . . . $\dfrac{£24\frac{15}{32}}{6\frac{3}{4}}$, (266, 3)

and $3\frac{4}{5}$ lbs. . . . . . $\dfrac{£24\frac{15}{32} \times 3\frac{4}{5}}{6\frac{3}{4}}$;

or 3 lbs. 9 oz. 12 dwts. . . . . . $£\dfrac{783}{32} \times \dfrac{19}{5} \times \dfrac{4}{27}$ or $£\dfrac{551}{40}$

or . . . . . . £13. 15. 6.

II. Time and Work, by Reduction to the Unit.

(1) If A can do 1 piece of work in 6 days,

A can do $\frac{1}{6}$ . . . . . 1 day.

(2) If A can do 1 piece of work in $\frac{2}{3}$ day,

A can do $\frac{1}{2}$ . . . . . $\frac{1}{3}$ day.

and A can do $\frac{3}{2}$ . . . . . 1 day,

or A can do $\dfrac{1}{\frac{2}{3}}$ . . . . . 1 day.

(3) If A can do 1 piece of work in $6\frac{3}{5}$ days or in $\frac{33}{5}$ days,

A can do $\dfrac{1}{\frac{33}{5}}$ . . . . . 1 day

or A can do $\dfrac{1}{6\frac{3}{5}}$ . . . . . 1 day.

Hence the number of pieces of work may always be found by dividing 1 by the *number* of days, whether this number be integral, or fractional, or mixed.

Conversely, and in like manner, the number of days may always be found by dividing 1 by the *number* of pieces of work, whether this number be integral, or fractional, or mixed.

Ex. 1. A can do a piece of work in $10\frac{5}{8}$ days, B in $9\frac{3}{8}$ days, and C in $5\frac{8}{11}$ days; in how many days can A, B and C working together do the piece of work?

A in 1 day can do $\dfrac{1}{10\frac{2}{3}}$ of the piece of work, B can do $\dfrac{1}{9\frac{3}{5}}$ and C $\dfrac{1}{5\frac{9}{11}}$;

$\therefore$ A, B, C working together can in 1 day do

$$\left(\frac{1}{10\frac{2}{3}}+\frac{1}{9\frac{3}{5}}+\frac{1}{5\frac{9}{11}}\right) \text{ of the piece of work,}$$

$$\text{or} \qquad \frac{3}{32}+\frac{5}{48}+\frac{11}{64} \quad \ldots\ldots\ldots\ldots$$

$$\text{or} \qquad \frac{71}{192} \quad \ldots\ldots\ldots\ldots$$

and $\therefore$ A, B and C working together can do the piece of work in

$$\frac{1}{\frac{71}{192}} \text{ days, or } \tfrac{192}{71} \text{ days, or } 2\tfrac{50}{71} \text{ days.}$$

Ex. 2. A and B can do a piece of work in 8 days, A and C in $10\frac{2}{3}$ days, and B and C in $9\frac{3}{5}$ days: in how many days can A alone do it?

Now A and B in 1 day can do $\frac{1}{8}$ of the piece of work,

and A and C . . . . . $\dfrac{1}{10\frac{2}{3}}$. . . . . . .

$\therefore$ A and B in 1 day and A and C in 1 day can do $\left(\dfrac{1}{8}+\dfrac{1}{10\frac{2}{3}}\right)$ of it,

or A in 2 days and B and C in one day can do $\left(\dfrac{1}{8}+\dfrac{1}{10\frac{2}{3}}\right)$ of it,

but B and C in 1 day can do $\dfrac{1}{9\frac{3}{5}}$ of it;

$\therefore$ A alone in 2 days can do $\left(\dfrac{1}{8}+\dfrac{1}{10\frac{2}{3}}-\dfrac{1}{9\frac{3}{5}}\right)$ of it,

or . . . . $\dfrac{1}{8}+\dfrac{3}{32}-\dfrac{5}{48}$ or $\dfrac{11}{96}$ of it.

$\therefore$ A alone in 1 day can do $\dfrac{11}{192}$ of the piece,

and $\therefore$ A can do the piece in $\dfrac{1}{\frac{11}{192}}$ days or $\frac{192}{11}$ or $17\frac{5}{11}$ days.

Ex. 3. A cistern can be filled by one tap in 2 hours 20 m., by a second in $3\frac{1}{2}$ hours, and can be emptied by a third in 1 hour 35 m.; if the cistern be empty and the three taps be opened, in how many hours will it be filled?

2 hours 20 min. $= 2\frac{1}{3}$ hrs.; 1 hour 35 m. $= 1\frac{7}{12}$ hrs.

The 1st tap will fill $\frac{1}{2\frac{1}{3}}$ or $\frac{3}{7}$ of the cistern in 1 hour,

2nd tap . . $\frac{1}{3\frac{1}{2}}$ or $\frac{2}{7}$ . . . .

and the 3rd tap will empty $\frac{1}{1\frac{7}{12}}$ or $\frac{12}{19}$ . . . .

∴ when all three taps are running they will fill

$\left(\frac{3}{7} + \frac{2}{7} - \frac{12}{19}\right)$ of the cistern in 1 hour,

or $\frac{11}{133}$ . . . .

∴ they will fill the cistern in $\frac{1}{\frac{11}{133}}$ or $\frac{133}{11}$ or $12\frac{1}{11}$ hours.

III. Hands of a Clock.

At what time after 2 o'clock will the minute and hour-hands of a watch first be at right angles to each other, or be fifteen minutes apart, and when will they next be at right angles again?

When the event first happens the minute-hand will have gained 25 m. on the hour-hand *since 2 o'clock;* and when it happens again will have gained 30 additional minutes, or will have gained 55 m. since 2 o'clock.

The minute-hand gains 11 m. on every 12 m. it advances

or . . . . 1 m. . . . $\frac{12}{11}$ . . . . .

∴ . . . . 25 m. . . . $25 \times \frac{12}{11}$ or $27\frac{3}{11}$ m.,

and . . . . 55 m. . . . $55 \times \frac{12}{11}$ or 60 m. .

∴ the event first happens at $27\frac{3}{11}$ m. past 2 o'clock, and again at 60 m. past 2, or at 3 o'clock.

IV. Least Common Multiple.

A, *B* and *C* start from the same point and travel in the same direction round an island 73 miles in circumference, *A* at the rate of 10, *B* at the rate of 14, and *C* at the rate of 16 miles a day: in how many days will they all come together again?

B gains 4 miles on *A* every day, therefore he gains 73 miles or a complete round in $\frac{73}{4}$ days, that is *A* and *B* are together at the end of every $\frac{73}{4}$ days; in like manner *A* and *C* are together at the end of every $\frac{73}{6}$ days; and therefore *A*, *B* and *C* are together at the end of any number of days which is a common multiple of $\frac{73}{4}$ and $\frac{73}{6}$;

but L.C.M. of $\frac{73}{4}$ and $\frac{73}{6}$ is $\frac{73}{2}$; (135)

therefore *A*, *B*, *C* are first together at the end of $\frac{73}{2}$ or $36\frac{1}{2}$ days.

V.

A post is divided into four parts; the first part is $\frac{3}{7}$ of the whole length, the second part is $\frac{2}{3}$ of the first, the third $\frac{7}{9}$ of the second, and the fourth is 1 yard 8 in.: find the length of the post.

The second part is $\frac{2}{3}$ of $\frac{3}{7}$, or $\frac{2}{7}$ of the whole length;

$\therefore$ the third part is $\frac{7}{9}$ of $\frac{2}{7}$, or $\frac{2}{9}$ . . . .

and $\therefore$ the first three parts are $\frac{3}{7}+\frac{2}{7}+\frac{2}{9}$ or $\frac{59}{63}$ . . . .

and $\therefore$ the fourth part is $\frac{4}{63}$ . . . .

but the fourth part is 1 yd. 8 in. or $1\frac{2}{9}$ yds.

$\therefore \frac{4}{63}$ of the whole length is $1\frac{2}{9}$ yds.;

and $\therefore$ the whole length is $\frac{1\frac{2}{9}}{\frac{4}{63}}$ yds. or $19\frac{1}{4}$ yds. (266, 2)

VI.

A person has a number of oranges to dispose of: he sells half of what he has and one more to *A*, half of the remainder and one more to *B*, half the remainder and one more to *C*, and half the remainder and one more to *D*: by which time he has disposed of all he had. How many had he at first?

When he had given *half* his oranges to *D* he had one left; therefore he had 2×1 or 2 before *D* came, and therefore he had 3 before he had given the 1 orange to *C*: but this is the number he had left when he had given half his oranges to *C*, therefore he had 2×3 or 6 before *C* came, and therefore he had 7 before he had given the 1 orange to *B*, and therefore (as before) he had 2×7 or 14 before *B* came; therefore he had 15 before he had given the 1 orange to *A*, and therefore he had 2×15 or 30 before *A* came: that is, he had thirty oranges at first.

EXERCISE 39.

I.

1. Reduce [illegible] of £1 to the fraction of a thaler, when $6\frac{2}{3}$ thalers are equal to 20*s*.

2. If $\frac{3}{5}$ of an estate be worth £220, what is the value of $\frac{5}{11}$ of the same?

3. *A* alone can do a piece of work in 10 days, *B* alone can do it in 14 days: how many days would the two together be in doing it?

4. At what time between 2 and 3 o'clock are the hands of a clock exactly together?

5. If $\frac{11}{12}$ of a number exceeds $\frac{4}{5}$ of half the number by $40\frac{1}{3}$, what must the number be?

6. Find the least sum of money that contains an exact number of thalers of 2*s*. $11\frac{3}{4}$*d*. each, and of dollars of 4*s*. $1\frac{1}{2}$*d*. each.

7. A legacy of £897. 15*s*. is to be divided among *A*, *B* and *C*. *A* is to receive $\frac{1}{5}$, *B* is to receive $\frac{3}{8}$, and *C* the remainder. Find what sum *B* will receive, and the fraction of the whole to be paid to *C*.

8. Sound travels at the rate of 1140 feet a second. If a shot be fired from a ship moving at the rate of 10 miles an hour, how far will the ship have moved before the report is heard $14\frac{1}{4}$ miles off?

9. *A* and *B* can do a piece of work in 6 days, *B* and *C* can do it in 7 days, and *A*, *B* and *C* can do it in 4 days. How long would *A* and *C* take to do it?

10. A hare starts 40 yards before a greyhound and is not seen by him till she has been up 30 seconds. She runs at the rate of 12 and the hound at the rate of 15 miles an hour: how long will the chase last, and what distance will the hound have run?

II.

11. What fraction of $\frac{15}{9}$ of a crown is $\frac{8}{15}$ of half-a-guinea? and what ratio does their difference bear to their sum?

12. If $\frac{3}{16}$ of a lottery ticket is worth £4. 10*s.*, what is the value of $\frac{4}{5}$ of the ticket?

13. A cistern is fed by a spout which can fill it in 3 hours; how long would it take to fill it, if the cistern has a leak which would empty it in 17 hours?

14. Find the value of $\dfrac{14 \text{ lbs. } 8 \text{ oz. } 18 \text{ dwts.}}{1 \text{ lb. } 10 \text{ dwts.}}$ of 5*s.* $2\frac{1}{4}$*d.*

15. There is a number to which 3 is added and $\frac{7}{15}$ of the result taken; to this 5 is added and $\frac{4}{15}$ of the result taken, giving $1\frac{1}{2}$: what is the number?

16. Find the least number of sovereigns that contains an exact number of 20-franc pieces of 15*s.* $11\frac{1}{4}$*d.* each.

17. Five brothers join in paying a sum of money: the eldest pays a third of it, and the others pay the remainder in equal shares, and thereby each of them pays £84 less than the eldest brother. What is the sum of money?

18. An elastic ball after striking the ground rises to $\frac{4}{7}$ of the height from which it fell. After striking the ground the third time it rises $3\frac{3}{8}$ inches: from what height did it fall at first?

19. *A* does $\frac{3}{8}$ of a piece of work in 4 hours, *B* does $\frac{3}{4}$ of what remains in 1 hour, and *C* finishes it in 20 minutes. How long would they have been doing the whole, if they had worked together?

20. Two boats row a race over a straight course 1 mile 995 yards long, their rates of speed being 12 miles and $11\frac{3}{8}$ miles an hour respectively. Assuming that sound travels at the rate of 1140 feet in a second, find how much the faster boat will be ahead of the other when the sound of the gun fired at starting is heard at the winning-post.

III.

21. What part of 405 oz. 7 dwts. 21 grs. is 90 oz. 1 dwt. 18 grs.; and how often is the latter quantity contained in the former?

22. Find the value of a ton and a third of sugar, when $\frac{3}{16}$ of a ton is worth £6. 5s.

23. If 3 men with 4 boys earn £5. 16s. in 8 days, and 2 men with 3 boys earn £4 in the same time: in what time will 6 men and 7 boys earn 20 guineas?

24. Find the average of $17\frac{1}{2}$, $25\frac{1}{4}$, $96\frac{3}{8}$, 10, 0, $42\frac{7}{8}$ and 56.

25. A screw advances $\frac{3}{32}$ inch at each turn: how many turns must be taken for it to advance $7\frac{5}{8}$ inches?

26. Find the least number of lbs. Av. that contains an exact number of ounces Av. and ounces Troy.

27. A person left $\frac{7}{18}$ of his property to his elder son, and $\frac{7}{11}$ of the remainder to his younger son, and the rest to his widow. The elder son received £514. 6. 8 more than the younger: how much did the widow receive?

28. A farmer paid a corn rent of 5 qrs. of wheat and 3 qrs. of barley, Winchester measure. What was the value of his rent when wheat was at 60s. and barley 54s. a quarter imperial measure, supposing an imperial gallon to be $\frac{33}{32}$ of a Winchester gallon?

29. If a piece of work can be done in 50 days by 35 men working at it together, and if after working together for 12 days 16 of the men leave the work, find the number of days in which the remaining men would finish the work.

30. *A* has three times as much money as *B*. They play together, and at the end of the first game *B* wins from *A* three-eighths of *A*'s money: what fraction of the sum which *B* now has must *A* win back in the second game that they may have exactly equal sums?

IV.

31. What fraction is 9 sq. poles 25 yds. 4 ft. 81 in. of 29 sq. poles 28 yds. 2 ft. 57 in.?

32. If $4\frac{3}{5}$ of a sum of money be equal to $\frac{11}{14}$ of £1. 11. $10\frac{1}{2}$, find what the sum must be.

33. A certain number of men mow 4 acres of grass in 3 hours, and a certain number of others mow 8 acres in 5 hours: how long will they be in mowing 11 acres, if all work together?

34. At what time after 5 o'clock are the hands of a watch exactly opposite to each other?

35. A man's debts amount to $\frac{1}{12}$ of his property, but before paying them he loses $\frac{3}{8}$ of his property; afterwards he recovers a portion equal to $\frac{1}{4}$ of what he has left, and then loses $\frac{1}{3}$ of what he has got. Can he pay his debts? What part of his property remains over?

36. Find the least sum of money that can be paid in coins of the respective value of £3. 17. $10\frac{1}{2}$ and £3. 12. $3\frac{3}{4}$.

37. *A* had 10*s*. in his purse, and *B* having paid *A* $2 \times \frac{3\frac{1}{7}}{1\frac{2}{5}}$ of £1. 11. 6 finds that he has remaining $\frac{1}{13}$ of the sum which *A* now has: what had *B* at first?

38. If from a thing an eighth part is taken away, and then from the remainder an eighth part, and so on; after how many operations will what is left be less than half the original thing?

39. *A* can do in 2 days as much work as *B* in 3 days, and *B* in 5 days as much work as *C* in 4 days; what time will *C* require to finish a piece of work which *A* can do in 9 days?

40. A tank can be filled by 3 separate pipes: by the first in 10, by the second in 9, and by the third in 8 hours. It is supplied by the first pipe till it is a quarter full, and then the second is also turned on till it becomes half full, and then all three begin to run. How long would it take to fill the tank?

v.

41. Find the value of $\frac{5}{7}$ of £1 multiplied by $6\frac{3}{5}$, and of $\frac{2}{3}$ of $\frac{3}{5}$ of £1 divided by $\frac{1}{4}$.

42. If $4\frac{5}{6}$ oz. of tea cost $8\frac{2}{3}$*s*., what will $30\frac{3}{7}$ lbs. cost?

43. The wages of *A* and *B* together for $22\frac{1}{2}$ days amount to the same sum as the wages of *A* alone for $38\frac{4}{7}$ days. For how many days will this sum pay the wages of *B* alone?

44. At what time after 12 o'clock are the hands of a clock exactly together?

45. If a man rows 10 miles in $2\frac{1}{2}$ hours against a stream, the rate of which is 3 miles an hour, how long will he be in rowing 5 miles with the stream?

46. Find the least number of sovereigns that contains an exact number of thalers and of dollars: 48 thalers being worth £7. 3s. and 8 dollars £1. 13s.

47. A met two beggars B and C, and having $\dfrac{3\frac{7}{11}}{4\frac{4}{5}}$ of $\dfrac{10\frac{5}{7}}{7\frac{1}{2}}$ of $\dfrac{77}{400}$ of a sovereign in his pocket, gave B $\frac{1}{4}$ of $\frac{3}{8}$ of that sum, and C $\frac{3}{4}$ of the remainder: what did each receive?

48. An excursion train a quarter of a mile long leaves a station at 8 h. 22 m., and travels at the rate of 40 miles an hour: the ordinary train which travels at the rate of 66 feet a second leaves the station at 8 h. 26 m. and follows the other. At what time may a collision be expected?

49. A can do half a piece of work in 3 hours, which is twice as much as B can do, and A, B and C together can do the whole in $2\frac{1}{3}$ hours. How long will it take B to do what C can do in 5 hours?

50. I have to be at a certain place at a certain time, and I find that if I walk at the rate of 4 miles an hour I shall be 5 minutes too late; if at the rate of 5 miles an hour, I shall be 10 minutes too soon. How far have I to go?

VI.

51. Add together $\frac{3}{5}$ of £1. 1s., $\frac{1}{6}$ of £1. 6. 4, and $\frac{1}{8}$ of 3s. 8d., and express their sum as a fraction of 6s. 8d.

52. If 3 cwts. 3 qrs. 21 lbs. $12\frac{2}{3}$ oz. cost £4. 8. 9; what is the price per cwt.?

53. If 6 men and 2 boys can reap 13 acres in 2 days, and 7 men and 5 boys can reap 33 acres in 4 days; how long will it take 2 men and 2 boys to reap 10 acres?

54. The time past noon is equal to $\frac{4}{11}$ of the time to midnight: what is the time?

55. A and B are travelling on the same road in opposite directions: A at the rate of $3\frac{3}{4}$ and B at the rate of $4\frac{2}{3}$ miles per hour: what is their rate of approach or separation per hour? If at starting they are 23 miles apart and they approach each other, in what time will they meet?

56. At what o'clock will a train which leaves London for Swindon at 2.45 P.M. and goes at the rate of 41 miles an hour meet a train which leaves Swindon for London at 1.45 P.M. and goes at the rate of 25 miles an hour; the distance between London and Swindon being 80 miles? and at what distance from London will they meet?

57. Express a degree of $69\frac{1}{2}$ miles in metres, supposing 32 metres to be equal to 35 yards.

58. How much ore must be raised, that on losing $\frac{11}{16}$ in roasting, and $\frac{18}{19}$ of the residue in smelting, there may result 506 tons of pure metal?

59. A cistern is supplied by two pipes, one of which would fill it in 5 hours and the other in 6 hours: there is a tap to the cistern which would empty it in 30 hours, and a leak that would empty it in 5 days: if while the cistern is being filled by the two pipes the tap and the leak are also running, what part of the cistern would remain empty in 3 hours?

60. Two boats start to row a race at 3 o'clock. The winning boat comes in at $6\frac{2}{3}$ min. past 3, 40 yards ahead of the other. At 4 min. past 3 the losing boat was 1140 yards from the winning-post. Find the length of the course, and the speed of the winning boat in miles per hour.

VII.

61. Add together $\frac{5}{8}$ of a guinea, $\frac{3}{7}$ of a pound, and $\frac{4}{5}$ of a shilling, and reduce their sum to the fraction of 13*s.* 6*d.*

62. Find the value of $\frac{5}{6}$ of $\frac{1}{7\frac{1}{2}}$ of 3 sq. yds. 6 ft. at $\frac{9}{25}$ of $\frac{1}{5\frac{1}{2}}$ of 4*s.* 2*d.* per sq. foot.

63. A man and his son can drink a barrel of beer in 15 days. They drink together for 6 days, and then the son alone drinks the remainder in 30 days: in what time can either alone drink the barrel?

64. At what times between 6 and 7 o'clock are the hands of a clock 15 minutes apart?

65. The $\frac{1}{2}$, $\frac{1}{4}$, $\frac{2}{3}$, and $\frac{3}{4}$ of a number are added together, and the sum is diminished by 139, giving 1343 as the difference: what is the number?

66. If standard gold which is worth £3. 17. $10\frac{1}{2}$ an oz. be still further alloyed so as to be worth only £3. 16. $1\frac{1}{2}$ an oz., find the least number of sovereigns made of the latter composition which shall be equal to an exact number made of standard gold.

67. The adult population of a country is 22815210; the adult females are $\frac{5}{13}$ of the whole population, and the adult males are $\frac{16}{17}$ of the adult females: find the whole population.

68. For 500 yards *A* can run at an average rate of $11\frac{3}{4}$ miles and *B* of $12\frac{1}{2}$ miles an hour: what start may *B* give to *A* in a race of 500 yards, that *B* may win by a yard?

69. A can by himself perform a certain quantity of work in 5 days, B twice as much in 7, and C four times as much in 11 days: in what time can A, B and C together perform 3 times the original work?

70. A has twice as much money as B. They play together, and at the end of the first game B wins from A one-third of A's money: what fraction of the sum which B now has must A win back in the second game that they may have exactly equal sums?

VIII.

71. Find the value of $1\frac{1}{3}$ of $\dfrac{4\frac{1}{8}}{5\frac{1}{4}}$ of $\dfrac{18s.\ 6\frac{3}{4}d.}{£1}$ of 3 days 2 hrs.

72. If a silver cup weighing 20 oz. 19 dwts. $2\frac{2}{11}$ grs. cost £5. 15. 3, what is the price per oz.?

73. Three gardeners working all day can plant a field in 10 days; but one of them having other employment can only work half-time: how long will it take them to complete the work?

74. Find the average of $11\frac{2}{3}$, $73\frac{1}{5}$, 0, $3\frac{13}{160}$, 82, $17\frac{5}{24}$, $5\frac{1}{4}$ and $9\frac{8}{15}$.

75. A and B are travelling on the same road in the same direction; A at the rate of 10 miles in 3 hours, and B at the rate of 19 miles in 5 hours: find their rate of approach or of separation per hour. If at starting B is $5\frac{3}{4}$ miles behind A, in how many hours will he come up with him?

76. Find the least number of lbs. Troy that contains an exact number of drams.

77. A and B have 18s. and 12s. respectively: if A give B $\dfrac{7\frac{2}{3}}{13\frac{6}{11}}$ of $\dfrac{2\frac{1}{5}}{4\frac{2}{7}}$ of twice the difference of their respective sums, and then $\frac{1}{17}$ of $\frac{4}{5}$ of A's present sum be added to $\frac{3}{4}$ of $\frac{1}{2}$ of B's, C's money will be $\frac{5}{8}$ of the result. What is the value of $\frac{3}{8}$ of C's money?

78. An elastic ball after each rebound rises to $\frac{4}{5}$ of the height from which it fell: after how many rebounds will the height to which it rises be less than $\frac{1}{10}$ of the height from which it fell at first?

79. A alone can perform a piece of work in 12 hours; A and C together can do it in 5 hours; and C's work is $\frac{3}{4}$ of B's. A begins work at 5 o'clock: at what time ought B and C to join him, so that all working together may complete the work by 12 o'clock?

80. I have to earn a certain sum of money by selling a certain number of nuts, and I find that if I sell at the rate of 40 a penny I shall earn 10d. too much, if at 50 a penny 5d. too little. How many have I to sell?

IX.

81. What fraction is 8 lbs. 1 oz. 19 dwts. 9 grs. of 13 lbs. 7 oz. 5 dwts. 15 grs.; and if the former quantity cost £10. 6. 6 what will the latter cost?

82. *A* was owner of $\frac{5}{17}$ of a privateer, and sold $\frac{3}{11}$ of $\frac{2}{9}$ of his share for £$12\frac{4}{33}$: what was the value of $\frac{1\frac{1}{4}}{5\frac{1}{2}}$ of $\frac{7}{12}$ of the vessel at the same rate?

83. *A* performs $\frac{3}{7}$ of a piece of work in 13 days, and with the help of *B* finishes it in 6 days; in what time could each of them do the piece of work separately?

84. At what time between 11 and 12 o'clock will the minute-hand be 20 minutes behind the hour-hand?

85. On a stream, *B* is intermediate to and equidistant from *A* and *C*: a boat can go from *A* to *B* and back in 3 hours 45 m., from *A* to *C* in $2\frac{1}{2}$ hours. How long would it take to go from *C* to *A*?

86. If 1 oz. of standard gold be worth £3. 17. $10\frac{1}{2}$, find the least number of ounces that can be coined (1) into an exact number of sovereigns, and (2) into an exact number of guineas.

87. The sea occupies $\frac{11}{15}$ of the surface of the globe. The surface of Asia is $\frac{121}{27}$ of that of Europe, of Africa is $\frac{22}{7}$, of America is $\frac{111}{28}$, and of Oceania is $\frac{3}{4}$; the surface of Africa is 12006522 sq. miles: find the surface of the globe.

88. On measuring a distance of 32 yards with a rod of a certain length it was found that the rod was contained 41 times with half an inch over: how many inches will there be over in measuring 44 yards with the same rod?

89. A vessel whose speed was $9\frac{1}{4}$ miles per hour started at 8 o'clock to go a distance of 74 miles. A second vessel, whose speed was to that of the first as 8 to 5, starting from the same place arrived 5 min. before the first. When did the second vessel start?

90. A man can do 4 times a certain work in 9 hours, a woman 3 times the work in 10 hours, and a child twice the work in 11 hours: if man, woman and child work together, in what time can they do 7 times the work?

91. Two boats start to row a race at 3 o'clock. The race is over at $6\frac{3}{4}$ min. past 3, the losing boat being 40 yards behind at the finish. At 4 min. past 3, this boat was 700 yards from the winning-post. Find the speed of each boat in miles per hour.

REDUCTION AND THE COMPOUND RULES.—DECIMALS.

REDUCTION.

267. In Reduction we have to consider the two following cases:—

(i) To reduce a decimal of one denomination to a lower denomination: and conversely

(ii) To reduce a quantity of one denomination to a decimal of a higher denomination.

I.

Proceeding as in Art. 256, we get in reality the same Rule: –

Multiply the decimal of the given denomination by the number which tells how many of the lower denomination make one of the given denomination.

Ex. Reduce ·549675 of a day to hours, to minutes, and to seconds.

$$\begin{array}{rl} \cdot549675 & \text{day.} \\ 24 & \\ \hline 13\cdot1922\ldots & \text{hours.} \\ 60 & \\ \hline 791\cdot532\ldots & \text{minutes.} \\ 60 & \\ \hline 47491\cdot92\ldots & \text{seconds.} \end{array}$$

For ·549675 of a day = ·549675 of 24 hours.
= ·549675 × 24 hours.
= 13·1922 hours, &c.

Hence ·549675 of a day = 13·1922 hrs. = 791·532 min. = 47491·92 sec.

II.

Proceeding as in Art. 257, we get in reality the same Rule:—

Divide the number of the given denomination by the number which tells how many of that denomination make one of the higher denomination.

Ex. Express $21\frac{3}{4}$ grains as a decimal of a dwt., an oz., and a lb.

$$\left.\begin{array}{r} 8 \\ 3 \end{array}\right\} \begin{array}{l})\ 21\cdot75 \text{ grs.} \\ )\ 2\cdot71875 \end{array}$$
$$\begin{array}{rl} 20\ ) & \cdot90625 \text{ dwt.} \\ 12\ ) & \cdot0453125 \text{ oz.} \\ & \cdot0037760416 \text{ lb.} \end{array}$$

We see at sight that $21\frac{3}{4} = 21\cdot75$, and to reduce 21·75 grs. to the decimal of a dwt. we divide by 24, to the decimal of an oz. we divide by 24 × 20, &c.

Hence $21\frac{3}{4}$ grs. = ·90625 dwt. = ·0453125 oz. = ·0037760416 lb. Tr.

III.

268. Sometimes in reducing a decimal of one denomination to a decimal of another denomination, we have to employ both the *descending* and the *ascending* process; for example

Reduce ·78936 of a guinea to the decimal of £1:

here the denomination *common* to guineas and to pounds is shillings; we therefore reduce the given quantity to shillings, and the result to the decimal of a pound; thus

$$\begin{array}{r} \cdot78936 \text{ guin.} \\ 21 \\ 20\ )\ 16\cdot57656\ s. \\ \pounds\cdot828828 \end{array} \qquad \therefore\ \cdot78936 \text{ guinea} = \pounds\cdot828828.$$

269. The preceding cases enable us

(i) *To reduce a decimal of one denomination to a compound quantity of lower denominations;* and

(ii) *To reduce a compound quantity to a decimal of a higher denomination.*

Ex. 1. Find the value of 5·34795 tons.

$$\begin{array}{r} 5\cdot34795 \text{ tons} \\ 20 \\ \hline 6\cdot959 \text{ cwts.} \\ 4 \\ \hline 3\cdot836 \text{ qrs.} \\ 28 \\ \hline 23\cdot408 \text{ lbs.} \\ 16 \\ \hline 6\cdot528 \text{ oz.} \\ 16 \\ \hline 8\cdot448 \text{ drs.} \end{array}$$

Since ·34795 tons = 6·959 cwts.

∴ 5·34795 tons = 5 tons 6·959 cwts.

In like manner 6·959 cwts. = 6 cwts. 3·836 qrs.

∴ 5·34795 tons = 5 tons 6 cwts. 3·836 qrs., &c.

Hence 5·34795 tons = 5 tons 6 cwts. 3 qrs. 23 lbs. 6 oz. 8·448 drs.

Ex. 2. Reduce 13*s*. 6¾*d*. to the decimal of £1: and express £8. 13. 6¾ in pounds only.

$$\begin{array}{r} 4\ )\ 3f. \\ \hline 12\ )\ 6\cdot75d. \\ \hline 20\ )\ 13\cdot5625s. \\ \hline \pounds\cdot678125 \end{array}$$

¾*d*. = ·75*d*., therefore 6¾*d*. = 6·75*d*.

6·75*d*. = ·5625*s*., therefore 13*s*. 6¾*d*. = 13·5625*s*.

13·5625*s*. = £·678125, ∴ 13*s*. 6¾*d*. = £·678125,

and £8. 13. 6¾ = £8·678125.

Ex. 3. Find the value of $\cdot 844791\dot{6}$ of £1.

```
  £                          £
·8447916                   ·8447916
      20                   16·895833̇s.
16·895833̇s.               10·75d.
      12                   3
10·75                ∴ Value required = 16s. 10¾d.
```

Ex. 4. Find what decimal £2. 15. $9\frac{3}{5}$ is of a guinea, and of $4\frac{1}{2}$ guineas.

```
     4 ) 3·4 f.
    12 ) 9·6 d.
21 { 3 ) 55·8 s.
   { 7 ) 18·6
       2·6571428 4/7 guineas
                 2        ∴ £2. 15. 9 3/5 = 2·6571428 4/7 guineas
     9 ) 5·3142857 1/7                   = ·5904761 of 4½ guin.
         ·5904761
```

Ex. 5. Reduce 5 furlongs 18 p. 3 yds. 2 ft. 11·4 in. to the decimal of a mile, and to the decimal of 2⅗ miles.

```
12 ) 11·4 in.
 3 ) 2·95 ft.
     3·983̇ yds.
         2
11 ) 7·96̇
40 ) 18·72̇4̇ po.
 8 ) 5·468106̇ fur.
     ·68351325̇7̇ mile
             5
13 ) 3·41756628̇7̇
     ·2628897144522 of 2⅗ miles.
```

Ex. 6. Express 12 oz. 15 drs. as a decimal of 1 lb. Troy.

```
       16 ) 15 drs.
       16 ) 12·9375 oz.
            ·80859375 lb. Av.
                 7000
24 { 8 ) 5660·15625 grains
   { 3 ) 707·51953125
      20 ) 235·83984375 dwts.
       12 ) 11·7919921875 oz. Troy
            ·982666015625 lb. Troy.
```

Exercise 40

Reduce

1. £·02375 to pence; ·03125s. and £·8947916 to farthings.
2. ·7859 cwt. to ounces; ·6197916 lb. Troy to grains.
3. 2·34954 miles to yards; 3·6874 acres to sq. yards.
4. ·0475 gallon to pints; ·678571428 week to minutes.
5. ·335s., 6·375d., ·4068f. to the dec. of £1.
6. 47·733 lbs. to the dec. of a ton; 10 drs. to the dec. of 1 lb.
7. 527·3994 yds. to the dec. of a mile; 37·9872 sec. to the dec. of a day.
8. 420·8138 sq. yds. to the dec. of an acre; 1 oz. to the dec. of 1 cwt.
9. ·625 of £1 to the dec. of a guinea, and of half-a-guinea.
10. 3·589 po. to the dec. of a chain; ·4326 yd. to the dec. of an ell.
11. 1 oz. Av. to the dec. of 1 oz. Troy; ·54375 lb. Troy to ounces Av.
12. ·67375 sq. chain to sq. yards; 1 perch to the dec. of 1 sq. furlong.

13. Find the decimal of a leap year which differs from a week by less than the ten-thousandth of a leap year.

14. Reduce 5¼d., ¾d., 8s. 11¾d., £1. 14. 10½ to the decimal of 1s.

15. Express 10s. 2¼d., 17s. 0⅝d., 1¾d., 1⅝d. ½, each as a decimal of £1.

16. Express as a decimal of £1 -£3. 18. 11¼, £2. 15. 9⅝, and £5. 7. 8¼.

17. Express 12s. 6¾d. as a decimal of £1, of £100, and of £·001.

18. Reduce 18s. 11¾d. to the decimal of a guinea; £1. 4. 4½ to the decimal of 27s.

19. Reduce £3. 15. 9¼ to the decimal of £9; 4⅝ guineas to the decimal of £50.

20. Reduce £2. 17. 9⅝ to the decimal of £3. 15s.; 10s. 0¾d. to the decimal of 3¾ guineas.

21. Reduce 3s. 3⅝·6855 to the decimal of £10; 5s. $1\frac{87}{1000}d.$ to the decimal of 1½ guinea.

22. Reduce £1. 15. 6¾ ¼ to the decimal of £5. 5s.; £13. 6. 8½ ¼ to the decimal of £50.

23. What decimal is £4. 6. 4½ of £5; £4. 18. 11¾ ½ of 6 guineas and a half; and £18. 19. 6¾ ¼ of 25 guineas?

24. Reduce 10 oz. 11 dwts. $21\frac{1}{3}$ grs. to the dec. of 1 lb. Troy; and of 1 lb. Av.

25. Reduce 4 cwts. 1 qr. $10\frac{1}{2}$ lbs. to the dec. of 1 cwt.; 17 cwts. 3 qrs. 17 lbs. $8\frac{7}{18}$ oz. to the dec. of a ton.

26. Express 11 yards, 3 furlongs 66 yards, and 6 yds. 2 ft. $7\frac{1}{2}$ in. each as a decimal of a mile.

27. Express 186 yards 2 ft. $8\frac{1}{25}$ in. as a dec. of a chain and of a link; and 2 qrs. 3 nls. $1\frac{5}{8}$ in. as a dec. of an ell.

28. Express 9 cwts. 13 lbs. 4 oz. 3·84 drs. as a dec. of a ton; and 1 gall. 3 qts. $1\frac{1}{2}$ pt. as a dec. of a gallon.

29. Express 5 days 12 h. 25 m. 37·92 s. as a decimal of a week.

30. Reduce 9 oz. $15\frac{7}{11}$ drs. to the dec. of 1 lb.; $14\frac{3}{8}$ oz. to the dec. of 1 oz. Troy; 10 lbs. 13 oz. 13 drs. to the dec. of 1 lb. Troy.

31. Express 17 lbs. 6 oz. 10 dwts. 21 grs. as a dec. of 1 cwt.; 29 days 12 h. 44 m. 2·82 s. as a dec. of a day; 3 qrs. 5 bush. 3 pk. $1\frac{3}{8}$ gall. as a dec. of a load.

32. Reduce 5 poles 4 yds. $2\frac{1}{2}$ ft. to the dec. of a furlong; 3 roods 31 p. $16\frac{1}{2}$ yds. to the dec. of an acre; 13 cu. feet 1323 cu. in. to the dec. of a cu. yard.

33. Reduce 5 cwts. 3 qrs. 6 lbs. 13 oz. $8\frac{8}{9}$ drs. to the dec. of a ton; 12 hours 55 m. $23\frac{1}{13}$ s. to the dec. of a day; 7 sq. chains 13 per. $13\frac{8}{9}$ yds. to the dec. of an acre.

34. Reduce 2 fur. 11 yds. 1 ft. 9 in. to the dec. of a mile; 272 days 4 h. 31 m. 36 s. to the dec. of a year of 365 days.

35. Find a dec. of a month which differs from 3 weeks 4 d. 5 h. 6 m. 7 s. by less than the millionth of a month.

Find the value of

36. ·0625 of a shilling; ·4375 of £1; ·334375 of £1.

37. ·97916 of 1*s*.; ·7635416 of £1; ·002083 of £1.

38. 3·45833*s*.; £5·79895833; 1·025 guinea.

39. ·52715 of a cwt.; ·3375 of an acre; 3·46875 qrs. (dry).

40. 1·28 ell; ·4765625 of a mile; 2·5384375 days.

41. ·8716 ton; ·10714285 of a cwt.; ·78409 of an acre.

42. ·01234375 ton; 2·275868055 lbs. Troy; ·0000355113636 mile.

43. £·0125 + ·0625*s*. + ·5*d*. + 4*s*. $7\frac{3}{4}$*d*.; ·91789772 acre.

44. ·175 ton + ·195 cwt. + ·145 qr. + ·15 lb.

MULTIPLICATION, DIVISION.

270. To multiply or divide a quantity by a decimal, or to find the value of a decimal of a quantity.

(1) *We may express the given quantity, when necessary, as a simple quantity, and perform the required operation:* or

(2) *We may reduce the decimal to a fraction in its lowest terms, and proceed as in fractions* (264). We must frequently have recourse to this method when the decimal is *repeating* and the result is required to be *exact.*

Ex. 1. Multiply £3. 14. $6\frac{3}{4}$ by 2·46875; or

Find the value of 2·46875 of £3. 14. $6\frac{3}{4}$.

£. s. d.		
3 . 14 . $6\frac{3}{4}$	2·46875	$2·46875 = 2\frac{46875}{100000} = 2\frac{15}{32}$.
20	3579	
74	2221875	£. s. d.
12	1728125	3 . 14 . $6\frac{3}{4}$
894	1234375	15
4	740625	4) 55 . 18 . $5\frac{1}{4}$
3579*f.*	4) 8835·65625	8) 13 . 19 . $7\frac{1}{4}\frac{1}{4}$
	12) $2208\frac{3}{4}$	1 . 14 . $11\frac{1}{4}\frac{21}{32}$
	20) 184 . $0\frac{3}{4}$	7 . 9 . $1\frac{1}{2}$
	£9 . 4 . $0\frac{3}{4}$·65625	£9 . 4 . $0\frac{3}{4}\frac{21}{32}$

Ex. 2. The circumference of a circle is 3·14159 times its diameter: find the diameter of a circle whose circumference is 13 yds. 2 ft. $7\frac{3}{4}$ in.

	yds.	yds.
12) 7·75	3·14159) 13·88194	(4·41876
3) 2·64583̇	1315 58	1·25628
13·88194	5894	3·07536
	2752	
	239	
	20	

∴ Quotient = 4 yds. 1 ft. 3·075...in.

Ex. 3. Find the value of 2·86805̇ of 3*s.* + ·83̇ of 4*s.* − 1·8 of 5*s.*

s.	*s.*	*s.*
2·86805̇	·83̇	1·8
3	4	5
8·60416̇ (161)	3·33̇	9·
3·33333̇		
11·9375 (160)		
9		
2·9375*s.*		
11·25		
1·		

∴ Value required = 2*s.* $11\frac{1}{4}$*d.*

Ex. 4. Find the value of $3\cdot\dot{3}$ of $\dfrac{4\cdot\dot{4}}{\cdot\dot{7}3\dot{5}}$ of 1 sq. ft. 3 in.

$$\text{Value required} = 3\tfrac{1}{3} \times \frac{4\frac{4}{9}}{\frac{735}{999}} \times 1\tfrac{1}{48} \text{ sq. ft.}$$

$$= \frac{10}{3} \times \frac{40}{9} \times \frac{999}{735} \times \frac{49}{48} \text{ sq ft.}$$

$$= \frac{370}{18} \text{ sq. ft.} = 20\tfrac{5}{9} \text{ sq. ft.}$$

$$= 20 \text{ sq. ft. } 80 \text{ in.}$$

Exercise 41.

Find the value of

1. ·78125 of £6; 2·3456 of £5; 3·9583 of £8.

2. ·7365 of 6*s.* 8*d.*; ·365 of £1. 0. 10; ·2345 of £2. 15*s.*

3. 32·015 of half-a-crown; ·59375 of 19*s.* 4*d.*; ·3481 of £4. 18. 8.

4. ·00390625 of £1. 12; ·0474609375 of £10. 13. 4; 4·6099 of £1. 13. 8.

5. ·3792 of £3. 18. 1½; ·0013 of £3. 17. 10½; ·00015740 of £81.

6. £874. 13. 4 × 1·875; £4. 15. 7½ × 24·5775.

7. £10. 15. 9 × 2·368; £14. 18. 7½ × 345·67.

8. £1205. 6. 8 ÷ 51·2; £23. 19. 2¼ ÷ 13·53.

9. £503. 12. 6¾ ÷ 26·312; £19. 0. 9·084 ÷ 5·07.

10. 27 lbs. 13 oz. 15 drs. × ·4352; 3 lbs. 9 oz. 8 dwts. 13 grs. × 46·802.

11. ·9765625 of 2 tons 18 cwts. 3 qrs. 14 lbs.

12. 20 po. 4 yds. 1 ft. 9 in. ÷ 15·625; 225 days 14 h. 36 m. ÷ 8·71846.

13. ⅓ × ·47 of £360. 2. 3; ·01 × ·101 of £74. 18. 6.

14. ·538461 of 3 hrs. 52 m. 16 s.; ·5357142 8 of 2 cwts. 3 qrs. 17½ lbs.

15. ·013 of 305 ac. 2 r. 20 p.; 13·263798 of 3 miles 7 fur. 22½ poles.

16. Reduce ·583 to a fraction, and find the number of grains it expresses when the unit is 3 oz. 5 dwts.

17. What does ·54186 represent when 12*s.* 4*d.* is taken as the unit?

Find the value of

18. $1\cdot6875$ of £3 − $\cdot0016875$ of £9. 9s. + $16\cdot875$ of 7s. 6d.

19. $1\cdot875$ of £1. 1s. + $1\cdot875$ of a crown + $1\cdot875$ of £$3\cdot625$.

20. $8\cdot71875$ of 8d. + $1\cdot46875$ of 6s. 8d. − $\cdot0625$ of 8 guineas.

21. $\cdot375$ of 6s. 8d. + $\cdot941875$ of 4s. + $1\cdot96971$ of 2s.

22. $\cdot625$ of £1. 1s. + $\cdot\dot{5}\dot{4}$ of 8s. 3d. + $\cdot0\dot{2}\dot{7}$ of £2. 15s.

23. $\cdot45$ of £1. 3. 9 + $\cdot2\dot{5}\dot{7}$ of £11. 5. 6 + $\cdot3125$ of £2. 10s

24. $\frac{3}{4}$ of $\frac{1}{9\frac{1}{2}}$ of £1. 18s. + $\frac{2}{3}$ of $\cdot\dot{3}75$ of 15s. + $\frac{2}{5}$ of $\cdot4\dot{2}\dot{9}$ of 8s. 3d.

25. $\cdot03125$ of £20 + $\cdot\dot{7}2\dot{9}$ of 6s. 2d. + $\cdot7\dot{2}\dot{9}$ of £2. 1. 3.

26. $\cdot\dot{8}5714\dot{2}$ of $2\cdot0625$ tons + $\cdot\dot{5}7142\dot{8}$ of $3\cdot375$ cwts. + $\cdot\dot{7}1428\dot{5}$ of $1\cdot25$ qrs. + $\cdot\dot{1}8571\dot{4}$ of $10\cdot5$ lbs.

27. Add $1\cdot275$ of a sq. yard to $3\cdot75$ of a square foot, and find the value at 3s. 4d. per foot.

28. Find the difference between $\cdot856$ of 20 guineas and $\cdot899$ of £20.

29. Find the difference between $3\cdot2\dot{4}$ of £236. 5s. and $3\cdot24$ of £236. 5s.

30. Find what $\dfrac{2\cdot8 \text{ of } 2\cdot2\dot{7}}{1\cdot1\dot{3}\dot{6}} + \dfrac{4\cdot\dot{4} - 2\cdot8\dot{3}}{1\cdot\dot{6} + 2\cdot\dot{6}1\dot{9}}$ of $\dfrac{6\cdot8 \text{ of } 3}{2\cdot25}$ represents, when £1. 14. $6\frac{3}{4}\cdot875$ is taken as the unit.

271. *To find what decimal one concrete quantity is of any other of the same kind.*

When the second quantity is *simple* we have already shewn how this process may always be effected by Reduction (269); when it is *compound* we find what *fraction* the first quantity is of the second (265) and reduce this fraction to a decimal.

Ex. 1. Reduce 3 roods 26 p. $28\frac{1}{2}$ yds. to the decimal of 3 acres 1 p. $9\frac{3}{4}$ yds.

rds. po. yds.	rds. po. yds.
3 . 26 . $28\frac{1}{2}$	12 . 1 . $9\frac{3}{4}$
40	40
146	481
$30\frac{1}{4}$	$30\frac{1}{4}$
$4408\frac{1}{2}$	$14439\frac{3}{4}$
$36\frac{1}{2}$	$120\frac{1}{4}$
4445 yds.	14560 yds.

But $\qquad \dfrac{4445}{14560} = \dfrac{889}{2912} = \dfrac{127}{416} = \dfrac{127}{8 \times 4 \times 13}.$

$$\begin{array}{r|l} 8 & 127 \\ \hline 4 & 15{\cdot}875 \\ \hline 13 & 3{\cdot}96875 \\ \hline & {\cdot}30528846153 \text{ decimal required} \end{array}$$

Ex. 2. Find what decimal $\frac{1}{4}$ of $\frac{1}{2\frac{4}{5}}$ of $\frac{4}{5}$ of 5 cwts. 2 qrs. 14 lbs. 7 oz. is of $\cdot\dot{4}2857\dot{1}$ of 15 tons 8 cwts. 1 qr. 14 lbs

cwts	qrs	lbs.	oz.		tons	cwts.	qrs.	lbs.
5	2	14	7		15	8	1	14
4					20			
22					308			
28					4			
630					1233			
16					28			
10087 oz.					34538 × 16 oz.			

$\therefore \frac{1}{4}$ of $\frac{1}{2\frac{4}{5}}$ of $\frac{4}{5}$ of 5 cwts. 2 qrs. 14 lbs 7 oz. $= \frac{1}{4} \times \frac{5}{14} \times \frac{4}{5} \times 10087$ oz.

$= 720{\cdot}5$ oz.

and $\cdot\dot{4}2857\dot{1}$ of 15 tons 8 cwts. 1 qr. 14 lbs. $= \dfrac{428571}{999999} \times 34538 \times 16$ oz.

$= \dfrac{3}{7} \times 34538 \times 16$ oz.

$= 2467 \times 96$ oz.

$$\begin{array}{r|l} 8 & 720{\cdot}5 \\ \hline 12 & 90{\cdot}0625 \\ \hline 2467 & 7{\cdot}5052083 \,(\,{\cdot}0030422409 \end{array}$$

```
             10420
               5528
                5943
                10093
                   22533
                     330
```

$\therefore$ Decimal required $= {\cdot}0030422409\ldots\ldots$

Exercise 42.

1. Reduce 3*s.* 11¾*d.* to the dec. of £1. 19. 4½; £1. 2. 3¼ to the dec. of £17. 16. 4; £1. 4. 0¾ to the dec. of £2. 10. 10½.

2. What decimal is 1$\frac{1}{2}$*d.* of 7*s.* 10$\frac{1}{2}$*d.* and 7*s.* 8$\frac{1842}{10000}$*d.* of 15*s.* 9*d.*?

3. What dec. is $\frac{7}{8}$ of 10*s.*, of 13*s.* 4*d.*? $\frac{7}{8}$ of £1. 1*s.*, of £2. 10*s.*? 3·45 of 10*s.* 6*d.*, of half a crown? $\frac{3}{4}$ of 2*s.* 6*d.*, of $\frac{5}{8}$ of a guinea and a half?

4. Express 3*s.* 5$\frac{11}{100}$*d.* as a decimal of a dollar of 4*s.* 1$\frac{1}{2}$*d.*

5. Express £5·456 as a decimal of a rupee of 1*s.* 10*d.*

6. Express ·0527 of £1. 7. 6 as a decimal of 13*s.* 4*d.*

7. Reduce 10 lbs. 11 oz. 12 dwts. 7 grs. to the dec. of 9 lbs. 8 oz. Av.

8. Reduce 3 hours 26 m. 37 s. to the decimal of 13 days 20 h. 23 m.

9. Reduce 3 roods 24 p. to the decimal of 2 acres 1 r. 36 p.

10. Reduce 1 cwt. 2 qrs. 3$\frac{1}{4}$ lbs. to the dec. of 1 ton 4 cwts. 1 qr. 24 lbs.

11. Add together £$\frac{1}{8}$, $\frac{3}{4}$ of 5*s.* and $\frac{3}{8}$ of £1. 1*s.*, and reduce the sum to the decimal of £25.

12. Add together £·0375, ·625*s.*, ·75*d.* and 3*s.* 3·5*d.*, and reduce the sum to the decimal of 7*s.* 6*d.*

13. Add together $\frac{5}{18}$ of 21*s.*, $\frac{3}{32}$ of £1, $\frac{7}{16}$ of 5*s.* and $\frac{5}{18}$ of 1*s.*, and reduce the sum to the decimal of £3. 15*s.*

14. Express $\frac{3}{8}$ of 7*s.* 6*d.* + ·625 of 10*s.* − ·545 of 9*s.* 2*d.* as a dec. of £10.

15. Add together $\cdot 2\dot{4}\dot{6}$ of 9*s.* 3*d.*, $\cdot 25\dot{9}$ of £1. 5*s.*, and ·02 of £3. 7. 6, and reduce the result to the decimal of £3.

16. Express £874. 13. 4 × 3·75 as a dec. of £1000.

17. Express the difference between $\frac{5}{8}$ of 5*s.* 9*d.* and $\frac{5}{7}$ of 6*s.* 4*d.* as the dec. of half a guinea.

18. Express a florin, a sixpence, and a fourpence as a decimal of 2$\frac{3}{4}$*d.*

If $\frac{1}{18}$ of 2$\frac{3}{4}$*d.* be the unit, what decimal will express a half-penny?

19. Reduce 8·4987 rupees, each worth 2*s.* 0$\frac{1}{2}$*d.*, to the decimal of £3. 13. 6.

20. What decimal of a crown is the difference between 6$\frac{1}{2}$ half-guineas and £3·525?

21. Reduce 13 perches 23$\frac{3}{4}$ sq. yds. to the decimal of 2·25 sq. chains.

22. Reduce 2$\frac{3}{8}$ of £1. 6. 5$\frac{3}{4}$ to the decimal of £18. 17. 10$\frac{3}{4}$.

23. Express ·101 of 1 lb. 5 oz. as a decimal of $\frac{1}{7}$ of 1 qr. 22 lbs. 8 oz.

24. Express $\frac{7}{11}$ of 5 ells 4 qrs. 2 nls. 1$\frac{1}{2}$ in. as a decimal of 10 po. 3 yds. 2$\frac{1}{4}$ ft.

CHAPTER XI.

DECIMAL MONEY: THE METRIC SYSTEM.

DECIMAL MONEY.

272. *To reduce any sum of money to the decimal of £1.*

(1) To reduce shillings to the decimal of £1 we divide by 20 (267, ii.), that is,—*We divide by 2, and move the decimal point one place to the left* (144); thus,

$$4s. = £·2, \qquad 11s. = £·55, \qquad 19s. = £·95.$$

(2) To reduce pence, &c. to the decimal of £1.

$$3\tfrac{1}{4}d. = 13f., \text{ and } £1 = 960f.;$$

$$\therefore 3\tfrac{1}{4}d. = £\frac{13}{960} = £\frac{13 \times (1 + \frac{1}{24})}{960 \times (1 + \frac{1}{24})} = £\frac{13\frac{13}{24}}{1000} = £·013\tfrac{13}{24},$$

where the number in the decimal part and in the numerator of the fraction is the number of farthings in $3\frac{1}{4}d.$ In like manner,

$$5\tfrac{1}{2}d. = £·022\tfrac{22}{24}, \qquad 6d. = £·024\tfrac{24}{24}, \qquad 7\tfrac{3}{4}d. = £·031\tfrac{31}{24}.$$

From 6*d.* upwards the fraction over is improper, but if we increase the number of farthings in the decimal part by 1, the numerator will then be the number of farthings above 6*d.*; thus,

$$6d. = £·025, \qquad 7\tfrac{3}{4}d. = £·032\tfrac{7}{24}, \qquad 10\tfrac{1}{2}d. = £·043\tfrac{18}{24}.$$

Lastly, to express the fraction as a decimal we divide mentally by 2 and then by 12; thus we write at sight

$3\frac{1}{4}d. = £·013,541\dot{6}$ where we divide 65 by 12;

$6d. = £·025$;

$10\frac{1}{2}d. = £·043,75$ where we divide 90 by 12;

and $11\frac{3}{4}d. = £·048958\dot{3}$ .... 115 by 12.

(3) If now the number of shillings be *even*, we write down their decimal of £1, and then write down the decimal of the pence, &c.; thus, $4s.\ 3\frac{1}{4}d. = £\cdot 213541\dot{6}$, $18s.\ 10\frac{1}{2}d. = £\cdot 94375$.

But if the number of shillings be *odd*, the 5 in the second place of their decimal must be added to the first figure of the decimal of the pence, &c.; thus,

$$7s.\ 3\frac{1}{4}d. = £\cdot 363541\dot{6}, \qquad 19s.\ 10\frac{1}{2}d. = £\cdot 99375.$$

In the same way we can write at sight

$$3s.\ 10\frac{3}{4}d. = £\cdot 194791\dot{6}, \qquad £5.\ 18.\ 11\frac{3}{4} = £5\cdot 948958\dot{3};$$

$$15s.\ 0\frac{3}{4}d. = £\cdot 753125, \qquad £8.\ 17.\ 10\frac{1}{2} = £8\cdot 89375;$$

and so for every other sum of money.

273. Conversely,—*To find the value of any decimal of £1.*

For the shillings double the figure in the first place of decimals, and if the figure in the second place be 5 or greater add 1; thus,

$$£\cdot 6345 = 12s. \ldots \qquad £\cdot 78456 = 15s. \ldots$$

Remove these figures and in the second example we have £·03456; but

$$£\cdot 03456 = £\frac{34\cdot 56}{1000} = £\frac{34\cdot 56 \times \frac{24}{25}}{1000 \times \frac{24}{25}} = £\frac{34\cdot 56 \times (1 - \frac{1}{25})}{960}$$

$$= 34\cdot 56 \times (1 - \tfrac{1}{25})f.;$$

or

$$= 34\cdot 56 \times (1 - \tfrac{4}{100})f.;$$

if therefore from 34·56 we subtract $\frac{4}{100}$ of 34·56 we shall have the result in farthings; that is, we must multiply 34·56 by 4, setting down 2 places to the right, and subtract; thus

$$\begin{array}{r} 34\cdot 56 \\ 1\cdot 3824 \\ \hline 33\cdot 1776 \end{array}$$

therefore $£\cdot 78456 = 15s.\ 8\frac{1}{4}d.\ \cdot 1776$.

In like manner we find the values of £3·948125 and of $£\cdot 6591\dot{6}$; thus—

$$\begin{array}{r} 48{\cdot}125 \\ \underline{1{\cdot}92500} \\ 46{\cdot}2 \end{array} \qquad\qquad \begin{array}{r} 9{\cdot}1\dot{6} \\ \underline{{\cdot}366\dot{6}} \\ 8{\cdot}8 \end{array}$$

$\therefore$ £3·948125 = £3. 18. 11$\frac{1}{2}$·2. $\qquad$ $\therefore$ £·6591$\dot{6}$ = 13*s*. 2$\frac{0}{4}$·8;

and similarly for every other decimal of £1.

274. *To reduce any sum of money to the decimal of £1, not going beyond 3 places.*

Now 3¼*d*. = £·013$\frac{13}{24}$ (272, 2) where the numerator of the fraction is the number of farthings in the given sum; hence up to 3*d*. this fraction is less than ½, above 3*d*. and up to 9*d*. it is greater than ½ and less than 1½, above 9*d*. and up to 12*d*. it is greater than 1½ and less than 2; therefore, that the third place of decimals may be as nearly correct as possible, we must up to 3*d*. neglect the fraction altogether, and the decimal will be given by the number of farthings; above 3*d*. and up to 9*d*. we must add 1; above 9*d*. and up to 12*d*. we must add 2: thus,

2¼*d*. = £·009 and 6*s*. 2¼*d*. = £·309,
7¾*d*. = £·032 ... 15*s*. 7¾*d*. = £·782,
11½*d*. = £·048 ... 17*s*. 11½*d*. = £·898.

275. Conversely,—*To find the value of any decimal of £1 of three places, to the nearest farthing.*

Remove the figures giving the shillings and suppose the remaining decimal to be £·034.

Now $\qquad$ £·034 = 34 × (1 − $\frac{1}{25}$) *f*. = 34 *f*. − $\frac{34}{25}$ *f*., $\qquad$ (273)

hence if the number formed by the figures in the second and third places of the remaining decimal be not greater than 12 the fraction to be subtracted is less than ½, and therefore this number will give the number of farthings; but from 13 to 37 the fraction is greater than ½ and less than 1½, therefore we must subtract 1; and from 38 upwards we must subtract 2. Thus,

£·807 = 16*s*. 1¾*d*., $\qquad$ £·432 = 8*s*. 7¾*d*.,
£5·399 = £5. 7. 11¾, $\qquad$ £7·790 = £7. 15. 9½.

276. In the *proposed decimal coinage*, a sovereign (£1) is taken as the unit (212), and its submultiples the florin, cent, and mil are respectively the tenth, hundredth, and thousandth part of the sovereign; so that in a decimal of £1 the figure in the first place of decimals represents florins, in the second cents and in the third mils. Hence to convert ordinary money into decimal money is simply to express ordinary money as a decimal of £1: and to convert decimal money into ordinary money is simply to express a decimal of £1 in ordinary money: and these processes both accurate and approximate have been pointed out in the preceding Articles (272—275).

Ex. 1. Reduce £15. 7*c.* 8*m.* to cents; £9. 8*f.* 6*c.* to mils; 25684 mils to pounds, &c.; and 5*c.* 6*m.* to the decimal of a florin.

To reduce pounds to cents we multiply by 100 (229); to reduce pounds to mils we multiply by 1000; to reduce mils to pounds we divide by 1000 (230); and to reduce cents to florins or the decimal of a florin we divide by 10 (267, ii): hence

(1) £15. 7*c.* 8*m.* = £15·078 = 1507·8*c.* (144)
= 1507*c.* 8*m.*

(2) £9. 8*f.* 6*c.* = £9·86 = 9860*m.*

(3) 25684*m.* = £25·684 (144)
= £25. 6*f.* 8*c.* 4*m.*

(4) 5*c.* 6*m.* = 5·6*c.* = ·56*f.*

Ex. 2. Find the sum of £12. 3*f.* 5*m.*, £23. 6*m.*, £18. 9*f.*, £25. 46*c.* and £37. 7*f.* 8*c.* 9*m.*

	£.
£12. 3*f.* 5*m.*	= 12·305
£23. 6*m.*	= 23·006
£18. 9*f.*	= 18·9
£25. 46*c.*	= 25·46
£37. 7*f.* 8*c.* 9*m.*	= 37·789
	117·460 = £117. 4*f.* 6*c.*

Ex. 3. Subtract £123. 7*f*. 8*c*. from £987. 6*c*. 5*m*.; and divide the difference by 5·643.

	£.
£987. 6*c*. 5*m*. =	987·065
£123. 7*f*. 8*c*. =	123·78

```
5·643)863·285 ( 152·9833
       29898
        16835
         55490
          4703
           189
            20
             3
```

∴ Quotient required = £152·9833.. = £152. 9*f*. 8*c*. 3·3..*m*.

Ex. 4. Express £3. 15. 7½ and £4. 18. 10¾ accurately in the proposed decimal coinage; that is in £. *f*. *c*. *m*.

At sight £3. 15. 7½ = £3·78125 (272)

= £3. 7*f*. 8*c*. 1·25*m*. (276)

At sight £4. 18. 10¾ = £4·9447916

= £4. 9*f*. 4*c*. 4·7916*m*.

Ex. 5. Express £2. 7*f*. 8*c*. 9*m*. and £1. 9*f*. 3*c*. 6·4583*m*. accurately in ordinary money: that is, in £. *s*. *d*.

£2. 7*f*. 8*c*. 9*m*.	= £2·789	39 1·56 37·44	(273)
	= £2. 15. 9¼·44.		
£1. 9*f*. 3*c*. 6·4583*m*.	= £1·9364583	36·4583 1·4583 35·	
	= £1. 18. 8¾.		

Ex. 6. Express £6. 8. 2¼ and £7. 13. 10¾ to the *nearest mil* in £. *f*. *c*. *m*.

At sight £6. 8. 2¼ = £6·409 (274)

= £6. 4*f*. 0*c*. 9*m*. (276)

At sight £7. 13. 10¾ = £7·695 = £7. 6*f*. 9*c*. 5*m*.

Ex. 7. Express £3. 4*f*. 5*c*. 6*m*., £8. 8*f*. 2*c*. 8*m*., £9. 9*f*. 9*c*. 6*m*. to the *nearest farthing* in £. *s*. *d*.

Now £3. 4*f*. 5*c*. 6*m*. = £3·456 = £3 . 9 . 1½ (275)

£8. 8*f*. 2*c*. 8*m*. = £8·828 = £8 . 16 . 6¾

£9. 9*f*. 9*c*. 6*m*. = £9·996 = £9 . 19 . 11

EXERCISE 43.

1. Reduce £45. 1*f.* 7*c.* to mils; and 876145 mils to £. *f.* *c.* *m.*

2. Reduce £25. 25*m.* to mils; and 3809 mils to florins.

3. Reduce £8. 4*f.* $3\frac{1}{2}$*c.* to mils; and $26\frac{1}{4}$*f.* to mils.

4. Find the sum of £327. 9*f.* 4*c.* 5*m.*, £89. 4*f.* 7*c.* 8*m.*, £5. 6*f.* 5*m.*, £479. 8*c.* 8*m.*, and £63. 75*c.*

5. Add together £78. 75*c.*, £14. $3\frac{1}{4}$*f.*, £9. 5*f.* $8\frac{1}{8}$*c.*, £35. 427*m.*, and 56384*m.*

6. Find the difference between £19. 9*f.* 9*m.* and £54. 2*f.* 3*c.* 4*m.*

7. Subtract £525. 7*f.* 6*c.* 3*m.* from £1000.

8. Subtract 5*f.* $3\frac{1}{2}$*c.* from £2. $43\frac{1}{5}$*c.*

9. Multiply £34. 2*f.* 8*c.* 9*m.* by 89; £5. 47*m.* by 5608.

10. Multiply $8\frac{3}{4}$*f.* by 1000; and £370. 5*m.* by 4376.

11. Divide £6852. 3*f.* 8*c.* 7*m.* by 8760.

12. Divide £230. 923*m.* among 77 persons: find each one's share.

13. How often is £5. 6*f.* 7*c.* 8*m.* contained in £4479. 9*f.* $4\frac{1}{8}$*c.*?

14. Reduce 7*f.* $3\frac{3}{8}$*c.* to the decimal of £1 and of £50.

15. Find the value of $\frac{1}{4}$ of $\frac{1}{2\frac{1}{3}}$ of $\frac{4}{5}$ of 5·4 of £67. 4*f.* 4*c.* 6*m.*

Express each of the following sums accurately in the decimal coinage; that is, in £. *f.* *c.* *m.*:—

16.	4*s.* 3*d.*;	9*s.* 6*d.*;	£3. 13. 9;	$4\frac{1}{2}$*d.*;	£1. 15. $10\frac{1}{2}$.
17.	3*s.* $0\frac{3}{4}$*d.*;	15*s.* $8\frac{1}{4}$*d.*;	£8. 18. $11\frac{1}{4}$;	$3\frac{3}{4}$*d.*;	£7. 19. $6\frac{3}{4}$.
18.	6*s.* 4*d.*;	2*s.* $1\frac{1}{4}$*d.*;	£1. 9. $4\frac{3}{4}$;	$5\frac{1}{2}$*d.*;	£2. 8. 5.
19.	13*s.* $0\frac{1}{4}$*d.*;	17*s.* $7\frac{3}{4}$*d.*;	£3. 0. $0\frac{1}{2}$;	10*d.*;	£2. 12. $11\frac{3}{4}$.
20.	15*s.* 8*d.*;	12*s.* $11\frac{3}{4}$*d.*;	£1. 0. $9\frac{1}{2}$;	$10\frac{3}{4}$*d.*;	£5. 17. $11\frac{1}{2}$.

Express each of the following sums accurately in ordinary money; that is, in £. *s.* *d.*:—

21. £5. 6*f.* 2*c.* 5*m.*; 4*f.* 8*c.* $7\frac{1}{2}$*m.*; 1*f.* 6·25*m.*; £2. 4*f.* 5*c.* 9·375*m.*

22. £3·621875; 9*f.* 9*c.* $6\frac{7}{8}$*m.*; 8*f.* 5*c.* $3\frac{1}{8}$*m.*; 7*f.* 8*c.* $3\frac{1}{2}$*m.*

23. £·1135416; £4·822916; £1. 7*f.* 9*c.* $6\frac{1}{8}$*m.*; £19. 8*f.* 7*c.* 6*m.*

24. £92·7739583; £5·9447916; 9*f.* 6·34*m.*; £2. 4*f.* 6*c.* 7·95*m.*

Express each of the following sums in £. *f.* *c.* *m.* to the *nearest mil*:—

25. 12*s.* 8*d.*; 15*s.* 4*d.*; £2. 12. 9½; 10¾*d.*; £3. 18. 6¾.

26. 17*s.* 4½*d.*; 14*s.* 3*d.*; £3. 18. 11; 3½*d.*; £7. 9. 4¼.

27. 15*s.* 0½*d.*; 17*s.* 9¼*d.*; £1. 13. 8; 9*d.*; £8. 19. 11¾.

Express each of the following sums in £. *s.* *d.* to the nearest farthing:—

28. 4*f.* 1*c.* 9*m.*; £3. 6*f.* 5*c.* 1*m.*; 8*f.* 3*c.* 2*m.*; £5·678.

29. £2. 3*f.* 4*c.* 5*m.*; £4. 2*f.* 6*c.*; 8*f.* 2*c.* 3*m.*; £1·073.

30. £45. 7*f.* 5*c.* 3*m.*; 7*f.* 8*c.* 9*m.*; £25·69; £87. 9*f.* 9*c.* 7*m.*

277. SOME APPLICATIONS OF THE PRECEDING RULES.

Ex. 1. Find the value of 427·46875 oz. of gold at £3. 14. 6¾ per oz.

Write down at sight £3. 14. 6¾ as £3·728125 (271, 3), and multiply, retaining 4 places of decimals, thus

```
   427·46875
    521 8273
   ---------
   1282 4063
    299 2281
      8 5494
      3 4197
         427
          85
          21
   ---------
£1593·6568 = £1593. 13. 1¾.
```

Ex. 2. Find the value of 427$\frac{15}{32}$ oz. of gold at £3. 7*f.* 2*c.* 8⅛ *m.* per oz.

427$\frac{15}{32}$ oz. = 427·46875 oz.,

£3. 7*f.* 2*c.* 8⅛*m.* = £3·728125;

hence, by the last example,

Value required = £1593·6568 = £1593. 6*f.* 5*c.* 7*m.*

Ex. 3. Find the cost of 87 tons 13 cwts. 3 qrs. 24 lbs. 13¾ oz. at £17. 4. 10¾ per ton.

```
   16 ) 13·625                         87·69859
      {4 ) 24·85156                    80724271
   28 {                                --------
      {7 )  6·21289                    8769859
    4 )  3·88755                       6138901
   20 ) 13·97188                        175397
        87·69859 tons.                   35079
                                          1754
                                           614
                                             7
                                    ----------
                          £1512·1611 = £1512. 3. 2¾.
```

Ex. 4. If a train travels 82 miles 7 fur. 26 po. 4 yds. in 3 hours 48 m. $51\frac{1}{2}$ s., find the average rate per hour.

```
   11 ) 8                        60 ) 51·5
   40 ) 26·72727                 60 ) 48·85833
    8 )  7·668181                      3·814305 hours.
        82·958522 miles.

   3·8,1,4,3,0,5 ) 82·958522 ( 21·749315 miles.
                    6672422              8
                    2858117       --------
                     188103        5·99452
                      35531             40
                       1202       --------
                         58        39·7808
                         20             5½
                          1        ---------
                                    39040
                                     3904
                                   ------
                                   4·2944
```

∴ Rate per hour is 21 miles 5 fur. 39 po. $4\frac{1}{4}$ yds.

Ex. 5. A tradesman's liabilities amount to £4375. 16. $9\frac{1}{4}$, and his assets realize £1957. 13. $4\frac{1}{2}$: how much will his estate pay in the pound, and how much will a creditor receive for a debt of £359. 15. $3\frac{3}{4}$?

```
   43,7,5,8,3,8,54 ) 1957·66875 ( ·4473814          359·765625
                       20733333                      4183744
                         322998                      -------
                          16690                      1439062
                           3563                       143906
                             63                        25183
                             20                         1079
                              1                          287
                                                           4
                                                   ----------
                                                   £160·9521
```

∴ the estate pays 8*s.* $11\frac{1}{4}$*d.* in the pound, and the creditor receives £160. 19. $0\frac{1}{2}$.

EXERCISE 44.

1. Find the value of 4785 articles at 12*s*. 6$\frac{3}{4}$*d*. each.
2. ....876$\frac{4}{5}$ .... £2. 17. 8$\frac{1}{2}$ each.
3. ....3089$\frac{1}{3}$............£4. 19. 5$\frac{1}{4}$ each.
4. ....594·76............£3. 11. 11$\frac{3}{4}$ each.
5. ....356·9375 bushels at 53*s*. 10*d*. a quarter.
6. .... 3479 things at 8*s*. 5$\frac{3}{4}$*d*. a dozen.
7. ....7857.............. £1. 9. 1$\frac{1}{2}$ a score.
8. ....26953 oranges at 4*s*. 7$\frac{1}{2}$*d*. a hundred.
9. Find the dividend on £565. 19. 3 at 10*s*. 11$\frac{1}{2}$*d*. in the £1.
10. ....£1375. 8. 8$\frac{1}{2}$ at 5*s*. 7$\frac{3}{4}$*d*. ....

Find the value of

11. 68 lbs. 10 oz. 13 drs. at 17*s*. 4$\frac{1}{2}$*d*. a lb.
12. 3 tons 17 cwts. 2 qrs. 21$\frac{1}{2}$ lbs. at £8. 9. 6$\frac{1}{2}$ a cwt.
13. 75 tons 16 cwts. 3 qrs. 23 lbs. at £15. 17. 3$\frac{3}{4}$ a ton.
14. 14 lbs. 10 oz. 15 dwts. 18 grs. of gold at £3. 17. 9 per oz.
15. 89 gallons 3 qts. 1$\frac{3}{4}$ pts. at 29*s*. 8*d*. a gallon.
16. 27 yds. 2 ft. 6$\frac{3}{4}$ in. at 13*s*. 10$\frac{1}{2}$*d*. a yard.
17. 46 acres 3 r. 35 p. at £2. 15. 6 an acre.
18. A bankrupt owes £1578. 18. 6; what must his assets realize to enable him to pay 15*s*. 4$\frac{1}{2}$*d*. in the £1?
19. If a tradesman fail for £3579. 8. 8, and his assets realize £1925. 13. 6$\frac{1}{4}$, how much can he pay in the £1?
20. Find the average speed of a train which travels 120 miles 57 chains 10 yards in 3 hours 35 m. 18 s.
21. If a cubic yard of earth weigh 1 ton 13 cwts. 3 qrs., find the weight of 345 cu. yards 19 ft. 875 in.
22. If a tradesman fail for £2789. 2. 5, and his assets amount to £2007. 8. 2, how much will a creditor on the estate for £197. 15. 9 receive?
23. A plot of building land containing 25 perches 27 yds. 8 ft. 88 in. is sold for £4589. 13. 8; find the price per sq. yard.
24. The annual value of a parish is £138624: a rate is to be laid which shall produce £19875. 15*s*.; at how much in the £1 to the nearest farthing must the rate be laid? How much will be paid on a house rated at £337. 12. 6?

25. A bankrupt owes *A* £515. 12. 6, *B* £407, and *C* £293. 6. 8; his estate is worth £911. 19. 4½: how much can he pay in the £1, and what will *A*, *B*, and *C* receive?

26. If 2 tons 4 cwts. 1 qr. 3½ lbs. cost £330. 14. 6¼, what will be the price of 1 cwt.?

27. If a bar of gold weighing 7 lbs. 5 oz. 12 dwts. 15 grs. be worth £330. 15. 5½, what is the price per oz.?

28. *A*, *B*, and *C* advance respectively £191. 12. 7¾, £61. 14. 8, and £122. 1. 9½ in a mining adventure which yields £511. 12. 6½. How much will this give for every £1 advanced, and how much of it will each of them receive?

MISCELLANEOUS.

29. Find as a decimal the average of $21\frac{2}{3}$, $73\frac{4}{5}$, 0, 3·065, 82, $17\frac{3}{16}$, $5\frac{1}{4}$, and $9\frac{4}{15}$.

30. In how many years will the error amount to a day in considering the year to consist of 365¼ days instead of 365·242218 days?

31. The gallon contains 277·274 cu. inches, and a gallon of water weighs 10 lbs., find the weight of a cu. foot of water. If mercury be 13·568 times heavier than water, find in ounces the weight of a cu. inch of mercury.

32. How many parcels of gold dust each weighing 17·36 grains can be made up out of 1 lb. 2 oz. 1 dwt. 3 grs.; and how much will remain over?

33. The average year of the Gregorian calendar is greater than the true year by 24·3648 seconds. Find, as a decimal of a day, the length of the true year. *The Gregorian calendar intercalates 97 days in 400 years.*

34. The circumference of every circle is 3·14159 times its diameter. The diameter of a carriage wheel is 4 ft. 8½ in.; find its circumference and how many revolutions the wheel will make in going 2 miles 7 f. 15 po.

35. From a rod 1·078 inches long, portions are cut off each equal to ·0037 of an inch long; find how many such portions can be cut off, and what length will remain over.

36. The weight of a cubic foot of sea-water is 2 qrs. 8 lbs. 8¾ oz.; a cask floating therein displaces 5 cu. feet 937½ cu. inches of water; find the weight of the cask, or, which is the same thing, of the water displaced.

THE METRIC SYSTEM.

278. The explanation of the Metric System given in Arts. 216—222 should be carefully read before commencing the following examples.

Ex. 1. Express $31415^{gr}\cdot92$ in succeeding denominations from the highest to the lowest.

Since 10 units of one denomination make 1 unit of the next higher denomination, and the units' figure represents grammes, the tens' figure will represent dekagrammes, the hundreds' figure hectogrammes, ... ; hence

$31415^{gr}\cdot92$ = 3 myriag. 1 kilog. 4 hectog. 1 dekag. 5 gr. 9 decig. 2 centig.

or = 31 kilog. 415 gr. 92 centigr.

or = 31 kilog. 415·92 gr.

Ex. 2. Express 9 hectares 25 ares 8 centiares as a decimal of a sq. kilometre.

A hectare is a sq. hectometre, and 100 sq. hectometres make 1 sq. kilometre (218): hence

9 hectares 25 ares 8 centiares = 9·2508 sq. hectom. (218)
= ·092508 sq. kilom.

Ex. 3. How many hectares are there in a field which contains 13 acres 3 r. 17 p.?

```
       40 ) 17
        4 ) 3·425
2·47114 ) 13·85625 ( 5·60723
           150055
             1787
               57
                8
                1
```

hence we see that 13 acres 3 r. 17 p. = 13·85625 acres,

and from the Table 1 hectare = 2·47114 acres;

$$\therefore \text{ no. of hectares} = \frac{13\cdot85625}{2\cdot47114} = 5\cdot60723.$$

Ex. 4. If a gallon of water weigh 10 lbs., find its volume in cu. centimetres.

2·2,0,4,6,2) 10·00000 (4·53593
118152
7921
1307
205
7
0

From the Table (226) 1 kilog. = 2·20462 lbs.

∴ 10 lbs. = 4·53593 kilog.

= 4535·93 grammes,

= 4535·93 cu. centimetres of water, (221)

that is, a gallon contains 4535·93 cu. centimetres.

Ex. 5. When silk is sold at 19 *f.* 25 *c.* the metre, find the corresponding price per yard in shillings and pence: supposing £1 to be equal to 25 *f.* 20 *c.*

$$19\,f.\ 25\,c. = \frac{19{\cdot}25}{25{\cdot}20} \text{ of } £1 = £\frac{1925}{2520} = £\frac{55}{72},$$

$$\text{and } 1 \text{ metre} = 39{\cdot}371 \text{ inches} = \frac{39{\cdot}371}{36} \text{ yard};$$

$$\therefore \frac{39{\cdot}371}{36} \text{ yard is sold for } £\frac{55}{72};$$

$$\text{or} \quad 1 \text{ yard} \ldots\ldots\ldots\ldots £\frac{55 \times 36}{72 \times 39{\cdot}371};$$

$$\text{or} \quad \ldots\ldots\ldots\ldots\ldots\ldots £\frac{27{\cdot}5}{39{\cdot}371}.$$

£
39·3,7,1) 27·5000 (·69848.
38774
3340
190
33
2

48·48
1·93
46·55

∴ price per yard is 13*s.* 11½·55.

EXERCISE 45.

Express

1. A length of $345678^{m}\cdot09$ in kilometres, and succeeding denominations.

2. A capacity of $207856^{lit}\cdot508$ in kilolitres, ...

3. A surface of $66789560^{sq.m}$ in sq. kilometres, ...

4. A volume of $357^{cu.m}\cdot08267$ in cu. metres, ...

5. A length of 345 kilometres 7 hectometres 6 metres 8 decimetres and 9 millimetres in metres and kilometres.

6. A weight of 3567 kilogrammes 8 hectogrammes 9 grammes and 5 decigrammes in tonneaux de mer, and in decigrammes.

7. An area of 708 hectares 9 ares and 5 centiares in hectares, and in centiares; also in sq. kilometres.

8. A volume of 457 cu. metres 24 cu. decimetres and 60 cu. centimetres in cu. metres.

9. A weight of 67 tonneaux de mer 8 quintaux 5 kilogrammes and 8 grammes in cu. metres, &c. of water.

10. Find the sum of 9 hectares 35 ares 8 centiares, 15 hectares 8 ares and 63 centiares, 23 hectares 85 centiares, and 17 hectares 80 ares 90 centiares.

11. From 4 dekagrammes 83 decigrammes subtract 29 grammes 687 milligrammes.

12. Find as a decimal of a kilogramme the value of

$$\cdot5678 \text{ millier} + 9\cdot3257 \text{ kilog.} - 349\cdot82 \text{ gr.}$$

13. Find the value of $3\frac{11}{12}$ of 87 cu. metres 62 cu. decimetres and 300 cu. centimetres.

14. What will be the price of 47 hectares 5 ares 65 centiares of land at 89 francs 76 centimes the are?

15. If $7^{hl}\cdot89$ weigh 107 kilog. 288·22 gr., find the weight of 1 litre.

16. If 8 metres 8 decimetres of cloth cost 253 francs 88 centimes, find the cost of 17 metres 64 centimetres.

17. The weight of a volume of mercury is 13·598 times that of an equal volume of water: find the weight of 567·859 cu. centimetres of mercury. (224.)

In the following examples we shall use these relations:

1 *metre* = 1·09363 *yds.* = 39·37079 *in.*

1 *sq. metre* = 1·19603 *sq. yds.*; 1 *cu. metre* = 1·30802 *cu. yds.*

1 *hectare or sq. hectometre* = 2·47114 *acres.*

1 litre = 1·76077 pints = ·22010 gallons.

1 kilogramme = 2·20462 lbs. Av. = 15432·3487 grains.

1 gramme = 1 cu. centimetre of distilled water.

18. Express a yard in terms of the metre, and an inch in terms of the centimetre.

19. Find the length of a tunnel 2 miles 63 chains 18 yards long in kilometres and metres.

20. Snowdon is 3571 feet above the level of the sea: express this height in metres, &c.

21. The length of the seconds pendulum in the latitude of London is 39·1393 inches: reduce this length to the decimal of a metre.

22. The mean diameter of the earth is 7912·409 miles: express this length in kilometres.

23. Find in miles, chains and yards the length of a railway 96 kilom. 315 metres long.

24. Mont Cenis tunnel is 12234 metres long: express its length in miles, chains and yards.

25. Mont Blanc is 4810·88 metres high: express this height in feet.

26. The standard height of the barometer at Paris is 76 centimetres: find this height in inches.

27. The metre is the ten-millionth part of the distance from the pole to the equator, measured on the surface of the ocean: find the earth's circumference in miles.

28. Express an acre in terms of the are; and a sq. mile in kilometres.

29. The area of England is 50387 sq. miles: find the area in sq. kilom.

30. Find in hectares, &c. the area of an estate which measures 387 acres 3 r. 24 p.

31. The area of France is 53027894 hectares: express this area in sq. miles.

32. How many acres, &c. are there in a field containing 7 hectares 25 ares 8 centiares?

33. In making a railway cutting 2558 cu. yards 25 cu. feet of earth have been removed: find this quantity in cu. metres, &c.

34. The volume of a room is 870 cu. metres 936 cu. decimetres: express this volume in cu. yards, feet and inches.

35. How many gallons, &c. are there in a cask containing 29 veltes—a velte being equal to 7·4505 litres?

36. How many hectolitres are there in 57 gallons $3\frac{1}{2}$ pints?

37. Express the lb. Troy in grammes; the lb. Av. as a decimal of a kilogramme; and the cwt. as a decimal of a millier.

38. A sea-service mortar weighs 4 tons 19 cwts. 2 qrs. 19 lbs.: find its weight in tonneaux and kilogrammes.

39. By how many kilogrammes is a French gun weighing 263 myriagrammes heavier than an English gun weighing 49 cwts. 3 qrs. 24 lbs.?

40. When cloth is sold at 15*s.* $9\frac{1}{2}$*d.* a yard, what is the corresponding price in francs and centimes per metre, if £1 be worth 25 fr. 25 c.?

41. When wheat is sold at 32 francs 50 centimes the hectolitre, what is the corresponding price in English money per bushel, if 25 fr. 10 c. equal £1?

42. When 325 sq. yards of building land is sold for £3850, find the corresponding price in francs per sq. metre, when £1 is equal 25 fr. 30 c.

43. The rent of a farm of 45 hectares 75 centiares is 3695 francs: find the rent per acre in £. *s.* *d.*, when 25 fr. is worth £1.

44. The pressure of the atmosphere is $14\frac{3}{4}$ lbs. to the sq. inch: find the pressure in kilogrammes to the sq. centimetre.

45. A shot projected with a charge of ·693 kilog. of powder had a range of 734 metres; express these quantities in grains and yards.

46. A 10-inch gun threw a solid shot of 134 lbs. $5\frac{1}{2}$ oz. a distance of 4875 yards; express the bore, weight and range in centimetres, kilogrammes and metres respectively.

47. In the year 1870, 14945 kilolitres 615 litres of brandy were imported into the United Kingdom, and paid a duty of 10*s.* 5*d.* a gallon: find the amount of duty paid.

48. A litre of alcohol, *i.e.* a cu. decimetre, weighs ·792 kilog.: find the weight of a cu. foot in lbs.

49. When the French Post-office allows 10 grammes, the English allows $\frac{1}{2}$ oz.; by how many grains is the latter weight within the former?

50. A cask of brandy containing 4·732 hectolitres is bought for 2835 fr. and duty is paid thereon at the rate of 10*s.* 5*d.* a gallon; find the price per bottle in shillings and pence, reckoning 6 bottles to the gallon, and 25 fr. 20 c. to the £1.

CHAPTER XII.

PRACTICE.

279. PRACTICE is the method of finding by means of Aliquot parts the value, weight, ... of any quantity, when the value, weight, ... of one unit of it is given. It is therefore another method of solving questions in Compound Multiplication. (247.)

280. To take aliquot-parts is to break up the value, weight, &c. of one unit into parts, which are either aliquot-parts (195) of a principal unit of it, or of each other. Thus if 12*s.* 9*d.* be the price of one unit, it can be broken up into the aliquot-parts 10*s.*, 2*s.* 6*d.*, and 3*d.*: for 10*s.* is $\frac{1}{2}$ of £1, 2*s.* 6*d.* is $\frac{1}{4}$ of 10*s.*, and 3*d.* is $\frac{1}{10}$ of 2*s.* 6*d.*

The facility with which questions in Practice can be performed depends in a great measure on the way in which the aliquot-parts are taken, for usually they can be taken in several ways. No rule can be given; the Student must rely on his own ingenuity and judgment.

281. The theory of Practice depends on the following principles—

(1) *If the price of an article be broken up into any number of parts, the value of any number of articles at the given price is the sum of their values at the various part-prices.* (62.)

Thus £3. 12*s.* 6*d.* is the sum of £3, 10*s.*, and 2*s.* 6*d.*; if now we give £3 for each of a given number of articles, and then 10*s.*, and then 2*s.* 6*d.*, we shall give £3. 12*s.* 6*d.* for each of them.

(2) *If the price of an article be the difference of two other prices, the value of any number of articles at the given price is the difference of their values at the two other prices.*

Thus £3. 12*s.* 6*d.* is the difference between £4 and 7*s.* 6*d.*; if now we give £4 for each of a given number of articles and receive back 7*s.* 6*d.* for each, we shall give £3. 12*s.* 6*d.* for each of them.

(3) *If one price be an aliquot-part of another price, the value of a given number of things at the first price is this aliquot-part of their value at the second price.*

Thus 5*s.* is $\frac{1}{4}$ of £1; the value of any number of things at 5*s.* is $\frac{1}{4}$ of their value at £1.

What has been said of *value* in this Art. applies to *weight*, *length*, *area*, &c.

282. Practice is Simple or Compound, according as the quantity whose value, weight, &c. is to be found is simple or compound.

283. SIMPLE PRACTICE.

i. Ex. 1. Find the value of 345 oz. of gold at £3. 17*s.* 10$\frac{1}{2}$*d.* per oz.

		£.					£.	*s.*	*d.*	
		345								
		3								
		1035			= value of 345 oz. at		3	0	0	per oz.
10*s.*	$\frac{1}{2}$ of £1	172	10	0	=			10	0	...
5*s.*	$\frac{1}{2}$ of 10*s.*	86	5	0	=			5	0	...
2*s.* 6*d.*	$\frac{1}{2}$ of 5*s.*	43	2	6	=			2	6	...
3*d.*	$\frac{1}{10}$ of 2*s.* 6*d.*	4	6	3	=				3	...
1$\frac{1}{2}$*d.*	$\frac{1}{2}$ of 3*d.*	2	3	1$\frac{1}{2}$	=				1$\frac{1}{2}$	...
		1343	6	10$\frac{1}{2}$	=		3	17	10$\frac{1}{2}$	...

The value of 345 oz. at £1 per oz. is £345, and therefore at £3 per oz. it is £345 × 3, or £1035. Since 10*s.* is $\frac{1}{2}$ of £1, the value of 345 oz. at 10*s.* is half that of 345 oz. at £1 (281, 3), and is therefore $\frac{1}{2}$ of £345, or is £172. 10*s.* In like manner, since 5*s.* is $\frac{1}{2}$ of 10*s.*, the value at 5*s.* is $\frac{1}{2}$ of £172. 10*s.*, or is £86. 5*s.*: and since 2*s.* 6*d.* is $\frac{1}{2}$ of 5*s.*, the value at 2*s.* 6*d.* is $\frac{1}{2}$ of £86. 5*s.*, or is £43. 2. 6; and since 3*d.* is $\frac{1}{10}$ of 2*s.* 6*d.*, the value at 3*d.* is $\frac{1}{10}$ of £43. 2. 6, or is £4. 6. 3; and lastly, since 1$\frac{1}{2}$*d.* is $\frac{1}{2}$ of 3*d.*, the value at 1$\frac{1}{2}$ is $\frac{1}{2}$ of £4. 6. 3, or is £2. 3. 1$\frac{1}{2}$.

The sum of all these results is £1343. 6. 10$\frac{1}{2}$; which is therefore the value of 345 oz. at £3. 17. 10$\frac{1}{2}$ per oz. (281, 1).

Or we may proceed thus :—

ii. £3. 17*s.* 10½*d.* is the *difference* between £4 and 2*s.* 1½*d.*: the difference therefore between the values of 345 oz. at £4 per oz., and at 2*s.* 1½*d.* per oz., will give the value at £3. 17*s.* 10½*d.* per oz. (282, 2).

		£.	£.	
		345	345	
2*s.*	$\frac{1}{10}$ of £1	34 . 10 . 0	4	
1½*d.*	$\frac{1}{16}$ of 2*s.*	~~8 . 12 . 6~~	1380	= value at £4 per oz.
		2 . 3 . 1½	36 . 13 . 1½ =	. . 2*s.* 1½*d.* ...
		36 . 13 . 1½	1343 . 6 . 10½ =	. . £3. 17. 10½...

To divide by 16 we have divided in succession by 4 and 4, striking out, of course, the first quotient.

iii. Precisely the same result may be obtained by introducing a *subsidiary* aliquot-part, from which we can find a required aliquot-part: thus, taking the preceding example, we have

		£.
		345
2*s.*	$\frac{1}{10}$ of £1	34 . 10
~~6*d.*~~	¼ of 2*s.*	~~8 . 12 . 6~~
1½*d.*	¼ of 6*d.*	2 . 3 . 1½
		36 . 13 . 1½

iv. Ex. 2. What is the cost of 456⅝ cwts. at 16*s.* 8¾*d.* per cwt.?

Since £⅝ is 12*s.* 6*d.*, the cost of 456⅝ cwts. at £1 is £456. 12. 6, we can therefore proceed as before, thus :—

		£. s. d.
		456 . 12 . 6.
10*s.*	½ of £1	228 . 6 . 3
4*s.*	⅕ of £1	91 . 6 . 6
2*s.* 6*d.*	¼ of 10*s.*	57 . 1 . 6¾
2½*d.*	$\frac{1}{12}$ of 2*s.* 6*d.*	4 . 15 . 1½ ¼
¼*d.*	$\frac{1}{10}$ of 2½*d.*	9 . 6¼ ⅝
		381 . 18 . 11¼ ⅞

v. Ex. 3. What must be given for 2864$\frac{3}{7}$ sacks at 9*s.* 10¾*d.* the sack?

Since £$\frac{3}{7}$ would introduce a fraction of a farthing, it will be better to find separately the cost of 2864 sacks and of $\frac{3}{7}$ of a sack, and then add.

		£. s. d.
		2864
5s.	$\frac{1}{4}$ of £1	716
4s.	$\frac{1}{5}$ of £1	572 . 16
10d.	$\frac{1}{6}$ of 5s.	119 . 6 . 8
~~6d.~~	$\frac{1}{10}$ of 5s.	~~71 . 12 . 0~~
$\frac{3}{4}$d.	$\frac{1}{8}$ of 6d.	8 . 19 . 0
		1417 . 1 . 8
		4 . $2\frac{6}{7}\frac{4}{7}$
		£1417 . 5 . $10\frac{6}{7}\frac{4}{7}$

s. d.
9 . $10\frac{6}{7}$
3
7) 29 . $8\frac{4}{7}$
4 . $2\frac{6}{7}\frac{4}{7}$

It would be still better to find the value of 2864 sacks at 10s. and at $1\frac{1}{4}$d., and then subtract (382, 2): thus,

		s.
		2864
1d.	$\frac{1}{12}$ of 1s.	238 . 8
$\frac{1}{4}$d.	$\frac{1}{4}$ of 1d.	59 . 8
		20) 298 . 4
		14 . 18 . 4

	£.
	2864
10s. is $\frac{1}{2}$ of £1	1432
	14 . 18 . 4
	£1417 . 1 . 8

vi. Ex. 4. What is the cost of 327 articles at 5s. $3\frac{1}{4}\frac{3}{8}$d. each?

		£.
		327
5s.	$\frac{1}{4}$ of £1	81 . 15 . 0
3d.	$\frac{1}{20}$ of 5s.	4 . 1 . 9
$1\frac{1}{3}$f.	$\frac{1}{9}$ of 3d.	0 . 8 . $2\frac{3}{4}\frac{1}{3}$
$\frac{3}{8}$f.	$\frac{1}{4}$ of $1\frac{1}{3}$d.	0 . 2 . $8\frac{1}{3}\frac{1}{2}$
		86 . 7 . $7\frac{3}{4}\frac{1}{8}$

		£.
		327
5s.	$\frac{1}{4}$ of £1	81 . 15
3d.	$\frac{1}{20}$ of 5s.	4 . 1 . 9
$\frac{1}{4}$d.	$\frac{1}{12}$ of 3d.	6 . $9\frac{3}{4}$
$\frac{3}{8}$f.	$\frac{1}{8}$ of 1f.	~~1 . $0\frac{1}{4}\frac{1}{8}$~~
	× 3	4 . $1\frac{1}{4}\frac{3}{8}$
		£86 . 7 . $7\frac{3}{4}\frac{1}{8}$

Ex. 5. Find the area of 145 allotments, each containing 2 ac. 3 r. 18 p.; or Multiply 2 ac. 3 r. 18 p. by 145.

		ac. r.
		145
		2
		290
2 r.	$\frac{1}{2}$ of 1 ac.	72 . 2
1 r.	$\frac{1}{2}$ of 2 r.	36 . 1
10 p.	$\frac{1}{4}$ of 1 r.	9 . 0 . 10
4 p.	$\frac{1}{10}$ of 1 r.	3 . 2 . 20
4 p.	....	3 . 2 . 20
		ac. 415 . 0 . 10

The manner in which the aliquot-part has been taken for the last 8 p. deserves attention.

Ex. 6. A tradesman fails and pays 11*s*. 9¾*d*. in the £1: how much will a creditor receive on an account of £545. 17*s*. 6*d*.?

If the tradesman paid in full, or £1 in the £1, the creditor would receive £545. 17*s*. 6*d*., hence if he paid 10*s*. in the £1, the creditor would receive ½ of £545. 17*s*. 6*d*., &c.

		£.	*s*.	*d*.
		545	17	6
10*s*.	$\frac{1}{2}$	272	18	9
1*s*. 3*d*.	$\frac{1}{8}$	34	2	$4\frac{1}{4}\frac{1}{2}$
3¾*d*.	$\frac{1}{4}$	8	10	$7\frac{0}{4}\frac{1}{8}$
3*d*.	$\frac{4}{5}$	6	16	$5\frac{2}{4}\frac{1}{2}$
		322	8	$1\frac{3}{4}\frac{1}{8}$

		£.
		545·875
10*s*.	$\frac{1}{2}$	272·9375
1*s*.	$\frac{1}{10}$	27·2937
6*d*.	$\frac{1}{2}$	13·6468
3*d*.	$\frac{1}{2}$	6·8234
¾*d*.	$\frac{1}{4}$	1·7058
		322·4072 = £322. 8. 1¾.

Ex. 7. What does a tax of 7*d*. in the £1 amount to on an income of £1285. 15. 10?

		£.	*s*.	*d*.
		1285	15	10
1*s*.	$\frac{1}{20}$	~~64~~	~~5~~	~~9½~~
6*d*.	$\frac{1}{2}$	32	2	10¾
1*d*.	$\frac{1}{6}$	5	7	$1\frac{3}{4}\frac{1}{6}$
		£37	10	$0\frac{1}{2}\frac{1}{6}$

(283, 3)

284. COMPOUND PRACTICE.

Ex. 1. Find the value of a bar of gold weighing 5 lbs. 10 oz. 12 dwts. 6¾ grs. at £3. 17*s*. 11*d*. per oz.

5 lbs. 10 oz. = 70 oz.

		£.	*s*.	*d*.
		3	17	11
				10
		38	19	2
				7
		272	14	2
10 dwts.	½ of 1 oz.	1	18	11½
2 dwts.	⅕ of 10 dwts.	0	7	9½
6 grs.	⅛ of 2 dwts.	0	0	$11\frac{2}{4}\frac{3}{4}$
¾ gr.	⅛ of 6 grs.	0	0	$1\frac{1}{4}\frac{27}{32}$
		£275	2	$0\frac{0}{4}\frac{19}{32}$

Ex. 2. What is the cost of 17 tons 12 cwts. 3 qrs. 18 lbs. of goods at £6. 15*s*. 9*d*. per cwt.?

17 tons 12 cwts. = 352 cwts.: and here we shall first find the cost of 352 cwts. by Simple Practice, and not by Compound Multiplication as in the last Example.

		£.			£. s. d.
		352			6 . 15 . 9
		6			352
		2112			2389 . 4 . 0
15s.	$\frac{1}{8}$ of £6	264	2 qrs.	$\frac{1}{2}$ of 1 cwt.	3 . 7 . $10\frac{1}{2}$
9d.	$\frac{1}{20}$ of 15s.	13 . 4	1 qr.	$\frac{1}{2}$ of 2 qrs.	1 . 13 . $11\frac{1}{4}$
		2389 . 4	14 lbs.	$\frac{1}{2}$ of 1 qr.	0 . 16 . $11\frac{1}{8}$...7
			4 lbs.	$\frac{1}{7}$ of 1 qr.	0 . 4 . $10\frac{3}{4}$...10
					£2395 . 7 . $7\frac{1}{2}\frac{3}{14}$

In the two last Examples it would have been easier to have expressed the price as a decimal of £1, and then to have proceeded as before; thus

		£.			£.
					6·7875
		3·89583			352
		70			13·5750
					339·375
		272·7083			2036·25
10 dwts.	$\frac{1}{2}$ of 1 oz.	1·9479	2 qrs.	$\frac{1}{2}$ of 1 cwt.	3·3937
2 dwts.	$\frac{1}{5}$ of 10 dwts.	·3895	1 qr.	$\frac{1}{2}$ of 2 qrs.	1·6968
6 grs.	$\frac{1}{8}$ of 2 dwts.	·0487	14 lbs.	$\frac{1}{2}$ of 1 qr.	·8484
$\frac{3}{4}$ gr.	$\frac{1}{8}$ of 6 grs.	·0061	4 lbs.	$\frac{1}{7}$ of 1 qr.	·2424
		275·1005			2395·3813
		= £275. 2. 0			= £2395. 7. $7\frac{1}{2}$.

Ex. 3. If 1 lb. standard is 1 lb. 2 oz. 11 dwts. 16 grs. Troy, what is the weight Troy, of 1 cwt. 2 qrs. 25 lbs. 10 oz. 6 drs.?

1 cwt. 2 qrs. 25 lbs. = 193 lbs. st.

		lb. Tr. oz. dwts. grs.		
		1 . 2 . 11 . 16	weight of	1 lb. st.
		12		
		14 . 7 . 0 . 0	. . .	12 lbs.
		8		
		116 . 8 . 0 . 0	. . .	96 lbs.
		116 . 8 . 0 . 0	. . .	96 lbs.
		1 . 2 . 11 . 16	. . .	1 lb.
8 oz.	$\frac{1}{2}$ of 1 lb. st.	7 . 5 . 20	. . .	8 oz.
2 oz.	$\frac{1}{4}$ of 8 oz.	1 . 16 . 11	. . .	2 oz.
4 drs.	$\frac{1}{8}$ of 2 oz.	4 . $13\frac{3}{8}$	. . .	4 drs.
2 drs.	$\frac{1}{2}$ of 4 drs.	2 . $6\frac{11}{16}$	. . .	2 drs.
		235 . 4 . 0 . $19\frac{1}{16}$	. . .	193 lbs. 10 oz. 6 drs.

EXERCISE 46.

Find the value of

1. 5087 articles at 3*s.* 4*d.*, at 3*s.* 9*d.* and at 3*s.* 10½*d.* each.
2. 3895 things at 7*s.* 4*d.*, at 9*s.* 2*d.* and at 5*s.* 7½*d.* each.
3. 25983 lbs. at 8*d.*, at 8¼*d.* and at 9¾*d.* a lb.
4. 365 days at 3¾*d.*, at 5*s.* 3*d.* and at £1. 4. 4½ a day.
5. 3119 things at £4. 7. 7 and at £5. 18. 5 each.
6. 1493 things at 12*s.* 11*d.*, at 16*s.* 4*d.* and at £10. 9. 9 each.
7. 2750 things at 4*s.* 5¼*d.* each; 653 things at 5*s.* 9⅜*d* each.
8. 862 things at 11*s.* 5½*d.* each; 276 things at 14*s.* 10¾*d.* each.
9. 567384 lbs. of cotton at 7⅜*d.* and at 8$\frac{11}{16}$*d.* a lb.
10. 2157 things at £2. 7. 4½ each; 14765 things at £1. 17. 8¼ each.
11. 313 things at £4. 6. 9⅜ each; 4321 things at £4. 17. 3⅝ each.
12. 3655 things at £2. 19. 5¾ and at £9. 16. 10½ each.
13. 4678 things at 12*s.* 0¾*d.* and at £2. 12. 2¾ each.
14. 65437 things at 9*s.* 7⅜*d.* and at £9. 18. 10¾ each.
15. 497 things at £2. 16. 8, at £3. 17. 6 and at £5. 8. 4 each (283, 2).
16. 969 things at 10¼*d.*, at 19*s.* 11¼*d.* and at £4. 19. 8¼ each.
17. 3546 things at £1. 6. 7⅜, at £3. 18. 10⅝ and at £5. 15. 7⅜ each.
18. 4567¼ things at £3. 15. 9½ each; 356⅓ tons at £1. 13. 5¼ a ton.
19. 7394⅝ things at £12. 8. 8¼ each; 3764⅜ things at £2. 14. 7¼ each.
20. 2611 yards at £2. 17. 7⅓ for a dozen yards.
21. 34897 things at 15*s.* 7⅜*d.* for 20; 85493 things at £4. 12. 6¾ for 100.
22. 178⅜ bushels at 6*s.* 7⅜*d.* each; 821⅜ things at 13*s.* 10⅝*d.* each.
23. 194$\frac{7}{11}$ things at £2. 12. 5⅜ each; 716⅔ acres at £44. 11. 3¼ an acre.
24. 169·875 qrs. at £2. 17. 10¼ a qr.; 8764·63 things at £10. 7. 7⅓ each.
25. 163 lbs. at 1*s.* 3⅜*d.* a lb.; 295 yards at 11*s.* 5$\frac{4}{7}$*d.* a yard.
26. 3 cwts. 2 qrs. 10 lbs. of tea at 2*s.* 4⅜*d.* a lb.
27. 2794 dwts. of fine gold at £4. 4. 11$\frac{4}{11}$ an oz.
28. 97 qrs. 5 bush. of wheat at 6*s.* 10⅝*d.* per bushel.
29. 17 bars of gold, each weighing 18 lbs. 8⅜ oz., at £3. 17. 10½ an oz.
30. Find the dividend on £5734. 16. 8 at 9*s.* 4½*d.* in the £1.
31. . . . . . £25962. 10*s.* at 7*s.* 11½*d.*

32. What will a rate of 2*s.* $5\frac{3}{8}$*d.* in the pound produce in a parish whose rental is estimated at £360817. 15*s.*?

33. Find the weight of 2697 packages, each weighing 19 lbs. 10 oz. 18 dwts. 22 grs.; and of 227 feet of iron when the weight of one foot is 9 lbs. $13\frac{11}{16}$ oz.

34. Find the produce of 157 acres at 3 qrs. 5 bush. $1\frac{7}{11}$ pk. per acre.

35. Find the lead in 453 cwts. of ore at 2 qrs. $15\frac{4}{11}$ lb. per cwt.

What is the cost of

36. 15 tons 4 cwts. 3 qrs. 21 lbs. at £11. 17. 6 a ton?

37. 1316 cwts. 2 qrs. 12 lbs. at £2. 15. $8\frac{1}{4}$ a cwt.?

38. 6 tons 7 cwts. 2 qrs. 17 lbs. at £3. 10. 7 a cwt.?

39. 2 tons 7 cwts. 22 lbs. 5 oz. at £16 a ton?

40. 23 lbs. 17 dwts. of silver at 8*s.* 9*d.* an oz.?

41. 9 yds. 2 ft. 10 in. at 5*s.* $7\frac{3}{4}$*d.* a yd.? 34 yds. $5\frac{1}{4}$ in. at 5*s.* 6*d.* a ft.?

42. 191 acres 3 r. 37 p. at £42. 3. 4 an acre?

43. 6231 cwts. 2 qrs. 11 lbs. 15 oz. at £3. 14. 8 a cwt.?

44. 57 cwts. 3 qrs. 24 lbs. at £4. 17. $3\frac{1}{2}$ per cwt.?

45. 35 perches $2\frac{5}{8}$ yards at £65340 per acre?

46. 12 qrs. 3 bush. 3 pk. at £2. 2. 8 a quarter?

47. 11 miles 3 fur. 55 yards at £32500 a mile?

48. 29 lbs. 9 oz. 17 dwts. 6 grs. of gold plate at £5. 15*s.* per oz.?

49. Find the rent of 215 acres 1 r. 19 p. at 13*s.* $2\frac{1}{4}$*d.* a rood.

50. Find the value of 23 sq. yds. 5 ft. 70 in. at 13*s.* 6*d.* a sq. yard.

51. Find the value of 7 gall. 1 qt. $1\frac{1}{2}$ pt. at 17*s.* 4*d.* a gallon.

52. If a pound of silver cost £3. 6*s.*, what is the price of a cup which weighs 10 lbs. 6 oz. 10 dwts., subject to a duty of 1*s.* 6*d.* per ounce, and also to a charge of 1*s.* 9*d.* per ounce for workmanship?

53. Find the area of an estate which can be subdivided into $347\frac{7}{8}$ fields, each containing 6 acres 3 r. 15 p.

54. Find in cu. feet and inches the volume of 12 tons 13 cwts. 1 qr. $12\frac{7}{8}$ lbs. of cast iron, when the volume of one ton is $8142\frac{7}{8}$ cu. inches.

55. Find the produce of 36 ac. 3 r. $17\frac{1}{4}$ p. at 3 qrs. 7 bu. 1 pk. an acre.

56. What is the weight of 62 gall. 3 qts. $1\frac{3}{4}$ pt. of oil, when the weight of one gallon is 8 lbs. $6\frac{2}{15}$ oz.?

57. What distance will a train travel in 3 hours 39 min. 22 s. at a speed of 49 miles 7 f. 52 yds. per hour?

For additional Examples see Exercise 44.

RULE OF THREE.

285. Two quantities are said to be *directly* proportional, or simply proportional, when any two values of the first quantity have to one another the same ratio as the corresponding values of the second.

Two quantities are *inversely* proportional when any two values of the first quantity have to one another the inverse ratio of the corresponding values of the second.

286. *When two quantities are connected in such a way, that when one is increased two, three, ... times, the other is also increased, two, three, ... times, they are in direct proportion.*

For example, if 1 lb. of sugar cost $5d.$, 2 lbs. will cost $2 \times 5d.$, 3 lbs. will cost $3 \times 5d.$, &c.

hence 7 lbs. will cost $7 \times 5d.$,

and 16 lbs. .... $16 \times 5d.$,

but 7 lbs. : 16 lbs. $= 7 : 16$ (186)

and $7 \times 5d. : 16 \times 5d. = 7 : 16$; (188)

$\therefore$ 7 lbs. : 16 lbs. $= 7 \times 5d. : 16 \times 5d.$,

that is, the *cost* of sugar is *directly* proportional to its *weight*.

In the same way (other things being equal)—

The rent of land is directly proportional to its area:

The cost of carriage is directly proportional to the weight carried:

The price of bread is directly proportional to the value of flour:

The quantity of oats consumed is directly proportional to the number of horses who consume:

The income arising from money at interest is directly proportional to the money, &c.

287. *When two quantities are connected in such a way, that when one is increased two, three, ... times the other is diminished two, three, ... times, they are inversely proportional.*

Thus if 1 man can mow a field in 24 days, two men can mow it

in half the time, or in $\frac{24}{2}$ days: three men can mow it in one-third of the time, or in $\frac{24}{3}$ days, &c.

$$\text{hence 4 men can mow it in } \frac{24}{4} \text{ days;}$$

$$\text{and 12 } \ldots\ldots\ldots\ldots \frac{24}{12} \text{ days;}$$

$$\text{but 4 men : 12 men} = 4 : 12,$$

$$\text{and } \frac{24}{4} \text{ days} : \frac{24}{12} \text{ days} = \frac{24}{4} : \frac{24}{12} = \frac{1}{4} : \frac{1}{12} \qquad (188)$$

$$= 12 : 4; \qquad (188)$$

$$\therefore 4 \text{ men : 12 men} = \frac{24}{12} \text{ days} : \frac{24}{4} \text{ days,}$$

that is, the number of men required to do a certain work is *inversely* proportional to the number of days, or *vice versa.*

In the same way (other things being equal):—

The length of carpet required to cover a given floor is inversely proportional to its breadth:—

The number of horses required to eat a given quantity of oats is inversely proportional to the number of days:—

The number of yards of cloth that can be bought for a given sum of money is inversely proportional to the price per yard:—

The weight of goods to be carried for a given sum is inversely proportional to the distance:—

The time required to travel a given distance is inversely proportional to the rate of travelling:—&c.

288. The preceding propositions will enable us, in simple cases, to determine whether a proportion is direct or inverse, but no more:—nor is it the province of Arithmetic to determine whether or in what way one quantity is proportional to another. Thus Geometry teaches us that the circumference of a circle is proportional to the diameter, but in Arithmetic the proportion is to be taken for granted.

Since then it is *assumed* in Arithmetic that every proportion is either *direct* or *inverse*, unless the contrary be distinctly stated, we need not apply the preceding tests in full but simply say:—If when

one quantity is increased, the other is also increased, the proportion is *direct;* and if when one is increased the other is decreased, the proportion is *inverse.*

289. RULE OF THREE is a process in which *three* things are given to find a fourth; two of the given things being of one kind, and the third and the answer of another kind, and the one kind of quantity being either directly or inversely proportional to the other kind.

For example: If 23 yards of cloth cost £3. 16. 8, how much must be given for 37 yards?

Here the *three* given quantities are 23 yards, £3. 16. 8, and 37 yards; and the *fourth* quantity, or one to be found, is the *cost* of 37 yards; of these quantities 23 yards and 37 yards are of one kind, and the £3. 16. 8 and *cost required* are of another kind. Also if the number of yards be increased 2, 3,... times, the cost will be increased 2, 3... times; that is the *length* and the *cost* are in *direct* proportion. (287.)

But the *corresponding* values to 23 yards and 37 yards are £3. 16. 8 and *cost required* respectively; therefore the ratio of 23 yards to 37 yards is the same as the ratio of £3. 16. 8 to cost required (285), and we have this proportion or (as it is usually called) statement—

$$23 \text{ yards} : 37 \text{ yards} = £3.\ 16.\ 8 : \text{cost required},$$

where it is arranged that the cost required is the *fourth* term, and therefore the given cost is the third; and since the proportion is direct, the first and second terms of the first ratio correspond to the first and second terms of the second ratio.

Replace the *quantities* in the first ratio by the *numbers* which measure them, and we have

$$23 : 37 = £3.\ 16.\ 8 : \text{cost required},$$

$$\text{therefore cost required} = \frac{£3.\ 16.\ 8 \times 37}{23}. \qquad (192)$$

Hence to find the *fourth* term, or cost required, we multiply the third term by the *number* in the second term, and divide by the *number* in the first term.

Again—If 1 cwt. 3 qrs. 16 lbs. be worth £23. 3. 9, what is the value of 5 cwts. 1 qr. 20 lbs.?

Here if the weight be *increased* the value will be *increased*, and therefore the proportion is *direct* (288); hence, as before, we have this statement :—

1 cwt. 3 qrs. 16 lbs. : 5 cwts. 1 qr. 20 lbs. = £23. 3. 9 : value required.

$$\begin{array}{l l}
\underline{4} & \underline{4} \\
7 & 21 \\
\underline{28} & 28 \\
212 & 608
\end{array}$$

But in order to replace the quantities in the first ratio by their corresponding numbers, we must bring them to a common denomination; bring them to lbs., and the statement becomes

212 lbs. : 608 lbs. = £23. 3. 9 : value required,

therefore 212 : 608 = £23. 3. 9 : value required; (185)

and therefore value reqd $= \dfrac{£23.\ 3.\ 9. \times 608}{212}$. (192)

Lastly—If 12 men can do a piece of work in 15 days, in how many days will 20 men do the same work?

Here if the number of men be *increased* 2, 3, ... times, the number of days will be *diminished* 2, 3, ... times, therefore the men and the days are in *inverse* proportion.

But 12 men correspond to 15 days

and 20 men .... *days required;*

therefore 15 days is to *days required* in the inverse ratio of 12 men to 20 men, that is, in the ratio of 20 men to 12 men: hence we have the statement

20 men : 12 men = 15 days : days required,

where, as before, the quantity required is the fourth term; and since the proportion is inverse, the *second* and *first* terms of the first ratio correspond to the *first* and *second* terms of the second ratio.

290. From these considerations we have deduced the following Rule:—

Take out the two given quantities of the same kind and corre-

sponding to them the other given quantity and the quantity required. *If the proportion be direct, make the ratio of the first two equal to the ratio of the second two: if inverse, make the inverse ratio of the first two equal to the ratio of the second two. If necessary, express the first and second terms in the same denomination. Multiply the third term by the* number *in the second term, and divide the product by the* number *in the first term; the result will be the "quantity required."*

Remark. The first ratio is not altered in value, if we multiply or divide both its terms by the same number (188); nor is the answer altered if we multiply or divide the first and third terms by the same number.

Ex. 1. If 29 tons of coal cost £25. 17. 2, what will 356 tons cost?

If the weight be increased 2, 3, ... times, the cost will be increased 2, 3, ... times, therefore the cost is *directly* proportional to the weight.

Now 29 tons correspond to £25. 17. 2,

and 356 tons . . . . . cost required;

therefore we have

29 tons : 356 tons = £25. 17. 2 : cost required.

```
                 20
                 ---
                 517
                  12
                 ----
                6206
                 356
               -----
               37236
              31030
             18618        12)
        29 ) 2209336 ( 76184d.
               179     20 ) 6348. 8
                53        £317. 8. 8.
                243
                 116
                   0
```

∴ Cost required = £317. 8. 8.

When we have reduced the third term to pence, we may consider the statement to stand thus:—

29 : 356 = 6206*d.* : cost required (in pence),

we therefore multiply the third term by 356, and then divide by 29, giving 76184*d.* which is equal to £317. 8. 8.

Ex. 2. If a bar of gold weighing 8 oz. 10 dwts. 15 grs. be worth £33. 3. 9, what will a bar weighing 3 lbs. 8 oz. 15 dwts. 3 grs. be worth?

Since as in Ex. 1 the value is *directly* proportional to the weight, we have

oz. dwts. grs.		oz. dwts. grs		£. s. d.	
8 . 10 . 15	:	44 . 15 . 3	=	33 . 3 . 9	: value required.
20		20		20	
170		895		663	
24		24		12	
695		3583		~~7965~~	
340		1790		1593	
~~4095~~		~~21483~~		341	
~~819~~		~~2387~~		1593	
~~91~~		341		6372	
13				4779	
				13) 543213	
				12) $41785\frac{1}{2}\frac{8}{13}$	
				20) 348.2 . 1	
				£174 . 2 . $1\frac{1}{2}\frac{8}{13}$	value reqd.

Here we reduce the 1st and 2nd terms to a common denomination (grs.), and for convenience we reduce the 3rd term to pence, so that we have

4095 : 21483 = 7965*d*. : value required (in pence).

Divide now the 1st and 3rd terms by 5; then divide the 1st and 2nd terms by 9, and then again by 7; and we shall have

13 : 341 = 1593*d*. : value required (in pence).

Proceeding now in the usual way, we find the value required to be $41785\frac{1}{2}\frac{8}{13}$*d*. or £174. 2. $1\frac{1}{2}\frac{8}{13}$.

Ex. 3. If 17 cwts. 3 qrs. 16 lbs. of barley cost £8. 18. 9, what weight may be bought for £2. 16. 3?

The weight is *directly* proportional to the money, therefore, stating as in Ex. 1, we have

				cwts. qrs. lbs.	
£8. 18. 9	:	£2. 16. 3	=	17 . 3 . 16	: weight required.
20		20		5	
178		56		89 . 1 . 24	
12		12		9	
~~2145~~		~~675~~	143	11) 805 . 0 . 20	
~~429~~		~~135~~		13) 73 . 0 . $22\frac{2}{11}$	
143		45		cwts. 5 . 2 . $14\frac{90}{143}$	weight reqd.

Here we reduce the 1st and 2nd terms to the common denomination, pence; and then dividing each of them, first by 5 and then by 3, we get

$$143 : 45 = 17 \text{ cwts. } 3 \text{ qrs. } 16 \text{ lbs. } : \text{weight required,}$$

and without reducing the 3rd term, we multiply by 45 or 5×9, and then divide by 143 or 11×13, giving the weight required.

Ex. 4. If $4\frac{7}{8}$ oz. of gold be worth £19. 12. 6, what is the value of 3 lbs. $11\frac{2}{3}$ oz.?

Here the value is *directly* proportional to the weight;

$$\therefore 4\tfrac{7}{8} \text{ oz. } : 47\tfrac{2}{3} \text{ oz.} = £19\tfrac{5}{8} : \text{value required;}$$

$$\therefore \text{Value required} = £\frac{157}{8} \times \frac{\overset{11}{\cancel{143}}}{3} \times \frac{8}{\underset{3}{\cancel{39}}} = £\frac{1727}{9}$$

$$= £191 \,.\, 17 \,.\, 9\tfrac{1}{3}.$$

Ex. 5. If for a given sum of money 4 tons 3 cwts. 48 lbs. can be carried $8\frac{5}{9}$ miles, how far can 5 tons 1 qr. 14 lbs. be carried for the same sum?

$$4 \text{ tons } 3 \text{ cwts. } 48 \text{ lbs.} = 83\tfrac{48}{112} \text{ cwts.} = 83\tfrac{3}{7} \text{ cwts.}$$

$$5 \text{ tons } 1 \text{ qr. } 14 \text{ lbs.} = 100\tfrac{42}{112} \text{ cwts.} = 100\tfrac{3}{8} \text{ cwts.}$$

Now if the weight be increased 2, 3,...fold, the distance must be decreased 2, 3,...fold: therefore the weight is *inversely* proportional to the distance.

But $83\frac{3}{7}$ cwts. correspond to $8\frac{5}{9}$ miles

and $100\frac{3}{8}$ cwts. . . . . . distance required;

therefore the *inverse* ratio of the two weights is equal to the ratio of their corresponding distances, thus

$$100\tfrac{3}{8} \text{ cwts. } : 83\tfrac{3}{7} \text{ cwts.} = 8\tfrac{5}{9} \text{ miles } : \text{distance required;}$$

$$\therefore \text{distance required} = \frac{\overset{11}{\cancel{77}}}{9} \times \frac{\overset{8}{\cancel{584}}}{\cancel{7}} \times \frac{8}{\underset{\cancel{73}}{\cancel{803}}} = \frac{64}{9} \text{ miles}$$

$$= 7\tfrac{1}{9} \text{ miles.}$$

Ex. 6. If 7 men can mow a field in $18\frac{3}{4}$ hours: in how many hours can 15 men mow the same field?

If the number of men be *increased*, the number of hours in which they can mow the field will be proportionally *decreased*, hence the *inverse* ratio of 7 men to 15 men equals the ratio of $18\frac{3}{4}$ hours to hours required; that is,

15 men : 7 men = $18\frac{3}{4}$ hours : hours required;

$$\therefore \text{ hours required} = \frac{75}{4} \times \frac{7}{15} = \frac{35}{4}$$
$$= 8\tfrac{3}{4} \text{ hours.}$$

Ex. 7. If a capital of £3250 realize a profit of £146. 5*s*., what profit will a capital of £100 realize at the same rate?

Though the three given quantities are all money, yet in stating the question we must distinguish between them: for £3250 and £100 represent *capital* laid out, while £146. 5*s*. and "profit required" represent *profit* arising therefrom: and as the profit is *directly* proportional to the capital laid out, we have

£3250 : £100 = £146. 5 : profit required.

$$\therefore \text{ Profit required} = £146\tfrac{1}{4} \times \frac{100}{3250}$$
$$= £4\tfrac{1}{2}.$$

291. When the quantities in the second and third terms are of the same kind, there is no objection to our alternating these terms (193*): thus in the preceding Example we have

£3250 : £100 = £$146\frac{1}{4}$: profit required,

therefore £3250 : £$146\frac{1}{4}$ = £100 : profit required,

which expresses that the ratio of the first capital to its profit is equal to the ratio of the second capital to its profit.

Such a mode of statement as this latter one is found very convenient in those classes of questions where 100, or some other fixed number, is taken as a standard.

Ex. 8. A public company whose capital was £50000 is wound up, and its assets are found to be £12187. 10*s*.: how much will be paid in the pound?

The dividend on £50000 is £12187·5, at the same rate what is the dividend on £1? hence we have (291)

£50000 : £12187·5 = £1 : dividend on £1;

$$\therefore \text{ Dividend on } £1 = £\frac{12187{\cdot}5}{50000} = £{\cdot}24375$$
$$= 4s.\ 11\tfrac{1}{4}d.$$

Ex. 9. A bankrupt's debts amount to £4357. 7. 8 and his assets to £929. 16. $5\frac{1}{2}$: how much will be received on a debt of £325. 14. $7\frac{1}{2}$?

Here the whole debt of £4357. 7. 8 is discharged by a payment of £929. 16. $5\frac{1}{2}$: and the question is by what payment will £325. 14. $7\frac{1}{2}$ be discharged at the same rate. Hence

```
£4357·383  :  £929·823 = 325·731  :  Payment required.
                           3 28929
                         293157900
                           6514620
                           1931579
                            260585
                              6515
                               977
          4357·383 ) 302872·176 ( 69·5078
                       4142919
                        221274
                          3405
                           355
                             7
```

∴ Payment required = £69. 10. $1\frac{7}{8}$.

Ex. 10. A person after paying an income-tax of 7*d.* in the £1 has a net income of £1247. 10. 5, what is his gross income?

For every 19*s.* 5*d.* of net income he has a gross income of £1, hence

```
19s. 5d. : £1247. 10. 5 = £1 : gross income required
    12                20
   ---
   233             24950
                      12
          233 ) 299405 ( 1285
                 664
                 1980
                  1165
                     0    ∴ Gross income = £1285.
```

Ex. 11. Two clocks are exactly together at 12 o'clock noon, on a certain day: one of them gains 7 sec. and the other loses 6 sec. in 12 hours. After what interval will one have gained half an hour on the other? and what o'clock will each then shew?

The one clock gains on the other at the rate of 13 sec. in 12 hours, or 26 sec. in 1 day; and the question is, after what interval will it have gained 30 min. or 30×60 sec. on the other. Now

$$26 \text{ sec.} : 30 \times 60 \text{ sec.} = 1 \text{ day} : \text{interval required};$$

$$\therefore \text{ Interval required} = \frac{30 \times 60}{26} \text{ days} = \frac{900}{13} \text{ days},$$

$$= 69 \text{ days } 5 \text{ h. } 32\tfrac{4}{13} \text{ m.}$$

and therefore the true time at the end of the interval is 5 h. $32\frac{4}{13}$ m. P.M.

But in $\frac{900}{13}$ days the first clock has gained $\frac{900}{13} \times 14$ sec. or $16\frac{2}{13}$ min., and the second has lost $\frac{900}{13} \times 12$ sec. or $13\frac{11}{13}$ min.

$\therefore$ the first clock will shew 5 h. $48\frac{6}{13}$ m. P.M.

and the second " " " 5 h. $18\frac{6}{13}$ m. P.M.

EXERCISE 47.

1. If 3 qrs. 7 lbs. of tobacco cost £17. 13. 6, what is the value of 5 cwts. 1 qr. 23 lbs.?

2. If 17 cwts. 2 qrs. 14 lbs. can be obtained for £8. 13. $3\frac{3}{4}$, what weight can be obtained for £21. 10. $2\frac{1}{2}$?

3. If 19 men can finish a work in 437 days, how long will it take 23 men?

4. The clothing of a regiment of 735 men costs £1398. 15. 6, what will the clothing of a regiment of 903 men cost at the same rate?

5. A person in 87 days spends £38. 19. $4\frac{1}{2}$, in how many days will he spend £163. 9. $9\frac{1}{2}$ at the same rate?

6. The interest on £271. 18. 4 for 77 days is £3. 3. $8\frac{1}{2}$: find the interest on the same sum for 245 days.

7. The interest on £232. 11. $7\frac{1}{2}$ for 10 months is £10. 2. $8\frac{1}{2}$: what sum will yield the same interest in 17 months?

8. How many ducats of 4*s.* $11\frac{3}{4}$*d.* each are equal in value to 55926 rix-dollars of 4*s.* $10\frac{1}{2}$*d.* each?

9. A draper having sold 147 yards of cloth at the rate of £1. 9. $3\frac{3}{4}$ for $1\frac{3}{4}$ yard, found that he had gained £16. 10. 9. What did the cloth cost him?

10. What do the taxes on a house rented at £317. 12. 6 come to, when the taxes on a house rented at 35 guineas are £6. 8. $7\frac{1}{2}$?

11. If 35 ells of velvet cost £13. 19. $8\frac{1}{4}$, what is the price of 63 yards?

12. When 19 yds. 2 qrs. 3 nls. of cloth cost £3. 12. $2\frac{1}{4}$, how many ells can be obtained for £10. 0. $0\frac{3}{4}$?

13. When the carriage of 3 cwts. 2 qrs. 14 lbs. for 37 miles is 18*s.* 5¼*d.*, what weight can be carried the same distance for £25. 16. 3?

14. If 1 ton 16 cwts. 3 qrs. 20 lbs. cost £4. 15. 8, how much will 3 tons 11¼ cwts. cost at the same rate?

15. A garrison of 638 men has provisions for 124 days; how long will the provisions last if the garrison be reinforced by 418 men?

16. A gang of reapers can reap 84 ac. 3 r. 14 p. in 13⅜ hours: in how many hours can they reap 401 ac. 8 p.?

17. If a sequin be worth 9*s.* 4½*d.*, and a carlino £5. 12. 3¼, how many sequins are equivalent to 450 carlini?

18. A grocer bought 2 tons 3 cwts. 3 qrs. of sugar for £120, and paid 50*s.* for expenses; how much must he charge per cwt. to have a clear profit of £61. 5*s.*?

19. If the 4*d.* loaf weighs 2 lbs. 3 oz. when wheat is at 7*s.* 1½*d.* a bushel, what should it weigh when wheat is at 7*s.* 11*d.* a bushel?

20. After payment of an income-tax of 7*d.* in the £1, a person has left £249. 19. 9½; find his full income.

21. If 2⅝ lbs. of tea cost 8*s.* 8¾*d.*, how much will $27\frac{5}{11}$ lbs. cost?

22. If 4⅜ oz. cost 1⅜*s.*, what weight can be had for £7⅔?

23. When ⅞ cwt. is worth £$3\frac{3}{16}$, what is the value of $\frac{6}{11}$ of a ton?

24. If 1¾ yard of silk cost 10*s.* 11¾*d.*, what will be the cost of 75⅝ yards? and how many yards can be got for £5?

25. If a person can walk 1 mile 2 f. 8½ p. in 20 minutes, how long will he be in walking 149 miles 2 f. 15 p.?

26. How many yards of cloth at 3*s.* 7½*d.* a yard must be given in exchange for 937½ yards of velvet at 18*s.* 1¾*d.* a yard?

27. A piece of cloth measuring 9 ells 1 qr. 3 nls. 1¼ in. cost £1. 12. 9⅝, what is the value of 8¾ yards?

28. If 5 cwts. 3 qrs. 25¾ lbs. be carried 234 miles for 17*s.* 6⅝*d.*, how far can 12 cwts. 1 qr. 10⅞ lbs. be carried for the same money?

29. A person paid £18. 15*s.* for a year's income-tax: but after the government increased the tax to 9*d.* in the pound he paid £52. 10*s.*: what was his income, and at what rate in the pound was the tax levied at first?

30. A watch which is 10 minutes too fast at 12 o'clock noon on Monday gains 3 m. 10 s. a day: what will be the time by the watch at a quarter past 10 on the morning of the following Saturday?

31. The price of ·0125 lb. of coffee is ·4583*s*., what is the value of ·075 of a ton?

32. What is the quarter's rent of 182·3 acres of land, at £4·65 per annum per acre?

33. If an ounce of gold be worth £4·139583, what is the value of ·3576 lb.?

34. What is the weight of a service of plate costing £1355·835, when one article weighing 7·4583 oz. costs £3·23194?

35. If a pipe of Port containing 115 gallons costs £97. 15*s*., what is the cost per dozen when 1 gallon fills 6·4 bottles?

36. If when the price of wheat is 55·5*s*. a quarter, the 6*d*. loaf weighs 3·4375 lbs., what is the price of wheat when the loaf weighs 2·8125 lbs.?

37. An English mile is ·2136 of a German mile. What time will a man who walks 4 English miles an hour take to walk a German mile?

38. A garrison of 1000 men was victualled for 28 days: after 11 days it was reinforced and the provisions were exhausted in 5 days; find the number of men in the reinforcement.

39. A person has a clear annual income of £398. 15. 2. During the first 17 weeks of the year 1868 he has been spending at the rate of £19. 2. 8 a fortnight: what reduction must he make in his daily expenditure so as to save £20 on the whole year?

40. A clock which was 1·4 min. fast at a quarter to 11 P.M. on Dec. 2 was 8 min. slow at 9 A.M. on Dec. 7; when was it exactly right?

41. If the rents of a parish amount to £15685. 15*s*. and a rate is granted of £2137. 8. 9, find to the nearest farthing at how much in the £1 the rate must be laid to realise this sum. How much must be paid by an estate valued at £453. 12. 8?

42. If £52. 18. 9 be given for 40 yards 2 ft. 10 in. of brickwork, how many yards, &c. can be built for £101. 15*s*.?

43. The weight of a 32-pounder gun being 3 tons 4 cwts., and that of an 18-pounder 2 tons 2 cwts., how many 18-pounders will be equal in weight to 189 32-pounders?

44. When the price of copper is 3*s*. 11½*d*. a lb. and the price of tin 1*s*. 7½*d*. a lb., how much copper must be given in exchange for 5 cwts. 3 qrs. 16 lbs. of tin?

45. A cup weighing 11 oz. 18 dwts. 8 grs. is worth £5. 18. 9: what is the value of a goblet weighing 3 lbs. 8 oz. 19½ dwts. at the same rate?

46. A floor can be covered with 31½ yards of drugget 7 quarters wide: how many yards of Brussels carpet 26 in. wide will cover the same room?

47. If 4 men working 15 hours, 3 men working 12 hours, and 8 men working 3 hours earn £3. 5*s*., what will a man's wages for 6 days come to if he works 11 hours a day?

48. A regiment of a thousand men are to have new coats: each coat is to contain 2¼ yards of cloth of 1½ yard wide, and to be lined with shalloon ⅞ yard wide: how many yards of shalloon will be required?

49. A clock which was 1·2 minute fast at a quarter to 11 P.M. on Nov. 28, was exactly right at 11·30 P.M. the following day. How many minutes was it slow at a quarter to 2 P.M. on Dec. 7?

50. Supposing the number of sheep in the country to be 13000000, what would be the value of the wool in a year, estimated at £8. 15*s*. per cwt., if 15 sheep yield 29 lbs. of wool?

51. If 7 women earn as much as 4 men, and 48 men assisted by 14 women earn £42. 7*s*., what number of women assisting 20 men will earn £17. 4. 6 in the same time?

52. If 7 oxen or 11 horses can eat the grass of a field in 37 days, in how many days will 5 oxen and 8 horses eat it?

53. If 3 lbs. 7⅝ oz. of cotton can be spun into a thread 264 miles 1010 yards long, what weight of this thread would be sufficient to reach round the globe, a distance of 25000 miles?

54. A room, 31 ft. 6 in. long and 23 ft. 10 in. broad, is to be covered with carpet ⅞ yard wide: what length of carpet will be required?

55. When 54 yards 2½ feet of carpet are required to cover a floor 23 ft. 6 in. long and 15 ft. 9 in. broad, what must be the width of the carpet?

56. If 12 oxen and 35 sheep eat 6 tons 7 cwts. of hay in 4 days, how much will it cost per week to feed 4 oxen and 6 sheep; the price of hay being £3. 15*s*. per ton, and 2 oxen eating as much as 5 sheep?

57. A tug 90 feet long, steaming at the rate of 8 miles an hour, is towing a ship 480 feet long by a hawser of 60 fathoms: a yacht whose length is 180 feet is sailing in the same direction at the rate of 10 miles an hour. What time will elapse between the yacht first coming up with the ship and finally clearing the tug?

58. A bankrupt's estate was calculated to give a dividend of 12*s*. 8*d*. in the £1; but by the unexpected recovery of a debt of £87. 18. 9 the dividend was raised to 13*s*. 10*d*. in the £1. Find the amount of the bankrupt's liabilities.

59. Two clocks, of which one gains 4 m. 17 s. and the other loses 3 m. 13 s. in 24 hours, were both within 2½ min. of the true time, the former fast and the latter slow, at noon on Monday last: they now differ from one another by half an hour; find the day of the week and the hour of the day.

60. In running a 3 mile race on a course $\frac{1}{4}$ of a mile round, *A* overlaps *B* at the middle of the seventh round. By what distance will *A* win at the same rate of running?

61. When the price of oats is 30*s.* a quarter, it costs 17*s.* 6*d.* a week to keep a horse, but when oats are 26*s.* a quarter it costs only 16*s.* $2\frac{1}{4}$*d.*; what quantity of oats does a horse consume in a year?

62. If 5 horses and 12 mules can draw a load of 25896 lbs. for a given distance, how many mules will be required with 9 horses to convey 337 cwts. 3 qrs. 20 lbs. the same distance; the load drawn by a mule being two-thirds of that drawn by a horse?

63. Two clocks strike 9 together on Tuesday morning. On Wednesday morning one wants 10 min. to 11 when the other strikes 11. How many minutes must the slower be put on, or the faster put back, that they may strike 9 together in the evening?

64. A merchant fails for £7852. 16*s.* and pays a first dividend of 11*s.* 8*d.* in the £1, and afterwards a second dividend of 8*s.* 9*d.* on what was then due. Find what his estate realised, and how much he paid altogether in the £1.

65. If 15 men 12 women and 9 boys can complete a piece of work in 50 days, how long would 9 men 15 women and 18 boys be in doing double the quantity, the part done by each in the same time being as the numbers 3, 2, and 1?

66. If 4 men or 6 women or 9 boys can perform a piece of work in $27\frac{1}{2}$ days, in what time can (1) 5 men and 9 women perform it? and (2) 5 men and 8 boys perform it?

67. A grain of gold can be beaten into a leaf of 56 sq. inches: how many of these leaves will make an inch in height, the weight of a cu. foot of gold being 10 cwts. 3 qrs. 11 lbs.?

68. How far must a carriage be driven that the fore wheel may make 560 more revolutions than the hind wheel; the diameters of the fore and hind wheels being $1\frac{1}{2}$ ft. and $2\frac{1}{2}$ ft. respectively, and the circumference of every circle being $3\frac{1}{7}$ times the diameter?

69. A watch was $6\frac{1}{11}$ min. slow at noon; it loses 12 min. in $20\frac{1}{4}$ hours: find the true time when its hands are together for the fourth time after noon.

70. In what time can a column of men clear a defile 3 miles in length, supposing the column to consist of 10 battalions each extending over 176 yards, and that the rate of marching over the last mile is reduced on account of the difficulty of the road from 75 paces of $2\frac{1}{2}$ feet each to 40 paces of $2\frac{2}{3}$ feet each per minute?

DOUBLE RULE OF THREE.

292. DOUBLE RULE OF THREE or COMPOUND RULE OF THREE is a process in which five quantities are given to find a sixth, and four of the five given quantities form two pairs of different kinds, and the fifth and the answer form a third pair of another kind, also the quantities of the first and second kind are directly or inversely proportional to the quantity of the third kind.

Take the example,—If 6 horses plough 21 acres in 5 days, in how many days will 16 horses plough 98 acres?
and arrange the quantities corresponding to the two hypotheses in two lines, putting those of the same kind under one another, thus

6 horses	21 acres	5 days
16 horses	98 acres	days required,

where it is seen that four of the given quantities form two pairs of different kinds—horses and acres; and the fifth and the answer form a third pair of another kind—days. Also the number of days is found to be inversely proportional to the number of horses (286), and directly proportional to the number of acres.

293. This definition of Double Rule of Three may be extended to cases where seven quantities are given to find an eighth; nine to find a tenth; &c.

294. *Every question in Double Rule of Three may be solved by Reduction to the Unit.*

Take the preceding Example, and remembering that the number of days is inversely proportional to the number of horses and directly proportional to the number of acres, we proceed thus:—

Since 6 horses plough 21 acres in 5 days,
1 horse ploughs 21 acres in 5×6 days,

and 1 horse ... 1 acre in $\frac{5 \times 6}{21}$ days,

$\therefore$ 1 horse ... 98 acres in $\frac{5 \times 6 \times 98}{21}$ days,

and 16 horses plough 98 acres in $\frac{5 \times 6 \times 98}{16 \times 21}$ days;

i.e. days required $= \frac{5 \times 6 \times 98}{16 \times 21} = 8\frac{3}{4}$.

295. We will now explain the ordinary method of solution which is by *two* or more Rule of Three statements, and by compounding them into one final statement: hence the name *Double* Rule of Three and *Compound* Rule of Three.

Taking the preceding Example, we first ask,—if 6 horses plough 21 acres in 5 days, in how many days can 16 horses plough them? and here, since the quantity of land to be ploughed is the same on the two hypotheses, the number of days will depend on the number of horses *only*, and since the number of days is *inversely* proportional to the number of horses, we have this statement,—

$$16 \text{ horses} : 6 \text{ horses} = 5 \text{ days} : \frac{5 \times 6}{16} \text{ days};$$

$$\text{or } 16 : 6 = 5 : \frac{15}{8}; \qquad \text{(A)}$$

that is, 16 horses can plough 21 acres in $\frac{15}{8}$ days.

Again, we ask,—if 16 horses can plough 21 acres in $\frac{15}{8}$ days, in how many days can they plough 98 acres?

and here, the number of horses that plough being the same on the two hypotheses, the number of days will depend on the number of acres *only*, and since the number of days is *directly* proportional to the number of acres, we have this statement:

$$21 \text{ acres} : 98 \text{ acres} = \frac{15}{8} \text{ days} : \text{days required};$$

$$\text{or } 21 : 98 = \frac{15}{8} : \text{number of days required.} \qquad \text{(B)}$$

Compounding the statements (A) and (B) we have (194)

$$16 \times 21 : 6 \times 98 = 5 \times \frac{15}{8} : \frac{15}{8} \times \text{number of days required};$$

$$= 5 : \text{number of days required.} \qquad (188)$$

But this last statement is very conveniently written thus:

$$\left.\begin{matrix} 16 : 6 \\ 21 : 98 \end{matrix}\right\} = 5 \text{ days} : \text{days required},$$

where the sign of multiplication between the factors of the terms of the first ratio is understood. Now we must remember that the

ratio 16 : 6 is got from the fact that the number of horses is inversely proportional to the number of days, when the number of *acres* is not considered; and that the ratio 21 : 98 is got from the fact that the number of acres is directly proportional to the number of days, when the number of *horses* is not considered.

Of course, the answer is got by multiplying the third term by the second 6×98, and dividing the product by the first term 16×21.

296. In the preceding Example the answer depends on *two* quantities only—*horses* and *acres*; but in many cases the answer depends on three or more quantities, and then we shall have as many separate statements, all of which, just as in the above Example, are compounded into one final statement.

297. From these considerations we deduce the following Rule:—

For convenience, arrange the quantities corresponding to the two hypotheses in two lines, placing those of the same kind under one another, and express them when necessary in the same denomination. Make the given quantity and "quantity required" of the same pair, the first and second terms of the second ratio. Take the first pair of given quantities of the same kind, and with them complete the statement (290): *do the same with the second pair, and with each succeeding pair, writing these ratios under one another. Multiply the third term by the product of the* numbers *in the second terms, and divide by the product of the* numbers *in the first.*

Ex. 1. If 3 tons 16 cwts. can be carried 25 miles for £11. 17. 6, what weight can be carried 52 miles for £5. 19. 2?

£11. 17. 6 = 2850*d*. £5. 19. 2 = 1430*d*.

Arranging corresponding quantities in two lines, placing those of a like kind under one another, we have

76 cwts.	25 miles	2850*d*.
Cwts. required.	52	1430

Make 76 cwts. and "cwts. required" the first and second terms of the second ratio.

Now if the miles the goods have to be carried are *increased*, the weight carried must be proportionally *decreased*, therefore the proportion is *inverse*, and must be in *inverse* order to the cwts., hence we must state 52 : 25.

Again, if the money paid for carriage is *increased*, the weight carried must be proportionally *increased*, therefore the proportion is direct, and follows the *same* order as the cwts.; hence we state 2850 : 1430.

Therefore the full statement is

$$\left.\begin{array}{r} 52 : 25 \\ 2850 : 1430 \end{array}\right\} = 76 \text{ cwts. : cwts. required.}$$

$$\therefore \text{cwts. required} = 76 \times \frac{25 \times 1430}{52 \times 2850} \text{ cwts.}$$

$$= 18\tfrac{1}{3} \text{ cwts.}$$

Ex. 2. If 288 men in 5 days of 11 hours each can dig a trench 264 yards long 5 feet wide and 2 feet deep, in how many days of 9 hours each can 112 men dig a trench 420 yards long 8 feet wide and 3 feet deep?

men.	days.	hours.	yards.	ft. wide.	ft. deep.
288	5	11	264	5	2
112	days reqd.	9	420	8	3

Make 5 days : days required, the second ratio.

If the number of men working at the trench be *increased*, the number of days in which it will be dug will be proportionally *decreased*, that is the proportion of men to days is *inverse*.

If the number of hours the men work per day be *increased*, the number of days they will have to work will be *decreased*, therefore the proportion of hours per day to days is inverse.

If the number of yards in the length of the trench be *increased*, the number of days in which it will be dug will be *increased*, or the proportion of length to days is direct.

If the number of feet in the width, or in the depth, be *increased*, the number of days will be *increased*, or the proportion of the width or the depth to the days is direct.

Hence in the first two statements we follow the *inverse* order of the days, and in the last three the same order, and our full statement is—

$$\left.\begin{array}{r} 112 : 288 \\ 9 : 11 \\ 264 : 420 \\ 5 : 8 \\ 2 : 3 \end{array}\right\} = 5 \text{ days : days required;}$$

$$\therefore \text{days required} = 5 \times \frac{288 \times 11 \times 420 \times 8 \times 3}{112 \times 9 \times 264 \times 5 \times 2} \text{ days}$$

$$= 60 \text{ days.}$$

Ex. 3. If 16 cannon firing 4 rounds in 7 minutes kill 270 men in an hour and a half, how many cannon firing 8 rounds in 9 minutes will kill 420 men in 40 minutes?

cannon.	rounds.	min.	men.	hours.
16	4	7	270	$1\frac{1}{2}$
cannon reqd.	8	9	420	$\frac{2}{3}$

If the number of rounds be *increased*, the number of cannon will be *decreased*—proportion inverse.

If the interval between each round be *increased*, the number of cannon will be *increased*—proportion direct.

If the number of men to be killed be *increased*, the number of cannon will be *increased*—proportion direct.

If the time during which the cannon fire be *increased*, the number of cannon will be *decreased*.

Therefore stating as directed in the last example, we have

$$\left.\begin{array}{rcl} 8 & : & 4 \\ 7 & : & 9 \\ 270 & : & 420 \\ \frac{2}{3} & : & \frac{3}{2} \end{array}\right\} = 16 \text{ cannon : cannon required;}$$

$$\therefore \text{cannon required} = 16 \times \frac{4 \times 9 \times 420 \times \frac{3}{2}}{8 \times 7 \times 270 \times \frac{2}{3}}$$
$$= 36.$$

Ex. 4. A garrison of 4500 men is supplied with provisions for 15 weeks at the rate of 13 oz. per day per man: how many men must leave that the same provisions may supply those that remain 27 weeks at 10 oz. per day per man?

Here we must find how many men the provisions will supply on the second hypothesis.

men.	weeks.	oz.
4500	15	13
men required	27	10

If the number of weeks be *increased*, the number of men must be *decreased*—proportion inverse.

If the number of ounces per day be *increased*, the number of men must be *decreased*—proportion inverse:—hence

$$\left.\begin{array}{rcl} 27 & : & 15 \\ 10 & : & 13 \end{array}\right\} = 4500 \text{ men : men required;}$$

$$\therefore \text{men required} = 4500 \times \frac{15 \times 13}{27 \times 10} = 3250;$$

$$\therefore \text{no. of men that must leave} = 4500 - 3250 = 1250$$

Ex. 5. A wall 1690 feet long has to be built in 30 days; and it is found that 7 men in 14 days have completed only 490 feet; how many additional men must be employed that the wall may be finished in the required time?

The question really is—how many men must be employed to complete the remaining 1200 feet in the remaining 16 days?

Now the length of the wall is *directly* proportional to the number of men; and the number of days is *inversely* proportional to the number of men: hence

$$\left.\begin{matrix}490 : 1200\\ 16 : 14\end{matrix}\right\} = 7 \text{ men} : \text{men required};$$

$$\therefore \text{ men required} = 7 \times \frac{1200 \times 14}{490 \times 16} = 15;$$

$$\therefore \text{ no. of additional men} = 15 - 7 = 8.$$

Exercise 48.

1. If 67 tons carried 87 miles cost £24. 5. 9, what will 73 tons carried 93 miles cost?

If 61 tons carried 81 miles cost £10. 11. 9, how far can 77 tons be carried for £31. 2. 5?

If 37 tons carried 57 miles cost £8. 15. 9, what weight can be carried 83 miles for £21. 15. 9?

2. If 939 men consume 364 quarters of wheat in 7 months, how many will consume 1404 quarters in 13½ months?

3. If 15 men take 17 days to mow 300 acres of grass, how long will 27 men take to mow 167 acres?

4. If £1375 put out at simple interest for 3 years produce £144. 7. 6, in what time will the interest on £2420 amount to £190. 11. 6 at the same rate?

5. If a tradesman with a capital of £1000 gains £90 in 7 months, how long will he be in gaining £20. 5*s*. with a capital of £315?

6. When wheat is at 15*s*. a bushel 8 men can be fed for 12 days at a certain cost. For how many days can 6 men be fed for the same cost when wheat is 12*s*. a bushel?

If 15 men are fed for 7 days at a certain cost when wheat is 12*s*. a bushel, what must be the price when 10 men are fed for 8 days at the same cost?

7. If a penny loaf weigh 6 oz. when wheat is 5*s*. 6*d*. a bushel, what should be the weight of the shilling loaf when wheat is 8*s*. 3*d*. a bushel?

If the penny loaf weigh 6 oz. when wheat is 5*s*. 6*d*. a bushel, what should be the price of a loaf weighing 4½ lbs. when wheat is at 8*s*. 3*d*. a bushel?

8. If 36 men can do a piece of work in 10 days working 10 hours a day, in how many days could 40 men complete the same work, working 9 hours a day?

If 27 men can do a piece of work in 18 days working 10 hours a day, how many hours a day must 12 men work to do the same in 45 days?

9. If a quantity of provisions would serve a besieged garrison of 2000 men for 15 weeks at the rate of 18 oz. a day for each man, how many ounces must each man receive that the garrison increased to 2500 may be able to hold out 3 weeks longer?

If a quantity of provisions will serve a besieged garrison of 1500 men for 12 weeks at the rate of 20 oz. a day for each man, how many men would the same provisions maintain for 20 weeks at the rate of 8 oz. a day for each man?

10. If 20 men could perform a piece of work in 12 days, find the number of men who would perform another work three times as great in one-fifth of the time.

11. If 9 men can reap 15 ac. 1 r. 28 p. in 5 days of $10\frac{1}{2}$ hours each, how many men will reap 401 ac. 8 p. in 7 days of $11\frac{1}{4}$ hours each?

12. If the carriage of 6 cwts. 3 qrs. for 124 miles cost £3. 4. 8, what weight would be carried 93 miles for £3. 0. $7\frac{3}{4}$?

If the carriage of 5 cwts. 1 qr. 12 lbs. for 39 miles be £2. 8. 6, what must be paid for the carriage of 7 cwts. 16 lbs. for $48\frac{3}{4}$ miles?

13. How many men would it employ for $5\frac{1}{2}$ days to cultivate a field of $2\frac{5}{8}$ acres, if each man completed 77 sq. yards in 9 hours, and the day consisted of 10 hours?

14. If 6 iron bars 4 ft. long 3 in. broad and 2 in. thick weigh 288 lbs., find the weight of 15 bars each $6\frac{1}{4}$ ft. long 4 in. broad and 3 in. thick.

15. If 5 men can reap a rectangular field whose length is 800 feet and breadth 700 feet in $3\frac{1}{2}$ days of 14 hours each, in how many days of 12 hours each can 7 men reap a field 1800 feet long and 960 feet broad?

If 5 men in 6 days can reap a field 1200 feet long and 800 feet broad working 6 hours a day, what is the breadth of a field whose length is 1280 feet which 6 men can reap in 5 days working 8 hours a day?

16. If 5 horses require as much corn as 8 ponies, and 15 quarters of corn last 12 ponies for 64 days, how long may 25 horses be kept for £41. 5*s*., when corn is 22*s*. a quarter?

If 21 horses and 217 sheep can be kept 10 days for £56. 8. 4, what sum will keep 9 horses and 60 sheep for 27 days, supposing that 3 horses eat as much as 50 sheep?

If 6 oxen or 13 sheep eat 26 cwts. 3 qrs. 20 lbs. of hay in 19 days, how much ought to be paid for the supply of hay to 17 oxen and 40 sheep during the month of March, hay being worth $4\cdot5\dot{3}$ guineas per ton?

17. If 10 cannon which fire 3 rounds in 5 minutes kill 270 men in an hour and a half, how many cannon which fire 5 rounds in 6 minutes will kill 500 men in one hour at the same rate?

18. If 25 labourers can dig a ditch 220 yards long 3 ft. 4 in. wide and 2 ft. 6 in. deep in 32 days when the day is 9 hours long, how many labourers would be able to dig a ditch half a mile long 2 ft. 4 in. deep and 3 ft. 6 in. wide in 36 days when the day is 8 hours long?

19. Two gangs of 6 and 9 men are set to reap two fields of 35 and 45 acres respectively. The first gang works 7 hours in the day, and the latter 8 hours. If the first gang complete their work in 12 days, in how many days will the second complete theirs?

20. If a beam which is 10 in. wide 8 in. deep and 5 ft. 6 in. long weigh 8 cwts. 1 qr., find the length of another beam the end of which is a sq. foot and which weighs a ton.

21. If the wages of 25 men amount to £76. 13. 4 in 16 days, how many men must work 24 days to receive £103. 10*s*., the daily wages of the latter being one-half those of the former?

22. If when copper is £7. 14. $4\frac{1}{3}$ per cwt. I can get 3 cwts. 2 qrs. 14 lbs. of brass for £27. 0. $3\frac{1}{4}$, how much brass shall I get for £153. 17. 6 when copper is at £$9\frac{1}{12}$ per cwt.?

23. If 17 cwts. 1 qr. can be carried 840 English miles for £15. 2. 6, how many Irish miles may 2 tons 6 cwts. be carried for £71. 10*s*.; the Irish mile being $1\cdot\dot{2}\dot{7}$ of an English mile?

24. If 5 labourers or 3 navvies can excavate 28 cu. yards of earth in 10 hours, in how many hours can 6 labourers and 8 navvies excavate 120 cu. yards, the soil in the latter case being one-fourth more difficult to work than in the former?

25. A person is able to perform a journey of 142·2 miles in $4\frac{1}{2}$ days when the day is 10·164 hours long: how many days will he be in travelling 505·6 miles when the days are 8·4 hours long?

26. If the 4*d*. loaf weigh 1 lb. $9\frac{1}{4}$ oz. when wheat is at 9*s*. 3*d*. a bushel, how much bread can be got for 5*s*. $7\frac{1}{2}$*d*. when wheat is at 76*s*. a quarter?

If the 6*d*. loaf weigh 4·35 lbs. when wheat is at 5·75*s*. a bushel, what ought to be paid for 49·3 lbs. of bread when wheat is at 9·2*s*. a bushel?

27. If 48 pioneers in 5 days of $12\frac{1}{4}$ hours long can dig a trench 139·75 yards long 4·5 yards wide and 2·5 yards deep, how many hours per day must 90 pioneers work during 42 days in order to dig a trench 1636·6875 yards long 4·875 yards wide and 3·2 yards deep?

28. An oz. of standard gold is worth £3. 17. $10\frac{1}{2}$ and contains ·916 of pure gold; find the value of (neglecting the alloy)

(1) 10 lbs. of jewellers' gold containing ·583 pure gold:

(2) a Mohur weighing 7 dwts. 23 grs. containing ·993 pure gold:

(3) a Star Pagoda weighing 2 dwts. $4\frac{1}{2}$ grs. containing ·792 pure gold.

29. A contractor engaged to finish a work of 3150 cu. yards in 50 days, and employed at once 60 men upon it, but at the end of 35 days he finds only 1800 cu. yards completed: how many more men must he put on to complete the work in the given time?

30. An engineer engages to complete a tunnel $3\frac{3}{4}$ miles long in 2 years 10 months; for a year and a half he employs 1200 men, and then finds he has completed only three-eighths of his work: how many additional men must he employ to complete it in the required time?

31. If 7 women earn as much as 4 men, and 48 men assisted by 14 women earn 121 guineas in 17 days, what number of women assisting 20 men will earn £21. 3. 6 in one-third of the time?

32. If 16 men in 18 days can perform a piece of work which occupies 36 boys for 24 days, how many men must help 21 boys to perform the same piece of work in 16 days?

33. Two cogged wheels, of which one has 15 cogs and the other 28, work in each other. If the first turn 16 times in $7\frac{1}{2}$ seconds, how often will the other turn in 21 seconds?

34. Two sets of men perform the same amount of work. Each man in the first set is stronger than each one in the second in the ratio of 7 to 6: the first set works 6 days a week for 10 weeks, and the second set 5 days a week for 7 weeks. If there are 9 men in the first set, how many are there in the second?

35. If 6 men can reap 15 acres in 3 days of 14 hours each, and 10 boys can reap $10\frac{5}{8}$ acres in 5 days of 9 hours each, find the ratio of the work of a man to the work of a boy. Find also how many acres 4 men and 7 boys can reap in 7 days, working 11 hours a day.

36. If 5 pumps each having a length of stroke of 3 ft. working 15 hours a day for 5 days empty the water out of a mine, how many pumps with a length of stroke of $2\frac{1}{2}$ ft. working 10 hours a day for 12 days will be required to empty the same mine, the stroke of the former set of pumps being performed 4 times as fast as those of the latter?

37. An engine of 40 horse-power can raise 53 tons 11 cwts. 1 qr. 20 lbs. through 11 ft. in one minute; how long will an engine of 30 horse-power take to fill a reservoir containing 900000 cu. feet, the water being raised through 50 feet, and a cu. foot of water weighing 1000 oz.?

38. In a boat-race the C. crew rowed 39 strokes per minute and the O. crew 41, but 19 strokes of the former were equivalent to 20 of the latter. The C. crew rowed over the course of 4 miles in 25 minutes; find the number of feet and the number of seconds by which the race was won.

39. When wheat was at 75*s.* a quarter the 4 lb. loaf was sold for $7\frac{1}{2}$*d.*, but when wheat rose 5*s.* a quarter the price of the 6 lb. loaf was raised to 1*s.* Suppose the cost of converting wheat into bread be at the rate of 1*s.* 4*d.* per cwt., how much would the bakers lose or gain on every £1 of their receipts by the alteration of prices?

40. A contractor engages to make a railway 40 furlongs in length in 4 months, and after employing 375 men 12 hours a day for 3 months he finds 25 furlongs have been finished. The rest of the work is one-third less laborious in character than that already done, but for the next month the men can work only 10 hours a day; how many extra men must he employ to complete the work in the specified time?

41. The cost of polishing a cubical block of marble whose edge is $2\frac{1}{2}$ ft. is 5 guineas: what ought to be paid for polishing another cubical block whose edge is $3\frac{3}{4}$ ft. when the price of labour has been reduced one-sixth? *The surface of a cube is proportional to the* SQUARE *of its edge.*

42. If 10 cannon which fire 7 rounds in 12 minutes discharge $12\frac{1}{2}$ cwt. of metal in one hour, what weight of metal will 15 cannon of double the bore of the former firing at the rate of 2 rounds in 5 minutes discharge in 1 hour 10 min.? *The weight of solid shot is proportional to the* CUBE *of its diameter.*

43. If 7 men build a wall $5\frac{1}{4}$ ft. high and $1\frac{1}{2}$ ft. thick about a square court containing 1 ac. 1 p. $29\frac{3}{4}$ yds. in 35 days of 12 hours each, and if 53 men of equal efficiency working 14 hours a day for 180 days build a wall $13\frac{1}{4}$ ft. high and $2\frac{1}{2}$ ft. thick enclosing a square park, find the *area* of the park. *The area of a square is proportional to the* SQUARE *of the length of its side.*

CHAIN RULE.

298. In CHAIN RULE we have a series of quantities of different kinds, and we have given the relations between the first and second, the second and third, the third and fourth, ..., to find what quantity of the last kind is equal to a given quantity of the first kind.

The following is an example:—If 3 lbs. of tea be worth 4 lbs. of coffee, and 6 lbs. of coffee be worth 20 lbs. of sugar, and 15 lbs. of sugar be worth 24 lbs. of rice; how many lbs. of rice are equal to 18 lbs. of tea?

We may either solve this question by three Rule of Three statements, or by Reduction to the Unit; take the latter method.

Since 3 lbs. tea $=$ 4 lbs. coffee, $\therefore$ 1 lb. tea $= \frac{4}{3}$ lbs. coffee,

and 6 lbs. coffee $=$ 20 lbs. sugar, $\therefore$ 1 lb. coffee $= \frac{20}{6}$ lbs. sugar,

and 15 lbs. sugar $=$ 24 lbs. rice, $\therefore$ 1 lb. sugar $= \frac{24}{15}$ lbs. rice;

$$\text{and } \therefore 18 \text{ lbs. tea} = 18 \times \frac{4}{3} \text{ lbs. coffee}$$
$$= 18 \times \frac{4}{3} \times \frac{20}{6} \text{ lbs. sugar}$$
$$= 18 \times \frac{4}{3} \times \frac{20}{6} \times \frac{24}{15} \text{ lbs. rice};$$
$$\text{or lbs. reqd rice} = \frac{18 \times 4 \times 20 \times 24}{3 \times 6 \times 15}.$$

If now at the head of the unreduced equations written down above we write an equation expressing the relation between "lbs. required rice" and the given quantity, we have

lbs. reqd rice $=$ 18 lbs. tea,
3 lbs. tea $=$ 4 lbs. coffee,
6 lbs. coffee $=$ 20 lbs. sugar,
15 lbs. sugar $=$ 24 lbs. rice,

and we see at once that "lbs. reqd rice" just found, is obtained by dividing the product of the numbers on the right-hand side of these equations by the product of the numbers on the left-hand side.

299. In the preceding equations the quantity on the right-hand side of one equation is of the *same kind* as that on the left-hand side of the next equation, and thus the Chain of quantities from rice to tea, tea to coffee, coffee to sugar, and sugar again to rice, is unbroken. And not only must they be of the *same kind* but also of the *same denomination*, for if not the one or more missing links must be supplied:—thus in the Example,

If 3 lbs. of currants be worth 5 oz. of tea, and 4 lbs. of tea be worth 9 lbs. of coffee, how many lbs. of coffee are worth 48 lbs. of currants?

we must either supply the missing link 16 oz. tea = 1 lb. tea, or we must express 5 oz. tea as $\frac{5}{16}$ lb. tea; so that we have

$$\begin{aligned} \text{lbs. reqd coffee} &= 48 \text{ lbs. currants}, \\ 3 \text{ lbs. currants} &= 5 \text{ oz. tea}, \\ 16 \text{ oz. tea} &= 1 \text{ lb. tea}, \\ 4 \text{ lbs. tea} &= 9 \text{ lbs. coffee}; \\ \therefore \text{lbs. reqd coffee} &= \frac{48 \times 5 \times 1 \times 9}{3 \times 16 \times 4} \\ &= 11\tfrac{1}{4}. \end{aligned}$$

$$\begin{aligned} \text{lbs. reqd coffee} &= 48 \text{ lbs. currants}, \\ 3 \text{ lbs. currants} &= \tfrac{5}{16} \text{ lb. tea}, \\ 4 \text{ lbs. tea} &= 9 \text{ lbs. coffee}; \\ \therefore \text{lbs. reqd coffee} &= \frac{48 \times \frac{5}{16} \times 9}{3 \times 4} \\ &= 11\tfrac{1}{4}. \end{aligned}$$

300. We deduce then the following Rule:—

Express the relations between the quantities in a series of equations where the first expresses the relation between the quantity required and the given quantity, and where the right-hand side of one equation and the left-hand side of the next equation and of the last equation and first equation, are quantities of the same kind and of the same denomination. The quantity required is found by dividing the product of the numbers on the right-hand side of these equations by the numbers on the left-hand side.

REMARK. It is unnecessary to name the quantity on the *left*-hand side of any equation; for it must be the same as the quantity on the right-hand side of the preceding equation.

Ex. 1. If $\frac{1}{5}$ of a sheep be worth £$\frac{2}{3}$, and $\frac{3}{7}$ of a sheep be worth $\frac{1}{14}$ of an ox, what must be given for 100 oxen?

$$
\begin{aligned}
£\text{s reqd} &= 100 \text{ oxen,}\\
\tfrac{1}{14} &= \tfrac{4}{7} \text{ sheep,}\\
\tfrac{1}{6} &= £\tfrac{2}{3};
\end{aligned}
$$

$$\therefore £\text{s reqd} = \frac{100 \times \frac{4}{7} \times \frac{2}{3}}{\frac{1}{14} \times \frac{1}{6}} = 100 \times \frac{2}{7} \times 70 = 2000.$$

Ex. 2. How much tea at 2*s.* 9*d.* a lb. ought to be given in exchange for 5 cwts. 3 qrs. 14 lbs. of tobacco at £19. 10*s.* per cwt.?

5 cwts. 3 qrs. 14 lbs. = 5 cwts. $3\frac{1}{2}$ qrs. = $5\frac{7}{8}$ cwts.

$$
\begin{aligned}
\text{Now lbs. reqd tea} &= 5\tfrac{7}{8} \text{ cwts. tobacco,}\\
1 &= £19\tfrac{1}{2},\\
1 &= 240d.,\\
33 &= 1 \text{ lb. tea};
\end{aligned}
$$

$$\therefore \text{lbs. reqd tea} = \frac{5\frac{7}{8} \times 19\frac{1}{2} \times 240}{33} = 833\tfrac{2}{11}.$$

Ex. 3. A Turkish day's journey is 10 hours of $3\frac{1}{2}$ English miles each hour, and 11 English miles is equal to 12 Roman miles: how many Roman miles are there in 25 Turkish days' journies?

$$
\begin{aligned}
\text{Roman miles reqd} &= 25 \text{ Turkish days' journies,}\\
1 &= 10 \text{ hours,}\\
1 &= 3\tfrac{1}{2} \text{ English miles,}\\
11 &= 12 \text{ Roman miles;}
\end{aligned}
$$

$$\therefore \text{Roman miles reqd} = 25 \times 10 \times \tfrac{7}{2} \times 12 \times \tfrac{1}{11} = 954\tfrac{6}{11}.$$

Ex. 4. If 1 lb. of standard gold, of which 11 parts out of 12 are fine, be worth £46. 14. 6, find the value of 550 Madras gold rupees each weighing 7 dwts. 12 grs. of which 916 parts out of 1000 are fine.

£46. 14. 6 = £$46\frac{29}{40}$, $\therefore$ 40 lbs. standard = £1869: hence

$$
\begin{aligned}
£\text{s reqd} &= 550 \text{ Madras rupees,}\\
2 &= 15 \text{ dwts. Madras standard,}\\
20 \times 12 &= 1 \text{ lb.} \;\; . \;\; . \;\; . \;\; . \;\; . \;\; . \;\; . \;\; .\\
1000 &= 916 \text{ lbs. fine,}\\
11 &= 12 \text{ lbs. English standard,}\\
40 &= £1869;
\end{aligned}
$$

$$\therefore £\text{s reqd} = \frac{550 \times 15 \times 916 \times 12 \times 1869}{2 \times 20 \times 12 \times 1000 \times 11 \times 40} = £802\tfrac{803}{1600}.$$

$$= £802.\ 10.\ 0\tfrac{3}{4}.$$

EXERCISE 49.

1. If 16 darics made 17 guineas, 19 guineas made 24 pistoles, 31 pistoles made 38 sequins; how many sequins were there in 1581 darics?

2. 121 Irish acres are equal to 196 Imperial acres, and 126 Imperial acres are equal to 100 Scotch acres; how many Scotch acres are equal to 385 Irish acres?

3. If 12 of *A* count for 13 of *B*, 6 of *B* for 18 of *C*, and 13 of *C* for 2 of *D*; how many of *A* count for 100 of *D*?

4. If 12 oxen be worth 29 sheep, 15 sheep worth 25 hogs, 17 hogs worth 3 loads of wheat, and 8 loads of wheat worth 13 loads of barley; how many loads of barley must be given for 340 oxen?

5. Required the relation between the metre and the Rhineland foot, supposing the metre equal to 39·371 inches, and that 37 English feet are equal to 36 Rhineland feet.

6. Suppose 10 lbs. of London equal 11 lbs. of Rome, and 26 marks of Spain equal 16 lbs. of London, what is the ratio between the Roman pound and the Spanish mark?

7. 1 lb. of standard silver, of which 222 parts out of 240 are pure, is coined into 66 shillings: what weight of pure silver is there in 20 shillings?

8. If 40 lbs. of standard gold, of which 11 parts out of 12 are fine, be coined into 1869 sovereigns; how many grains of pure gold are there in 1 sovereign?

9. If 100 francs are equal in value to 27 thalers, and 33 thalers are equal to £5, how many francs and centimes are equal to £1?

10. How many pounds of tea at 2*s.* 4½*d.* a lb. must be given in exchange for 2 tons 3 cwts. of sugar at £2. 7. 6 per cwt.?

11. The stadium contained 600 Greek feet; and the Roman mile contained 1000 paces of 5 Roman feet each: 24 Greek feet were equal to 25 Roman feet, and 12 Roman miles to 11 English miles: find the length of the stadium in yards.

12. A sovereign standard gold weighs 5·136 dwts., a shilling standard silver weighs $\frac{1}{66}$ of a pound Troy: what weight of standard silver is equal in value to 4 oz. of standard gold?

13. If 22 oz. of fine gold make 24 oz. of standard gold, and the Mint gives 1869 sovereigns for 40 lbs. of standard gold, how many sovereigns ought to be given for 67 lbs. 10 oz. of fine gold?

14. The Cologne mark of fine silver weighs 3609 grains: find its value when 37 oz. of fine silver make 40 oz. of English standard silver, and 1 oz. of standard silver is worth 5*s.* 1½*d.*

15. From a Cologne mark of fine silver weighing 3609 grains are coined 14 thalers; find the value of a thaler, when an ounce of English standard silver, of which 37 parts out of 40 are fine, is worth 5*s.* $1\frac{3}{8}$*d.*

16. If $47\frac{1}{2}$ South German florins, which are $\frac{9}{10}$ fine, weigh 500 grammes, and a kilogramme be equal to 15432 grains; what is the value of this florin in English money when standard silver $\frac{37}{40}$ fine is worth 5*s.* $1\frac{3}{8}$*d.* per oz.?

17. If 1 lb. of standard silver, of which 37 parts out of 40 are fine, be worth 66*s.*, find the value of an Arcott Rupee weighing 7 dwts. 9 grs., of which 941 parts out of 1000 are fine.

18. If 1 lb. of standard gold, of which 11 parts out of 12 are fine gold, be worth £46. 14. 6, find the value of 595 gold rupees of Bombay, each weighing 7 dwts. $10\frac{1}{2}$ grains, of which 187 parts are fine gold and 13 alloy.

19. The length of the minute hand of the clock of the Palace at Westminster is 11 feet: what distance will the end of it travel through in a year of $365\frac{1}{4}$ days, if 7 times the circumference of a circle be 22 times its diameter?

20. The distance from Paris to Lyons by railway is 506 kilometres and the first-class fare is 56 fr. 80 c.; find the rate per mile in pence, &c. supposing 5 francs to be equal to 4*s.* and 8 kilometres to be equal to 5 miles.

21. The distance by rail from Turin to Venice is 435 kilometres and the first-class fare is 51 lire 45 centimes; find at the same rate, in English money, the fare from London to Edinburgh a distance of 401 miles; reckoning 5 lire equal to 4*s.* and 8 kilometres to 5 miles.

22. The distance by railway from London to Bristol is 118 miles, and the fare 20*s.* 10*d.*: find at the same rate the fare in florins and cents from Innsbruck to Verona a distance of 31·5 German miles, when 117 English miles equal 25 German and 20*s.* equal 12 fl. 30 cent.

23. When cloth is sold at 15*s.* $9\frac{1}{2}$*d.* a yard, what is the corresponding price in francs and centimes per metre; a metre being equal to 39·371 inches and £1 to 25 fr. 25 c.?

24. When wheat is sold at 22 fr. 50 c. the hectolitre; find the price per bushel in English money, supposing the hectolitre equal to 22 gallons and 25 fr. 10 c. equal to £1.

25. The rent of a farm of 45 hectares 75 centiares is 3695 francs; find in English money the rent of 87 acres 3 r. 25 p. at the same rate; supposing 100 hectares equal to 247 acres, and 25 francs equal to £1.

PROPORTIONAL PARTS. PARTNERSHIP.

301. To divide a given quantity into PROPORTIONAL PARTS is to divide it into parts which shall have the same ratio to each other that certain given numbers have.

For example—Divide £735 among A, B, C, and D, so that their shares may be proportional to the numbers 3, 5, 7, and 9.

Now $3+5+7+9=24$: if therefore we divide the given quantity into 24 equal parts, and give 3 of these equal parts to A, 5 to B, 7 to C, and 9 to D, we shall have given away the whole quantity, and the shares of A, B, C, and D will be respectively

$$\frac{£735}{24}\times 3,\quad \frac{£735}{24}\times 5,\quad \frac{£735}{24}\times 7,\quad \text{and}\quad \frac{£735}{24}\times 9,$$

and will therefore be as $3 : 5 : 7 : 9$ (188).

302. If the given numbers are fractions we may follow the same method; but it will be more convenient to find integral numbers proportional to the given numbers. For example,

Divide £735 among A, B and C, so that their shares may be proportional to $1\frac{1}{2}$, $2\frac{2}{3}$ and $3\frac{3}{4}$.

$$\text{Now } 1\tfrac{1}{2} : 2\tfrac{2}{3} : 3\tfrac{3}{4} = \frac{3}{2} : \frac{8}{3} : \frac{15}{4}$$
$$= 18 : 32 : 45. \qquad (188)$$

But $18+32+45=95$; therefore

$$A\text{'s share} = \frac{£735}{95}\times 18,\quad B\text{'s} = \frac{£735}{95}\times 32,\quad \text{and } C\text{'s} = \frac{£735}{95}\times 45.$$

303. We have then the following Rule:—

Divide the given quantity by the sum of the given numbers expressing the ratios of the parts; multiply the quotient by each of these numbers, and the products will give the parts required.

Ex. 1. Divide 837 into three parts proportional to the numbers 5, 9 and 13.

$$5+9+13=27;$$

$$\therefore \text{ 1st part} = \frac{837}{27}\times 5 = 31\times 5 = 155;$$

$$\text{2nd part} = \frac{837}{27}\times 9 = 31\times 9 = 279;$$

$$\text{3rd part} = \frac{837}{27}\times 13 = 31\times 13 = 403.$$

Ex. 2. Divide £392. 10 among three persons so that their shares may be to each other in the ratio of $\frac{1}{1\frac{1}{2}}, \frac{1}{2\frac{2}{3}}, \frac{1}{3\frac{3}{4}}$.

$$\frac{1}{1\frac{1}{2}} : \frac{1}{2\frac{2}{3}} : \frac{1}{3\frac{3}{4}} = \frac{2}{3} : \frac{3}{8} : \frac{4}{15}$$
$$= 80 : 45 : 32.$$

But
$$80+45+32=157;$$

$$\therefore \text{ 1st person's share} = \frac{£392.\ 10}{157}\times 80 = £\frac{5}{2}\times 80 = £200;$$

$$\text{2nd} \; . \; . \; . \; . = \qquad = £\frac{5}{2}\times 45 = £112.\ 10;$$

$$\text{and 3rd} \; . \; . \; . \; . = \qquad = £\frac{5}{2}\times 32 = £80.$$

Ex. 3. Divide £28. 17. 6 among *A*, *B*, and *C* so that *B*'s share may be half as much again as *A*'s, and *C*'s share one-third as much again as both *A*'s and *B*'s.

Half as much again of a quantity is the quantity and half the quantity, or $\frac{3}{2}$ of the quantity. In like, one-third as much again of a quantity is $\frac{4}{3}$ of that quantity.

$$\therefore B\text{'s share} = \frac{3}{2} \text{ of } A\text{'s share},$$

$$\text{and } \therefore A\text{'s share and } B\text{'s share} = \frac{5}{2} \text{ of } A\text{'s share},$$

$$\text{and } C\text{'s share} = \frac{4}{3} \text{ of } \frac{5}{2} \text{ of } A\text{'s share} = \frac{10}{3} \text{ of } A\text{'s share},$$

$$\therefore A\text{'s share} : B\text{'s} : C\text{'s} = 1 : \frac{3}{2} : \frac{10}{3}$$
$$= 6 : 9 : 20.$$

But $$6+9+20=35;$$

$$\therefore A\text{'s share} = \frac{£28\frac{7}{8}}{35}\times 6 = £\frac{33}{40}\times 6 = 16s.\ 6d.\times 6 = £4.\ 19.\ 0,$$

$$B\text{'s}\ldots\ldots = \qquad\qquad = 16s.\ 6d.\times 9 = £7.\ 8.\ 6,$$

$$\text{and } C\text{'s}\ldots\ldots = \qquad\qquad = 16s.\ 6d.\times 20 = £16.\ 10.\ 0.$$

Ex. 4. Divide 1000 guineas among A, B, C and D so that A's share may be to B's as 2 : 3, B's share to C's as 4 : 5, and C's to D's as 6 : 7.

$$A\text{'s share} : B\text{'s} = 2 : 3 = 16 : 24, \qquad (188)$$

$$B\text{'s}\ldots\ldots : C\text{'s} = 4 : 5 = 24 : 30,$$

$$C\text{'s}\ldots\ldots : D\text{'s} = 6 : 7 = 30 : 35;$$

$$\therefore A\text{'s} : B\text{'s} : C\text{'s} : D\text{'s} = 16 : 24 : 30 : 35.$$

But $$16+24+30+35=105,$$

$$\therefore A\text{'s share} = \frac{£1050}{105}\times 16 = £10\times 16 = £160; \text{ \&c.}$$

Ex. 5. £15 is to be given to 12 men, 10 women and 18 boys, in such a way that a woman is to receive half as much again as a boy, and a man as much as a woman and boy together: what do the men receive?

10 women receive as much as 15 boys,

and 12 men receive as much as 12 women and 12 boys,

as 18 boys . . . . . .

as 30 boys;

$$\therefore \text{men's share} : \text{women's} : \text{boys'} = 30 : 15 : 18 = 10 : 5 : 6.$$

But $$10+5+6=21;$$

$$\therefore \text{men's share} = \frac{£15}{21}\times 10 = £\frac{5}{7}\times 10 = £\frac{50}{7} = £7.\ 2.\ 10\tfrac{2}{7}.$$

Ex. 6. Three districts are to provide according to their population a contingent of 182 men. The population of the first district is 2456, of the second 735, and of the third 4361: find as exactly as possible the number of men to be provided by each district.

$$\begin{array}{r} 2456 \\ 735 \\ 4361 \\ \hline 7552 \end{array}$$

$\therefore$ First district's share $= \dfrac{182}{7552} \times 2456 = 59{\cdot}18,$

Second . . . . $= \dfrac{182}{7552} \times 735 = 17{\cdot}71,$

Third . . . . $= \dfrac{182}{7552} \times 4361 = 105{\cdot}09.$

But $59 + 17 + 105 = 181$ only; therefore we have now to find which district must provide another man. The *absolute* increase in providing another man by each district is

$$\cdot 82; \qquad \cdot 29; \qquad \cdot 91;$$

but the *relative* increase is

$$\frac{\cdot 82}{2456}; \quad \frac{\cdot 29}{735}; \quad \frac{\cdot 91}{4361};$$

$$\text{or} \quad \cdot 033; \quad \cdot 039; \quad \cdot 0208;$$

that is, the *relative* increase is *least* when the additional man is provided by the third district; and therefore the men to be provided by each district respectively are 59, 17 and 106.

PARTNERSHIP.

304. PARTNERSHIP or FELLOWSHIP is a direct application of the Rule of Proportional Parts.

Partnership is either *Simple* or *Compound*. It is Simple, when the capital of each partner has been subscribed for the same time: for then it is understood that the gain or loss arising from the partnership shall be divided among the partners in proportion to the capital subscribed by each of them. It is Compound, when the capital of each partner has *not* been subscribed for the same time: for then it is understood that the gain or loss arising from the partnership shall be divided among the partners not only in proportion to the *capital* subscribed by each, but also to the *time* for which it has been subscribed.

Ex. 7. A, B, C and D form a partnership: A subscribes £350, B £420, C £240 and D £600. At the end of 9 months they dissolve, and share the profits, amounting to £364. 10. 3: what will be the share of each?

The capital employed to realise the profit is

$$£350 + £420 + £240 + £600 = £1610,$$

and of this capital A contributes £350;

$$\therefore A\text{'s share of the profits} = \frac{£364.\ 10.\ 3}{1610} \times 350$$

$$= £79.\ 4.\ 10\tfrac{0}{4}\tfrac{1}{23}.$$

Similarly B's share $$= \frac{£364.\ 10.\ 3}{1610} \times 420 = \&c.$$

Ex. 8. A and B enter into partnership with a capital of £5000, of which A contributes £3300, and B the remainder. At the end of 3 months they admit C with a capital of £1500, the next month they admit D with a capital of £1950. Their profits at the end of the year amount to £1729. 13. 9:—how much will each of the partners receive?

A has had £3300 in the partnership for 12 months, which is to be considered as equal to £3300 × 12 for 1 month. In like manner B's capital is to be considered as equal to £1700 × 12 for 1 month; C's as equal to £1500 × 9 for 1 month; and D's as equal to £1950 × 8 for 1 month.

These equivalent capitals are for the *same* time, and therefore the division of profits will be in proportion to these several capitals only.

The work may be arranged thus:—

£.	£.	
3300 × 12	= 39600	A's equivalent capital.
1700 × 12	= 20400	B's . . . . .
1500 × 9	= 13500	C's . . . . .
1950 × 8	= 15600	D's . . . . .
	89100	

$$\therefore A\text{'s share} = \frac{£1729.\ 13.\ 9}{89100} \times 39600 = £768.\ 15.\ 0;$$

$$B\text{'s share} = \frac{£1729.\ 13.\ 9}{89100} \times 20400 = \&c.$$

Ex. 9. A and B engage in trade, their capitals being in the ratio of 7 : 11. At the end of 3 months A withdraws $\frac{1}{4}$ of his capital, and a month afterwards B adds twice as much as A has withdrawn. How should a profit of £337. 7. 6 be divided at the end of the year?

If we represent A's capital by 7, B's will be represented by 11: the unit being a matter of indifference. At the end of 3 months A withdraws $2\frac{1}{3}$, and therefore has remaining $4\frac{2}{3}$ for 9 months; and at the end of 4 months B adds $2 \times 2\frac{1}{3}$ or $4\frac{2}{3}$, and therefore has invested $15\frac{2}{3}$ for the next 8 months. Proceeding as in the last Example:—

$$\left.\begin{array}{l} 7 \times 3 = 21 \\ 4\frac{2}{3} \times 9 = 42 \end{array}\right\} = 63 \quad A\text{'s equivalent capital,}$$

$$\left.\begin{array}{l} 11 \times 4 = 44 \\ 15\frac{2}{3} \times 8 = 125\frac{1}{3} \end{array}\right\} = \underline{169\frac{1}{3}} \quad B\text{'s} \;.\;.\;.\;.\;.\;.$$

$$232\frac{1}{3}$$

$$\therefore\ A\text{'s share} = \frac{£337.\ 7.\ 6}{232\frac{1}{3}} \times 63 = £\frac{337.375 \times 189}{697}$$

$$= £91.\ 9.\ 7\tfrac{3}{4}.988$$

$$= £91.\ 9.\ 8 \text{ very nearly;}$$

and $\therefore$ B's share $= £245.\ 17.\ 10$ . . .

EXERCISE 50.

1. Divide 28561 into parts proportional to 1, 1, 4, 7; and also into parts proportional to $\frac{1}{12}, \frac{1}{6}, \frac{1}{3}, \frac{1}{2}$.

2. Divide 15123 into parts which have the same ratio to one another as $\frac{5}{6}, \frac{7}{8}, \frac{9}{16}, \frac{5}{32}, \frac{4}{15}$.

3. A, B and C contribute respectively to an undertaking £96, £175 and £284, and they gain £129. 10*s*.: how shall this gain be divided between them?

4. Copper coins are composed of 95 parts copper 4 tin and 1 zinc. What weight of each metal would be required to coin copper coins of the value of £1000? 3 *pennies weigh* 1 *oz. Av.*

5. In England gunpowder is compounded of 75 parts nitre 10 sulphur and 15 charcoal; in France of 77 parts nitre 9 sulphur and 14 charcoal: if a ton of each gunpowder be mixed, what weight of each ingredient will there be in the mixture?

6. Sugar is composed of 49.856 parts oxygen, 43.265 carbon and 6.879 hydrogen: how many lbs. of each of these materials is there in 11 cwts. 2 qrs. 12 lbs. of sugar?

7. A certain kind of brass is compounded of 325 parts copper 165 zinc 8 lead and 2 tin; what weight of each metal is there in 43 kilog. 850 gr. of this brass?

8. Divide £593. 8. 6 into three parts which shall be to one another as $2\cdot\dot{3} : 3\cdot\dot{4}\dot{5} : 5\cdot\dot{6}2\dot{9}$.

9. A person in his will directed that $\frac{1}{3}$ of his property should be given to *A*, $\frac{1}{3}$ to *B*, $\frac{1}{4}$ to *C*, and $\frac{1}{6}$ to *D*: shew that this disposition cannot be carried out. If his property amount to £471. 12. 6, dispose of it so that their shares may have to one another the relation he intended.

10. Divide a sovereign between *A*, *B*, and *C*, so that *B* may have one-third as much again as *A*, and *C* one-fourth as much again as *B*:—and again, so that *B* may have one-fourth as much again as *A*, and *C* may have two-thirds of what *A* and *B* have together.

11. The sum of £328. 3. $6\frac{1}{4}$ is to be divided between *A*, *B*, *C* and *D* in such a way that for every £3 given to *A*, *B* is to receive £5, *C* £8, and *D* £9. What sum did each receive?

12. *A*, *B* and *C* had each a cask of whiskey containing respectively 36, 54 and 78 gallons. They blended their whiskeys and then refilled their casks from the mixture: how much of the whiskeys of *A* and *B* are contained in *C*'s cask?

13. Four merchants *A*, *B*, *C* and *D* trading with a capital of £23800, find after a certain time their respective shares increased by £26. 11. 8, £37. 4. 4, £53. 3. 4 and £63. 16*s*. How much did they respectively subscribe to the original capital?

14. *A* and *B* hold a pasture in common for which they pay £13. 10*s*., *A* puts in 23 horses for 27 days, and *B* 21 horses for 39 days; how much ought each to pay of the rent?

15. *B* and *J* become partners, *B* bringing £500 and *J* £300. At the end of 4 months *B* doubles his capital and a new partner *R* is introduced who brings £350; and at the end of 6 months *J* trebles his capital. The year's profits amount to £750: how ought it to be divided between them?

16. *A* starts business with a capital of £240 on the morning of 19th March, and on the 16th July admits a partner *B* with a capital of £180. The profits at the end of the year amount to £94. 8*s*.: what is each person's share of them?

17. *A* and *B* enter into partnership with capitals as 4 : 5. At the end of 3 months they withdraw respectively $\frac{1}{2}$ and $\frac{1}{3}$ of their capitals. When the year closes they find their profit to be £436. 9. 6: how must it be divided between them?

18. A debt of £34. 15. 10 is paid in crowns, shillings, and pence, whose numbers are as 4 : 7 : 10. Find the number of each coin.

19. A and B rent a field for 21 guineas. A puts in 10 horses for $1\frac{1}{2}$ months, 30 oxen for 2 months and 100 sheep for $3\frac{1}{4}$ months; B 40 horses for $2\frac{1}{2}$ months, 50 oxen for $1\frac{1}{4}$ months and 115 sheep for 3 months. If the food consumed in the same time by a horse, an ox and a sheep be in the ratio 3 : 2 : 1, what portion of the rent must each pay?

20. The sum of £765 is to be divided between 10 men 32 women and 48 children: if each man's share is to be equal to the share of 2 women, and the 32 women are to have twice as much as the 48 children, how much will each woman receive?

21. The sum of £177 is to be divided among 15 men 20 women and 30 children in such a manner that a man and a child together may receive as much as 2 women, and all the women together may receive £60. What will a man and a child receive?

22. Divide 23 cwt. 1 qr. 9 lbs. into two parts which shall be to each other as $11\frac{1}{4}$ cu. feet is to $3\frac{4}{7}$ cu. yards.

23. The sum of three fractions is $\frac{183}{242}$; and 22 times the first, 23 times the second and 24 times the third give equal products. Find the fractions.

24. Divide 32 gall. 3 qts. $1\frac{1}{2}$ pt. into four measures, so that the first shall be to the second as 9 to 14, the second to the third as 21 to 25, and the third to the fourth as 20 to 33.

25. 1 lb. of tea, of coffee, and of sugar together cost 5*s.* $8\frac{1}{2}$*d.*: find the price of each having given that 7 lbs. of tea cost as much as 16 lbs. of coffee, and 3 lbs. of coffee as much as 11 lbs. of sugar.

26. The total amount paid in wages to 6 men 8 youths and 11 women is £23. 9*s.* How much does each person receive, when for every half-crown earned by a man a woman earns 1*s.* 9*d.*, and for every shilling earned by a youth a woman earns $10\frac{1}{2}$*d.*?

27. Find the three highest integral numbers whose sum is under a million, so that the first may be to the second as 5 yds. 2 ft. 6 in. is to 5 yds. 3 qrs. $2\frac{1}{2}$ nls. and the second may be equal to $\frac{3 \text{ cwt. } 3 \text{ qrs. } 12 \text{ lbs.}}{4 \text{ cwt. } 2 \text{ qrs. } 16 \text{ lbs.}}$ of the third.

28. A rents a house for a year for £93. 12*s.*, and at the end of 4 months takes in B as a co-tenant, and they admit C in like manner for the last $2\frac{1}{2}$ months: what portion of the rent must each of them pay?

29. Five cantons are to furnish a contingent of 200 men according to their population. The population of the first canton is 28300, of the second 25750, of the third 16432, of the fourth 8454, and of the fifth 23648. Find with the greatest exactness the number to be furnished by each canton.

ALLIGATION.

305. ALLIGATION, or the mixing of things of the same kind, but of different qualities, may be considered under the following cases:—

Case I. When the quantity, and price, of each of the things composing the mixture are given, to find the price of the mixture.

Case II. When the price of each of the things which are to compose the mixture is given, to find what quantity must be taken of each, in order that the mixture may be of a given price.

306. Case I. This case is equivalent to finding an *average* or *mean* price (255, ii., vii.). *We multiply the number of each quantity expressed in the same denomination by its price, and divide the sum of these products by the sum of the numbers.*

Ex. A wine merchant blends 60 gallons of sherry at 24*s.* a gallon, 50 gallons at 26*s.* a gallon, and 70 gallons at 32*s.* a gallon: find the price of a gallon of the mixture.

				s.
60 gallons	at	24*s.*	a gallon	= 1440
50	„	26*s.*	„	= 1300
70	„	32*s.*	„	= 2240

∴ 180 gallons of the mixture = 4980

and ∴ 1 gallon of the mixture $= \frac{4980}{180} s. = \frac{83}{3} s.$
$= 27s.\ 8d.$

307. Case II. We shall first suppose that only two kinds enter into the mixture, and from the result we shall shew how a solution can easily be obtained when three or more kinds enter.

Ex. 1. How must a grocer mix tea at 2*s.* 4*d.* a lb. and 2*s.* 11*d.* a lb. to make a mixture worth 2*s.* 8*d.* a lb.?

In making the mixture at 2*s.* 8*d.* a lb.

1 lb. at 2*s.* 4*d.* brings a gain of 4*d.*,
and 1 lb. at 2*s.* 11*d.* „ loss of 3*d.*

In order therefore that the gain in using the former may be equal to the loss in using the latter, for every 3 lbs. of the former we must take 4 lbs. of

the latter, for then the gain would be $3 \times 4d.$ and the loss $4 \times 3d.$ We must therefore take the quantities in the ratio of 3 and 4; that is, in the inverse ratio of the differences of the two prices and the mean price.

Ex. 2. How must a grocer mix teas at 2*s.* 3*d.*, 2*s.* 7*d.*, and 2*s.* 10*d.* a lb., to make a mixture worth 2*s.* 6*d.* a lb.?

30	27	1 + 4 lbs. at 2*s.* 3*d.*
	31	3 ,, 2*s.* 7*d.*
	34	3 ,, 2*s.* 10*d.*

For convenience, arrange the prices under one another in order, and write the mean price on the left. Now one of the given prices is under the mean price, and the other two above it: we can therefore by the last Example mix teas at the first price and the second price, and at the first price and the third price, so as to get mixtures at the mean price: and the sum of these mixtures will give a mixture of all three at the mean price.

But in mixing at the first and second price we must take 1 lb. of the first and 3 lbs. of the second, and in mixing at the first and third price we must take 4 lbs. of the first and 3 lbs. of the third, hence mixing at all three prices we must take

5 lbs. of the first, 3 lbs. of the second, and 3 lbs. of the third;

or any equimultiples of these quantities.

Ex. 3. How must a grocer mix teas at 2*s.* 6*d.*, 2*s.* 9*d.*, 3*s.* 1*d.* and 3*s.* 4*d.* a lb., to form a mixture worth 2*s.* 10*d.* a lb.?

34	30	3 lbs. at 2*s.* 6*d.*	34	30	6 lbs. at 2*s.* 6*d.*
	33	6 ,, 2*s.* 9*d.*		33	3 ,, 2*s.* 9*d.*
	37	4 ,, 3*s.* 1*d.*		37	1 ,, 3*s.* 1*d.*
	40	1 ,, 3*s.* 4*d.*		40	4 ,, 3*s.* 4*d.*

Arranging as before, we see that there are two prices under the mean price and two above it:—hence we can mix tea at either of the former prices with tea at either of the latter prices so as to get a mixture at the mean price, and this is usually expressed by saying *link* a price under the mean, with one above it. Now linking the first and third, we must take 3 lbs. of the first and 4 lbs. of the third: and linking the second and fourth we must take 6 lbs. of the second and 1 lb. of the fourth:—or, linking the first and fourth, and the second and third, we must take 6 lbs. of the first, 4 lbs. of the fourth, 3 lbs. of the second and 1 lb. of the third: and these sets of quantities, and their equimultiples, will give the mixture required.

308. We deduce then the following Rule:—

Write the given prices under one another in order, and to the left write the mean price. Link all the prices, so that one under *and one* above *the mean price shall always go together: and put against each price the difference between the price with which it is linked and the mean price;—these differences, or any equimultiples of them, will give the quantities required.*

REMARK. It is from this process of *linking* that the name Alligation is derived.

309. Sometimes it is required to make the mixture, so that there shall be a given quantity of one kind:—Thus in Ex. 3 suppose everything else the same, but that we must take 20 lbs. of the tea at 2*s.* 6*d.* Now the first solution shews that a mixture can be made at the required price, by taking the following quantities in order, or any equimultiples of them—

3 lbs., 6 lbs., 4 lbs., 1 lb.

multiplying then each of them by $\frac{20}{3}$, we get

20 lbs., 40 lbs., $26\frac{2}{3}$ lbs. $6\frac{2}{3}$ lbs.

Sometimes, again, the quantity to be contained in the whole mixture is given: thus in Ex. 3, suppose everything else the same, but that the whole mixture must weigh 20 lbs. Now the first solution gives a whole mixture of 14 lbs.: if therefore we multiply each of the quantities composing it by $\frac{20}{14}$ or $\frac{10}{7}$, their sum will be 20 lbs.

EXERCISE 51.

1. A grocer mixes 47 lbs. of tea at 2*s.* $1\frac{3}{4}$*d.* a lb., 25 lbs. at 2*s.* 4*d.* a lb., and 20 lbs. at 2*s.* $10\frac{1}{2}$*d.* a lb., what is the price of 1 lb. of the mixture? If he had also added 8 lbs. of sloe-leaves at $3\frac{1}{4}$*d.* a lb., what then would be the price?

2. A goldsmith melts together 27 oz. of standard gold (22.), 19 oz. of gold 15 carats fine, 20 oz. $18\frac{3}{4}$ carats fine, and $3\frac{3}{4}$ oz. of copper: what is the fineness of the mixture? *Pure gold is 24 carats fine; hence standard gold is 22 carats fine.*

3. A [illegible]er buys wheat at 39*s.* per quarter and some of a superior quality at 6*s.* per bushel: in what proportion must he mix the two so as to sell the mixture at 46*s.* per quarter?

4. A person wishes to melt equal quantities of gold 943 and 827 milliemes fine, with alloy, so as to get a gold 467 milliemes fine; what quantities of each must he take? 943 *milliemes fine means that* 943 *parts out of* 1000 *are fine.*

5. A person buys some tea at 6*s.* per lb. and some at 4*s.* per lb. In what proportion must he mix them so that by selling his tea at 5*s.* 3*d.* per lb., one-sixth of his receipts may be clear profit?

6. Mix spirit at 8*s.*, wine at 7*s.* and cider at 1*s.* a gallon with water, so that the mixture may be worth 5*s.* a gallon.

7. It is required to mix teas at 2*s.* 10½*d.*, 2*s.* 6*d.*, and 2*s.* 3*d.* a lb., with sloe-leaves at 3*d.* a lb., so that the mixture being sold at 2*s.* 10*d.* a lb., one-fourth of the receipts may be clear profit.

8. How much gold at £4. 5*s.* per oz., silver at 5*s.* an oz. and copper considered as of no value comparatively, may be melted together that the compound may be worth £2. 15*s.* per oz.?

9. We wish to mix wines at 35 c. and at 55 c. the litre to form a mixture at 42 c. the litre: how much of the second wine must we take for 182 litres of the first? How much must we take of each kind to have a mixture of 640 litres?

10. A grocer having four sorts of tea at 1*s.* 9*d.*, 2*s.*, 2*s.* 6*d.* and 2*s* 9*d.* a lb. would have a mixture of 81 lbs. at 2*s.* 4*d.* a lb. What quantity must he take of each sort?

11. A person bought 80 lbs. of sugar of two different sorts for £1. 5. 4. The better sort cost 5*d.* per lb., and the worse 3½*d.* per lb. Find how many lbs. there were of each sort.

12. A greengrocer sells potatoes at 2*s.* 8*d.*, 2*s.* 11*d.* and 3*s.* 3*d.* a bushel: what quantities of each kind must he sell that the average price obtained shall be 3*s.* a bushel?

13. What quantities of coffee at 1*s.* 7½*d.* a lb. and of chicory at 5¾*d.* a lb. must a person take to make a mixture of 33 lbs. worth 1*s.* 2½*d.* a lb.?

14. The specific gravity of lead is 11·324, of cork ·24 and of fir ·45: how much cork must be added to 60 lbs. of lead that the united mass may weigh as much as an equal bulk of fir?

15. A silversmith gave £48. 10. 10 for 16 lbs. 8 oz. of silver, giving 5*s.* 2½*d.* an oz. for one part and 4*s.* 4½*d.* for the rest: how many oz. of each kind did he buy?

16. The price of gold is £3. 17. 10½ per oz.; a composition of gold and silver weighing 18 lbs. is worth £637. 7*s.*, but if the proportions of gold and silver were interchanged it would be worth only £259. [illegible] Find the proportion of gold and silver in the composition, and the price of silver per oz.

CHAPTER XIII.

PER-CENTAGE.

310. PER CENTUM or PER CENT. means for a hundred: thus, 5 per cent. means 5 for a hundred.

311. Public companies, bankers, merchants, &c., regulate their transactions, and calculate their profits and losses with reference to 100 as a standard. In this way they can readily accommodate their dealings to the varying circumstances connected with them, and at once compare the results of their several undertakings.

Thus, suppose a company with a capital of £50000 makes a profit of £4125, and another with a capital of £32500 makes a profit of £2637. 10*s.*, it is not easy to see which of the two is the more prosperous; but reducing them to a common standard of £100, the profit of the first is £8¼ and of the second £7½.

312. Tables recording the increase or decrease of population, the number of persons engaged in trade, agriculture, under education, &c. are for the same reason constructed with reference to 100.

Suppose the population of a town has increased during the last 10 years from 34575 to 37341; the increase is 2766 on 34575, or 8 on 100, or 8 per cent.

313. Again, when a quantity is made up of several parts, it is usual to consider the whole quantity as composed of 100 units (of weight, measure, value, &c.), and then each of the parts will contain a certain number of these units; that is, a certain per cent. of the given quantity. Thus, 15 per cent. of gunpowder is charcoal; that is, if a mass of gunpowder contain 100 units of weight, 15 of these units, when the mass is decomposed, will be charcoal.

314. The 100 we refer to may be £100, 100 persons, 100 oz., &c. and the number giving the per cent. is so many units of the same kind; but since

$$£100 : £5 = 100 \text{ persons} : 5 \text{ persons} = 100 \text{ oz.} : 5 \text{ oz.},$$
$$= 100 : 5,$$

we usually consider the 100 as an abstract number, and therefore the number giving the per cent. will also be an abstract number; but if necessary we can at once pass to any corresponding concrete numbers.

315. Every question in Per-Centage can be readily solved by Rule of Three, and the rules given below can be immediately deduced from the Rule of Three statement.

316. CASE I. *To find the value of a given per cent. of a given quantity.*

5 per cent. of a quantity means 5 parts for a hundred of the quantity, and is therefore equal to $\frac{5}{100}$ of the quantity. In like manner 7 per cent. or 12 per cent. of a quantity is $\frac{7}{100}$ or $\frac{12}{100}$ of the quantity. Hence we have this Rule (128),—

Multiply the quantity by the number expressing the rate per cent. and divide by 100.

Ex. 1. A rent collector receives £1365. 17. 6, and he retains $3\frac{1}{2}$ per cent. for his trouble: how much does his per-centage amount to?

$$\begin{array}{rrr}
£. & s. & d. \\
1365 \,. & 17 \,. & 6 \\
 & & 3 \\
\hline
4097 \,. & 12 \,. & 6 \\
\frac{1}{6}\;\; 682 \,. & 18 \,. & 9 \\
\hline
47,80 \,. & 11 \,. & 3 \\
20 & & \\
\hline
16,11 & & \\
12 & & \\
\hline
1,35 & & \\
4 & & \\
\hline
1,40 & &
\end{array}
\qquad\qquad
\begin{array}{r}
£. \\
1365\cdot875 \\
3 \\
\hline
4097\cdot625 \\
\frac{1}{6}\;\; 682\cdot9375 \\
\hline
47\cdot805625 \\
= £47.\ 16.\ 1\frac{1}{4}.
\end{array}$$

$\therefore$ Per-Centage $= £47.\ 16.\ 1\frac{1}{4}\frac{2}{5}$.

Ex. 2. How many persons are engaged in agriculture, when they constitute 17 per cent. of a population of 6457312?

$$\text{No. reqd} = \frac{17}{100} \times 6457312 = 1097743\cdot04 = 1097743.$$

317. Case II. *To find how much per cent. one quantity is of another quantity.*

For example,—How much per cent. is 19 parts out of 45? that is, how many parts must be taken out of 100, when 19 are taken out of 45? hence

$$45 : 19 = 100 : \text{per cent. reqd};$$

$$\therefore \text{per cent. reqd} = \frac{19}{45} \times 100.$$

Again,—What per cent. will a profit of £4375 give on a capital of £50000? that is, what will be the profit on 100 when £4375 is the profit on £50000? hence

$$£50000 : £4375 = 100 : \text{per cent. reqd};$$

$$\therefore \text{per cent. reqd} = \frac{4375}{50000} \times 100.$$

We may therefore adopt this Rule,—*Find what fraction the first quantity is of the second, and multiply by* 100.

Ex. 3. How much per cent. is 9*d.* in the pound?

$$\text{Since } £1 = 240d.;$$

$$\therefore \text{per cent. reqd} = \frac{9}{240} \times 100 = 3\tfrac{3}{4}.$$

Ex. 4. The population of a town has increased during the last 10 years from 38851 to 44565: find the increase per cent.

Since $44565 - 38851 = 5714$, there is an increase of 5714 persons on 38851 persons;

$$\therefore \text{per cent. reqd} = \frac{5714}{38851} \times 100$$

$$= 14\cdot707\ldots$$

```
38851 ) 571400 ( 14·707
        182890
         27486
           290
            18
```

318. Case III. *The per-centage on a quantity being given and the rate per cent., to find the quantity.*

Suppose the rate per cent. to be $4\frac{3}{4}$; that is, suppose the per-centage on 100 to be $4\frac{3}{4}$, then

$$4\tfrac{3}{4} : 100 = \text{given per-centage} : \text{quantity reqd};$$

$$\therefore \text{quantity reqd} = \frac{100}{4\frac{3}{4}} \times \text{given per-centage}.$$

Hence this Rule,—*Divide* 100 *by the rate per cent., and multiply the quotient by the given per-centage.*

Ex. 5. The profits of a Company during the past year amount to £2457. 11. 3, which will just allow of a dividend of $3\frac{3}{4}$ per cent.; find the amount of the Company's capital.

Since the dividend is equal to $3\frac{3}{4}$ per cent. of the capital,

$$\therefore \text{Company's capital} = £2457\tfrac{9}{16} \times \frac{100}{3\frac{3}{4}}$$

$$= £\frac{39321}{16} \times \frac{400}{15}$$

$$= £65535.$$

Exercise 52.

1. How much is $12\frac{3}{8}$ per cent. on £568. 6. 8? and how much is $1\frac{1}{4}$ per cent. on the result?

2. Find 5 per cent. on £621. 13. 9; and from the result deduce $4\frac{3}{4}$ per cent. on the same sum.

3. The population of York in 1861 was 40433, and it increased 8·32 per cent. between 1861 and 1871; find the population in 1871.

4. The population of the City of London decreased 33·11 per cent. between 1861 and 1871: in 1861 it was 113387; find what it was in 1871.

5. The population of Huddersfield increased 101·43 per cent. between 1861 and 1871; in 1871 it was 70253, find what it was in 1861.

6. The population of a town increased 35 per cent. between 1851 and 1861, and 19 per cent. between 1861 and 1871; the population in 1871 was 93177, find the population in 1851.

7. *A* bought goods to the value of £345. 15*s*. and sold them to *B* at a gain of 15 per cent. on his outlay, and *B* sold them to *C* at a loss of 15 per cent. on his outlay; how much did *C* give for them?

8. *A* sells goods to *B* at a gain of $22\frac{1}{2}$ per cent., and *B* sells the same goods to *C* at a gain of $7\frac{1}{2}$ per cent.: *C* gave £263. 7. 6 for the goods, how much did *A* give for them?

9. How much per cent. is 1 part out of 8; 7 parts out of $24\frac{1}{2}$; 37 of 75; $43\frac{3}{4}$ of 165; $869\frac{3}{8}$ of 4655?

10. What rate per cent. is equivalent to 6*d.* in the £1; 3*s.* $10\frac{1}{2}$*d.* in the £1; £2. 16. 8 in £24. 5. 10; £34. 17. $9\frac{3}{4}$ in £607. 1. $11\frac{1}{4}$?

11. How much in the £1 is 5 per cent., is $12\frac{1}{2}$ per cent., is $41\frac{2}{3}$ per cent.?

12. The population of the United Kingdom on 3 April, 1871, was made up as follows:—

England and Wales	22704108
Scotland	3358613
Ireland	5402759
Isle of Man and Channel Islands	144430
Army, Navy, and Seamen abroad	207198

find what per cent. each of these parts is of the whole United Kingdom.

13. The population of England and Wales in 1801 was 8892536, in 1821 was 12000236, in 1851 was 17927609, and in 1871 was 22704108; find the increase per cent. between each of these periods, and also between the first period and last.

14. The population of Ireland in 1851 was 6551970, in 1861 was 5764543, and in 1871 was 5402759: find the decrease per cent. for each decennial period.

15. In 1871 the population of Scotland was 3358613, and the number of males was 1601633; find what per cent. the females are of the whole population, and what per cent. the males are of the females.

16. If a tradesman's pound weight is 13 drams too light, find his gain per cent. from this source alone.

17. With 17 cwts. 3 qrs. 19 lbs. of nitre, 3 cwts. 2 qrs. 15 lbs. of charcoal, and 2 cwts. 1 qr. 7 lbs. of sulphur a mass of gunpowder is made; what per cent. of each ingredient enters into the composition of the gunpowder?

18. 1 lb. of standard silver is coined into 66 shillings; find the profit per cent. when standard silver is worth 5*s.* $0\frac{3}{4}$*d.* per oz.

19. Find the value of the goods imported when an *ad valorem* duty of $8\frac{3}{4}$ per cent. produces £3863. 2. 6.

20. The population of a town has increased by 5455 persons between 1861 and 1871, and this increase is 9·68...per cent. of the population in 1861: find the population in 1871.

21. A deduction is made for a debt of £1373. 6. 8, and £1308. 2s. is accepted in discharge of it: at what rate per cent. is the deduction made?

22. If a debt after a deduction of 3 per cent. becomes £210. 3. 4, what would it have become if a deduction of 4 per cent. had been made?

23. A person buys a farm of 150 acres for £4624, and after repairing the buildings lets it at 30s. an acre, thereby getting a return of $4\frac{1}{2}$ per cent. for his money: how much did he expend on repairs?

24. A builder buys half an acre of land at 15s. 9d. a square yard, and builds a house upon it at a further cost of £1094. 5s.: what rent per annum must he obtain to realize 9 per cent. on his outlay?

25. After deducting a charge of $8\frac{3}{4}$ per cent. on a certain sum, and then a charge of $6\frac{1}{4}$ per cent. on the remainder, the result is £310. 5s. Required the original sum.

26. A person derives an income of £364. 10s. from a sum of money put out at $4\frac{1}{2}$ per cent.: what is his capital, and what diminution of it would make the same reduction of income as an income-tax of 5d. in the pound?

27. What must be the gross produce of an estate that after paying a 10 per cent. income-tax, and a rate of 2s. $1\frac{1}{2}$d. in the £1 on the residue, there may remain £1574 per annum?

28. Out of the profits of a Joint-Stock Company 7d. in the £1 is paid for income-tax; and out of the remainder the manager takes $3\frac{1}{3}$ per cent. for his salary. His salary amounts to £436. 17. 6; find the gross profits of the Company.

29. A man allows his agent 5 per cent. on his gross income for collecting his rents. He spends one-seventh of his net income in insuring his life, and this part is exempted from income-tax. His income-tax, which is laid at 10d. in the £1, amounts to £38. 19s.; find his gross income.

30. The capital of a Company is £64875, and the profits of the year amount to £5143. 11. 8; find to the nearest quarter the highest dividend that can be paid, and how much will be carried forward.

31. A wine which contains $7\frac{1}{2}$ per cent. of spirit is frozen, and the ice which contains no spirit being removed, the proportion of spirit in the wine is increased to $8\frac{3}{4}$ per cent. How much water in the shape of ice was removed from 504 gallons of the original wine?

32. The stuff out of a lead-mine contains at first 15·9 per cent. of lead. After washing, by which process the amount of lead ore is not diminished, the stuff contains 87·45 per cent. of lead. How much rock was washed away out of 216 tons 5 cwts. of the original stuff?

II. COMMISSION, BROKERAGE, INSURANCE.

319. COMMISSION is the charge made by an agent for buying or selling goods, and is usually a per-centage on the value of the goods bought or sold.

320. BROKERAGE is the charge made by a broker for buying or selling Stocks, bills of exchange, ships, cargoes, goods, &c.; and is usually a per-centage on the full amount of the transaction.

321. INSURANCE is a contract by which one party (the insurer) undertakes to pay a specified sum, on the happening of a particular event, in consideration of the other party (the insured) paying year by year, or once for all, a certain per-centage of that sum.

The consideration paid by the insured is called the *premium*, and the instrument containing the contract the *policy*.

The ordinary kinds of insurance are *life*, *fire* and *marine*. Life and fire insurances are undertaken by companies; marine both by companies and by private persons.

In a *life insurance*, the insurer undertakes to pay a sum there specified at the death of the person insured; and the premium is usually paid year by year.

In a *fire insurance*, the insurer undertakes to indemnify the insured up to the sum there specified against any loss that may occur to his property by fire; and the premium is usually paid year by year. It varies in amount as widely as from 1*s*. 6*d*. to £5. 5*s*. for every £100 specified in the policy.

In a *marine insurance*, the insurers undertake to indemnify the insured up to the sum there specified against any loss that may occur to ship, or cargo, or freight, any or all of them, during a particular voyage, or for a certain period not exceeding 12 months; and one premium only is paid.

A marine insurance is usually undertaken by several parties, and each of them writes his name *under* or at the foot of the policy, and undertakes on his own account to indemnify the insured, to the extent of the sum set opposite to his name; and on this account marine insurers are called *underwriters*.

When a man insures, so as to recover not only his property, but the premium and all other expenses connected with its insurance, it is said to be *covered.*

322. DISCOUNT, without reference to time, is an allowance which merchants and tradesmen make to such of their customers as are willing to pay ready money. This allowance is usually a per-centage on the amount of the account.

323. Since Commission, Brokerage,... is a per-centage on a given sum of money, to find its amount (316)—

We multiply the sum by the number expressing the rate per cent. and divide by 100.

Ex. 1. An agent sells goods to the value of £583. 10*s.*, on which he receives a commission of 3¾ per cent.: how much does his commission amount to?

```
      £.    s.  d.                    £.
     583 . 10 . 0                    583·5
                 4 - ¼                   4
    ----------------                ------
    2334 .  0 . 0                    2334·0
¼ |  145 . 17 . 6              ¼ |   145·875
    ----------------                --------
    21,88 .  2 . 6                   21·88125 = £21. 17. 7½.
        20
    -----
    17,62
       12                  ∴ His commission is £21. 17. 7½.
    -----
     7,50
```

Ex. 2. A person at the age of 36 insures his life for £3125 at the rate of £2. 15. 9 for every £100: find the premium that must be paid year by year.

```
              £.                                 £.
             3125                               3125
                2                                  2
            -----                              -----
             6250                               6250
10s. ½  |   1562 . 10               10s. ½ |   1562·5
 5s. ½  |    781 .  5                5s. ½ |    781·25
 6d. 1/10 |   78 .  2 . 6            6d. 1/10 |   78·125
 3d. ½  |     39 .  1 . 3            3d. ½ |     39·0625
            -------------------            ---------------------------
            87,10 . 18 . 9                 87·109375 = £87. 2. 2¼
               20
            -----
             2,18
               12                  ∴ Yearly Premium = £87. 2. 2¼.
            -----
             2,25
```

Ex. 3. A cargo is valued at £5270. 6*s.*; the premium on insurance is at the rate of 5 guineas per cent., policy duty at 4*s.* per cent., and commission $\frac{7}{16}$ per cent.: what sum must be insured to cover the cargo and the expenses of insurance, and what premium must be paid?

Here	Premium	= £5. 5. 0	per cent.
	Duty	= 4. 0	. . .
	Commission	= 8. 9	. . .
∴ whole expense of insurance		= £5. 17. 9	. . .

In case of loss, for every £100 received from the underwriters £5. 17. 9 is for expenses of insurance, and the remaining £94. 2. 3 is for cargo: hence to recover both cargo and expense of insurance we must insure £100 for every £94. 2. 3 of cargo: therefore

£94. 2. 3 : £5270. 6 = £100 : sum to be insured.

Also, the expenses of insurance are at the rate of £5. 17. 9 for every £94. 2. 3 of cargo; therefore

£94. 2. 3 : £5270. 6 = £5. 17. 9 : expenses of insurance;

and £94. 2. 3 : £5270. 6 = £5. 5. 0 : premium to be paid.

Exercise 53.

1. What does a factor receive for selling goods to the amount of £375. 16. 8 at a commission of $1\frac{3}{4}$ per cent.?

2. Find the brokerage on the purchase of £1545. 17. 6 Consols at 2*s.* 6*d.* per cent.

3. What annual premium must a person 28 years old pay on a policy of insurance for £4325 at the rate of £2. 8. 7 per cent.?

4. What is the ready money payment of an account amounting to £359. 14. 9, allowing a discount of $2\frac{1}{2}$ per cent.?

5. A person insures his houses valued at £3570 as a common insurance at 1*s.* 6*d.* per cent.; and his shops and warehouses, valued at £2830, as a hazardous insurance, at 2*s.* 6*d.* per cent.: find the amount of his premiums.

6. What sum must be paid to insure a cargo worth £2585, the premium being 35*s.*, policy duty 2*s.*, and brokerage 2*s.* 6*d.* per cent. respectively?

7. What premium must be paid on the insurance of goods worth £[illegible]48. 10*s.* at 65*s.* per cent. that in case of total loss the owner may recover both the value of the goods and the premium?

8. An agent sells goods to the value of £79653. 12. 6, on which he receives a commission of $3\frac{3}{8}$ per cent., while his office and other expenses amount to $22\frac{1}{2}$ per cent. of his commission. How much clear profit does he make, and how much does he remit to his principal?

9. A broker at the public sales buys 5 chests of indigo weighing 18 cwts. 3 qrs. 22 lbs. nett, at 5*s.* 10*d.* a lb.: find the brokerage at $\frac{5}{8}$ per cent.

10. A person at the age of 45 insures his life in each of two offices for £3850; the premiums being at the rate of £3. 19. 10 and £3. 14. 7 per cent. respectively. Find his annual payment.

11. At what rate per cent. is discount allowed when a tradesman deducts £4. 0. $9\frac{3}{4}$ from a bill of £89. 15. 10?

12. A tradesman insures his warehouse and goods for £12500; $\frac{3}{8}$ of this sum being for his warehouse and $\frac{5}{8}$ for his goods. The warehouse is insured at 12*s.* 6*d.* and the goods at 24*s.* 6*d.* per cent.: find the amount of his premium.

13. For what sum must a merchant insure a cargo worth £1517. 8. 6 at $3\frac{3}{4}$ per cent., so that in case of loss both cargo and premium may be *covered?*

14. A ship worth £15325 is to be insured, so that its value and all the expenses connected with its insurance may be covered. The premium is $2\frac{1}{2}$ guineas per cent., policy duty 4*s.* per cent., and brokerage $\frac{1}{2}$ per cent.; what is the amount of the whole expense paid on insurance?

15. What sum must be paid on the insurance of a cargo of the value of £3457. 10. 6 so that in case of loss the cargo and all expenses of insurance may be recovered? The premium is at the rate of $4\frac{1}{2}$ guineas per cent., policy 4*s.* per cent., and agent's commission $\frac{1}{2}$ per cent.

III. PROFIT AND LOSS.

324. Under the head of PROFIT AND LOSS, we estimate a profit or a loss not absolutely, but *in relation to the cost price.* If one article costs 5*s.* and is sold for 6*s.*, and another article costs 10*s.* and is sold for 11*s.*, the absolute gain is in both cases the same, but relatively to the cost price the first gain is double of the second.

325. Men of business adopt 100 as a *standard cost price*, and reduce the gain or loss on a particular cost price to the correspond-

ing gain or loss on 100; that is, to a gain or loss of so much per cent. This may be effected by the statement (291).

Cost price : gain or loss thereon = 100 : gain or loss per cent. (A.)

Again, when the cost price is represented by 100, the selling price is represented by 100 + gain per cent., or 100 − loss per cent., according as a gain or loss has been made, and we have the statement

$$\text{Cost price : selling price} = 100 : \begin{cases} 100 + \text{gain per cent., or} \\ 100 - \text{loss per cent.} \end{cases} \quad \text{(B).}$$

Lastly, if an article be sold at two different prices, these selling prices will be to one another as their representative selling prices; for from (B) we have (193)

1st selling price : cost price = 1st repve selling price : 100,

and cost price : 2nd selling price = 100 : 2nd repve selling price; and, compounding these statements (194), we have

1st selling price : 2nd selling price (C).

= 1st repve selling price : 2nd repve selling price.

326. But although questions in Profit and Loss can always thus be solved by Rule of Three, yet it is often useful to remember that since a gain of 15 per cent. means a gain of 15 on 100, where 100 represents the cost price, it is a gain of $\frac{15}{100}$ of the cost price. And in like manner a loss of 9 per cent. means a loss of $\frac{9}{100}$ of cost price.

Ex. 1. Tea is bought at 3*s*. 6*d*. a lb. and sold at 3*s*. 10½*d*. What is the gain per cent.?

The gain on 3*s*. 6*d*. is 4½*d*., hence (A)

3*s*. 6*d*. : 4½*d*. = 100 : gain per cent.;

$$\therefore \text{gain per cent.} = \frac{4\frac{1}{2}}{42} \times 100 = \frac{9}{84} \times 100$$

$$= 10\tfrac{5}{7}.$$

Ex. 2. If cloth be bought at 16*s.* 8*d.* a yard, and sold at a loss of $12\frac{1}{2}$ per cent., what price did it fetch?

A loss of $12\frac{1}{2}$ per cent. means that if the cost price be represented by 100 the selling price will be represented by $100 - 12\frac{1}{2}$ or by $87\frac{1}{2}$, therefore (B)

$$100 : 87\tfrac{1}{2} = 16s.\ 8d. : \text{selling price};$$

$$\therefore \text{selling price} = \frac{87\frac{1}{2}}{100} \times 16s.\ 8d. = \frac{7}{8} \times 16s.\ 8d.$$

$$= 14s.\ 7d.$$

Or we may proceed thus:—

$$\begin{array}{rl} & \begin{array}{rcl} s. & & d. \\ 16 & . & 8 = \text{cost price}, \end{array} \\ 12\frac{1}{2} \text{ per cent.} = \frac{1}{8} & \underline{\begin{array}{rcl} 2 & . & 1 = \text{loss}, \end{array}} \\ & \begin{array}{rcl} 14 & . & 7 = \text{selling price}. \end{array} \end{array}$$

Ex. 3. If a cwt. of sugar cost £2. 6. 8, at what price per lb. ought it to be retailed, to gain 15 per cent.?

From (B) $100 : 115 = £2.\ 6.\ 8 :$ selling price per cwt.;

$$\therefore \text{selling price per cwt.} = \frac{115}{100} \times £\frac{7}{3} = £\frac{23}{20} \times \frac{7}{3}$$

$$= £\frac{161}{60};$$

$$\therefore \text{selling price per lb.} = \frac{\frac{161}{\cancel{60}} \times \overset{4}{\cancel{240}}}{112} d.$$

$$= 5\tfrac{3}{4}d.$$

Ex. 4. By selling a watch for £34. 10*s.* there is a loss of 8 per cent.; what will be the loss or gain per cent. by selling it for £38?

The two selling prices are £34. 10*s.* and £38, and the first representative selling price is 100 − 8 or 92, therefore (C)

$$£34.\ 10 : £38 = 92 : \text{2nd repve selling price};$$

$$\therefore \text{2nd repve selling price} = \frac{38}{34\frac{1}{2}} \times 92 = \frac{76}{\underset{3}{\cancel{69}}} \times \overset{4}{\cancel{92}} = \frac{304}{3}$$

$$= 101\tfrac{1}{3};$$

$$\therefore \text{Gain per cent.} = 1\tfrac{1}{3}.$$

Ex. 5. A clockmaker by selling a clock for £4. 12. 6 loses $7\frac{1}{2}$ per cent.; at what price should he have sold it to gain $6\frac{1}{4}$ per cent.?

The representative selling prices are $100-7\frac{1}{2}$ and $100+6\frac{1}{4}$, or $92\frac{1}{2}$ and $106\frac{1}{4}$, hence (C)

$$92\tfrac{1}{2} : 106\tfrac{1}{4} = £4.\ 12.\ 6 : \text{selling price reqd};$$

$$\therefore \text{selling price reqd} = \frac{106\frac{1}{4}}{92\frac{1}{2}} \times £4\tfrac{5}{8} = £\frac{\overset{85}{\cancel{425}}}{\underset{\underset{2}{\cancel{10}}}{\cancel{370}}} \times \frac{\cancel{37}}{8} = £\frac{85}{16}$$
$$= £5\tfrac{5}{16}$$
$$= £5.\ 6.\ 3$$

Ex. 6. If $5\frac{1}{2}$ per cent. be gained by selling butter at £5. 5. 6 per cwt., how much per cent. will be gained by selling it at 1*s.* 3*d.* a lb.?

$$\text{Selling price per cwt.} = 112 \times 1\tfrac{1}{4}s. = 140s. = £7;$$

$$\therefore \text{from (C) } £5.\ 5.\ 6 : £7 = 105\tfrac{1}{2} : 100 + \text{gain per cent.};$$

$$\therefore 100 + \text{gain per cent.} = \frac{7}{5\frac{11}{40}} \times 105\tfrac{1}{2} = \frac{7 \times 40}{211} \times \frac{211}{2}$$
$$= 140;$$

$$\therefore \text{Gain per cent.} = 40.$$

Ex. 7. If a person purchases pins 18 in a row and sells them at the same price 11 in a row, how much does he gain per cent.?

The price of the pins is *inversely* proportional to the number in a row;

$$\therefore \text{cost price} : \text{selling price} = 11 : 18.$$

But $\quad \text{cost price} : \text{selling price} = 100 : \text{repve selling price};$

$$\therefore 11 : 18 = 100 : \text{repve selling price};$$

$$\therefore \text{repve selling price} = \frac{18}{11} \times 100$$
$$= 163\tfrac{7}{11};$$

$$\therefore \text{Gain per cent.} = 63\tfrac{7}{11}.$$

Ex. 8. A merchant buys 1260 quarters of corn, one-fifth of which he sells at a gain of 5 per cent., one-third at a gain of 8 per cent.,

and the remainder at a gain of 12 per cent.; if he had sold the whole at a gain of 10 per cent. he would have obtained £23. 13*s.* more: what was the prime cost per quarter?

$\frac{1}{5}$ of 1260 qrs. = 252 qrs.; $\frac{1}{3}$ of 1260 qrs. = 420 qrs.; ∴ remdr = 588 qrs.

Gain on 252 qrs. at 5 per cent. = $\frac{5}{100}$ of 252 qrs. at *cost* price;

∴ actual gain = $\frac{5}{100}$ of 252 qrs. + $\frac{8}{100}$ of 420 qrs. + $\frac{12}{100}$ of 588 qrs.

$$= \frac{1260}{100} \text{ qrs.} + \frac{3360}{100} \text{ qrs.} + \frac{7056}{100} \text{ qrs.} = \frac{11676}{100} \text{ qrs.}$$

$= 116\frac{19}{25}$ qrs. at cost price;

and hypothetic gain = $\frac{10}{100}$ of 1260 qrs. = 126 qrs.;

∴ Difference of two gains = $9\frac{6}{25}$ qrs. at cost price;

∴ Cost price of $9\frac{6}{25}$ qrs. = £23. 13. = 473*s.*;

$$\text{and ∴ Cost price of 1 qr.} = \frac{473}{9\frac{6}{25}} s. = \frac{473 \times 25}{231} s. = \frac{43 \times 25}{21} s.$$

$= 51s.\ 2\frac{2}{7}$.

EXERCISE 54.

1. An article is bought for 3*s.* $7\frac{3}{4}$*d.* and sold for 4*s.* 8*d.*; find the gain per cent.

2. A horse was bought for £31. 5*s.* and sold at a loss of 12 per cent.; at what price was he sold?

3. Cloth is sold at 6*s.* $1\frac{1}{2}$*d.* per yard at a loss of $12\frac{1}{2}$ per cent.; find the prime cost.

4. Goods are bought for £5. 18. 9 and sold for £6. 7. $5\frac{1}{4}$; what per cent. is gained by the transaction?

5. A horse was bought for £34 and sold for £27. 12. 6; what was the loss per cent.?

6. By selling an article for 3*s.* 9*d.* a person loses 5 per cent.; at what price must he sell it to gain $4\frac{1}{2}$ per cent.?

7. The cost of a 38-gallon cask of wine was £25, and 8 gallons are lost by leakage; at what price per gallon must the remainder be sold to realize 10 per cent. on the outlay?

8. A person having bought goods for £40 sells half of them at a gain of 5 per cent.; for how much must he sell the remainder so as to gain 10 per cent. on the whole?

9. Sugar is bought at £2. 7. 11 a cwt. and retailed at 5¾*d.* a lb.; what is the gain or loss per cent.?

10. Figs are bought at £5. 16. 8 a cwt.; at what price per lb. must they be sold to realize a profit of 15 per cent.?

11. By selling an article for £25. 10*s.*, 6¼ per cent. is lost; what per cent. is gained or lost by selling it for £33. 15*s.*?

12. If a tradesman gains 4*s.* 10½*d.* on an article which he sells for 16*s.* 3*d.*, what is his gain per cent.?

13. An article is sold for 6*s.* 9*d.* at a gain of 17 per cent.; what will be the gain per cent. when the price is reduced 4½*d.*?

14. If 100 lbs. of tea be bought at 2*s.* 2*d.* a lb. and sold at 2*s.* 6*d.*, and 100 lbs. of sugar be bought at 4¼*d.* a lb. and sold at 5¾*d.*, what profit per cent. will be realized on the outlay?

15. A contractor bought 250 sheep and sold them for £532. 5. 10 at a gain of 16⅔ per cent.; what was the cost price of each sheep?

16. A stationer sold quills at 11*s.* a thousand, clearing three-eighths of the money; what would he clear per cent. by selling them at 13*s.* 6*d.* a thousand?

17. A draper having bought 500 yards of calico at 8*d.* a yard, sells half of it at 9½*d.*, a quarter at 9*d.*, and the rest at 6¾*d.* a yard; what is his whole gain, and his gain per cent.?

18. A draper bought 3672 yards of linen at 3*s.* 2*d.* a yard. He sells ⅓ of it at 3*s.* 6*d.* a yard, and ⅓ of the remainder at 3*s.* 11*d.*; at what price per yard must he sell the rest to gain on the whole 11¼ per cent.?

19. A person buys 3½ cwts. of tea at 2*s.* 8½*d.* per lb. and 4½ cwts. at 1*s.* 7⅝*d.* per lb., and mixes them; he sells 5 cwts. at 2*s.* 3*d.* a lb.: at what price per lb. must he sell the remainder to gain 20 per cent. on his outlay?

20. A bankrupt's stock was sold for £520. 10*s.* at a loss of 17 per cent. on the cost price; had the stock been sold in the ordinary course of trade it would have realized a profit of 10 per cent. How much was it sold under the trade price?

21. A lb. of jeweller's gold is worth £36. 7. 6; from this gold a chain is made weighing 29 dwts. 8 grs. and £4. 10*s.* is paid for workmanship; at what price must the chain be sold to gain 37½ per cent. on the outlay?

22. A farmer sold 250 bushels of wheat at 6*s*. 8*d*. a bushel at a loss of $7\frac{1}{2}$ per cent.; afterwards he sold 150 more at a gain of $12\frac{1}{2}$ per cent.: what is his profit on the latter transaction, and how much does he gain on the whole?

23. A person buys shares in a railway when they are at £$19\frac{1}{2}$, £15 having been paid, and sells them at £32. 9*s*. when £25 has been paid; how much per cent. does he gain?

24. If eggs be bought at 21 for a shilling, how many must be sold for a guinea to give a profit of $12\frac{1}{2}$ per cent.?

25. £6. 2. 6 was spent in buying apples at 2*s*. 11*d*. a bushel. When they came to be sold part of them were worthless, but the rest, on being sold at a profit of 30 per cent., realized £6. 16. 6: how many bushels were there of worthless ones?

26. A merchant buys 3150 yards of cloth. He sells $\frac{1}{7}$ of it at a gain of 6 per cent., $\frac{2}{9}$ at a gain of 8 per cent., $\frac{1}{5}$ at a gain of 12 per cent., and the remainder at a loss of 3 per cent. Had he sold the whole at a gain of 5 per cent. he would have received £12. 1. 6 more than he did: what was the prime cost of 1 yard?

27. With a gallon of rum which costs 15*s*., a man mixes a quart of water and sells it at 16*s*. a gallon, with a gallon of gin at 11*s*. he mixes $2\frac{1}{2}$ pints of water and sells it at 12*s*. a gallon, and with a gallon of brandy which costs 22*s*. he mixes 3 pints of water and sells it at 23*s*. a gallon: how much per cent. profit does his business yield him, supposing him to sell twice as much rum as gin, and twice as much gin as brandy?

28. A merchant sells cloth at the same price per yard as he bought it per ell; what is his gain per cent.? If he sells at the same price per ell as he bought per yard, what is his loss per cent.?

29. A farmer purchased 749 sheep and sold 700 of them for the price he paid for the whole, and afterwards sold the remaining sheep at the same price per head as the others: find the gain per cent.

30. A person sold 72 yards of cloth for £8. 14*s*., his profit being the cost of 11·52 yards: how much did he gain per cent.?

31. Of a pipe of wine containing 126 gallons 10 are lost by leakage, and the rest is afterwards bottled off so that a dozen bottles contain $2\frac{1}{2}$ gallons: by selling them as imperial quarts how much per cent. is gained or lost on the whole transaction?

32. A tradesman's prices are 25 per cent. above cost price; if he allows a customer 12 per cent. on his bill, what profit does he make?

33. A watch is bought for 25 guineas; at what price must it be sold to secure a clear profit of 30 per cent. after allowing a discount of $2\frac{1}{2}$ per cent. to the purchaser?

34. Silk is bought at *14s.* $5\frac{1}{4}$*d.* a yard; at what price per yard must it be sold to clear $17\frac{1}{2}$ per cent., after allowing a discount to the purchaser of $3\frac{3}{8}$ per cent.?

35. A grocer mixes two kinds of tea which cost him *1s. 8d.* and *2s. 1d.* per lb. respectively in the proportion of 5 to 2; what must be the selling price of the mixture that he may gain 33 per cent. on his outlay?

36. A grocer buys coffee at the rate of £8. *10s.* per cwt., and chicory at £2. *10s.* per cwt., and mixes them in the proportion of 5 parts chicory to 7 parts coffee; at what rate per lb. must he sell the mixture so as to gain $16\frac{2}{3}$ per cent. on his outlay?

37. How many lbs. of tobacco at *5s. 8d.* a lb. must a tobacconist mix with 4 lbs. at *6s. 6d.* that he may sell the mixture at *7s. 10d.* a lb. and gain $33\frac{1}{3}$ per cent. upon his outlay?

38. A person buys some tea at *3s.* a lb., and some at *2s.* a lb.: in what proportion must he mix them so that by selling his tea at *2s.* $7\frac{1}{5}$*d.* a lb. he may gain 20 per cent. on each lb. sold?

39. A merchant made a mixture of wine at *28s.* a gallon with brandy at *42s.* a gallon, and he found that by selling the mixture at *35s.* a gallon he gained 15 per cent. on the price of the wine, and 20 per cent. on the price of the brandy: in what ratio were the wine and the brandy mixed together?

40. A wine-merchant mixes two kinds of wine and sells the mixture so as to gain 8 per cent. on what the wine cost him. Had he sold each kind of wine at the same price per gallon as he sells the mixture he would have gained 10 per cent. and 6 per cent. respectively on the cost price of each. In what proportion were the two kinds of wine mixed together?

INTEREST.

327. INTEREST is money paid for the use of money lent.

The sum lent is called the *Principal;* and the rate at which £100 is lent for *one* year is called the *Rate per cent.*

328. When the principal on which interest is reckoned remains the same during the whole time of the loan, the interest is *simple;* but when the interest as soon as it becomes due is added to the principal and forms a new principal for the next year, the interest is *compound.*

329. The sum of the principal and the interest at the end of any time is called the *amount*.

SIMPLE INTEREST.

330. In SIMPLE INTEREST, the interest is *directly proportional* (287) to the principal, and to the rate per cent., and to the time. The amount is directly proportional to the principal; but *not* to the rate per cent., *nor* to the time.

Every question in interest involves the consideration of *principal, rate per cent., time*, and *interest* or *amount:* and three of these quantities are always given, to find the fourth. There are then *four* cases, according as the quantity to be found is (1) *Interest* or *Amount;* (2) *Principal;* (3) *Rate per cent.;* (4) *Time.*

331. CASE I. *Having given the principal, rate per cent., and time, to find the interest or amount.*

For example,—Find the interest of £735. 12. 6 at $4\frac{3}{4}$ per cent. for $3\frac{1}{2}$ years.

Since the interest is directly proportional to the principal, and the interest on £100 for 1 year is £$4\frac{3}{4}$, we have

$$£100 : £735.\ 12.\ 6 = £4\tfrac{3}{4} : \text{int. for 1 year,}$$

$$\therefore \text{int. for 1 year} = \frac{£735.\ 12.\ 6 \times 4\frac{3}{4}}{100}; \qquad (290)$$

$$\therefore \text{int. for } 3\tfrac{1}{2} \text{ years} = \frac{£735.\ 12.\ 6 \times 4\frac{3}{4}}{100} \times 3\tfrac{1}{2}; \qquad (330)$$

$$\text{or int. reqd} = \frac{£735.\ 12.\ 6 \times 4\frac{3}{4} \times 3\frac{1}{2}}{100} \qquad (139)$$

$$= \frac{\text{Principal} \times \text{rate p. c.} \times \text{time}}{100}.$$

Hence to find the interest we have this Rule,—

Multiply the principal by the rate per cent., the product by the time in years, and divide the result by 100. ***To find the amount, add the interest to the principal*** (329).

REMARK (1). When decimals are used, it is sufficient that the result be correct to the *third* place (275).

REMARK (2). Sometimes it is convenient to multiply the principal by the *product* of rate and time, instead of by each in succession.

REMARK (3). When the time is given in *months and days*, 12 months are reckoned to the year, and 30 days to the month.

REMARK (4). When the time is given in *weeks*, multiply by the number of weeks and divide by 52×100 or 5200.

REMARK (5). When the time is given in *days*, multiply by the number of days and divide by 36500 : or better, multiply by *double* the rate, and divide by 73000.

REMARK (6). To divide by 73000 we may proceed thus: First multiply divisor and dividend by $1 + \frac{1}{3} + \frac{1}{30} + \frac{1}{300}$; that is, to each number add $\frac{1}{3}$ of itself, then $\frac{1}{10}$ of this result, and then $\frac{1}{10}$ of this new result: the divisor will be found to be 100010, for

$$\begin{array}{r|r} & 73000 \\ \frac{1}{3} & 24333\frac{1}{3} \\ \frac{1}{10} \text{ of } \frac{1}{3} & 2433\frac{1}{3} \\ \frac{1}{10} \text{ of } \frac{1}{30} & 243\frac{1}{3} \\ \hline & 100010 \end{array}$$

and, considering this divisor as 100000, the quotient is found approximately by removing the decimal point in the new dividend 5 places to the left (144).

If we had decreased the new divisor and dividend each by $\frac{1}{10,000}$ of itself the divisor would be almost accurately 100,000: hence we may consider the preceding result as too great by $\frac{1}{10,000}$ of itself, which in money is nearly equivalent to $\frac{1}{4}d.$ in £10.

We have then the following Rule, sometimes called *the third, tenth, and tenth* Rule :

Divide the dividend (neglecting the decimal part) by 3, *the quotient by* 10, *and this new quotient by* 10 : *add these quotients and*

the dividend together, and point off 5 places of decimals. Subtract $\frac{1}{4}d.$ for every £10 of interest.

Ex. 1. Find the simple interest and amount of £240. 12. 6 at $2\frac{1}{2}$ per cent. for $8\frac{3}{4}$ years.

	£.	s.	d.
	240	12	6
			2
	481	5	0
$\frac{1}{2}$	120	6	3
	601	11	3
			9 − $\frac{1}{4}$
	5414	1	3
$\frac{1}{4}$	150	7	$9\frac{3}{4}$
	52,63	13	$5\frac{1}{4}$
	20		
	12,73		
	12		
	8,81		
	4		
	3,25		

	£.
	240·625
	2
	481·250
$\frac{1}{2}$	120·3125
	601·5625
	9
	5414·0625
$\frac{1}{4}$	150·3906
	52·636719 = £52. 12. $8\frac{3}{4}$;

∴ interest = £52. 12. $8\frac{3}{4}\frac{1}{4}$
and principal = £240. 12. 6
∴ the amount = £293. 5. $2\frac{3}{4}\frac{1}{4}$

Ex. 2. Find the simple interest of £512. 16. 8 at $4\frac{5}{8}$ per cent. for 3 years 7 months 21 days. See Remark (3).

			£.	s.	d.
			512	16	8
					4
			2051	6	8
$\frac{1}{8}$		$\frac{1}{8}$	256	8	4
$\frac{1}{32}$		$\frac{1}{4}$	64	2	1
			2371	17	1
					3
			7115	11	3
6 mo.		$\frac{1}{6}$	1185	18	$6\frac{1}{2}$
1 mo.		$\frac{1}{6}$	197	13	$1\frac{0}{4}\frac{1}{3}$
15 days		$\frac{1}{2}$	98	16	$6\frac{1}{2}\frac{1}{6}$
6 days		$\frac{2}{5}$	39	10	$7\frac{1}{4}\frac{2}{3}$
			86,37	10	$0\frac{1}{3}\frac{1}{3}$
			20		
			7,50		
			12		
			6,00		
			4		
			,02$\frac{1}{6}$		

$\frac{2\frac{1}{6}}{100} = \frac{13}{600}$

		£.
		512·8333
		4
		2051·3333
$\frac{1}{8}$	$\frac{1}{8}$	256·4166
$\frac{1}{32}$	$\frac{1}{4}$	64·1041
		2371·8540
		3
		7115·5620
6 mo.	$\frac{1}{6}$	1185·9270
1 mo.	$\frac{1}{6}$	197·6545
15 da.	$\frac{1}{2}$	98·8272
6 da.	$\frac{2}{5}$	39·5309
		86·375016 = £86. 7. 6

∴ Interest = £86. 7. $6\frac{0}{4}\frac{13}{600}$

Ex. 3. What is the simple interest of £320. 17. 6 at $4\frac{1}{2}$ per cent. for 17 weeks? See Remark (4).

```
              £.
           320·875
                 4
          1283·500
           160·4375
          1443·9375
                17
5200 { 400 ) 245,46·9375
     {  13 )  61·3673
               4·7205 = £4. 14. 4 3/5.
```

Ex. 4. Find the simple interest of £420. 13. 8 at $4\frac{1}{2}$ per cent. from 19th Mar. 1868 to 8th Sept. 1870. See Remarks (5) and (6).

The day on which the money is lent and the day on which it is paid make only one day: hence we reckon only 12 days in Mar. but 8 in Sept., giving 173 days: therefore the whole time is 2 years 173 days, or 903 days.

```
Mar.   12
April  30
May    31
June   30
July   31
Aug.   31
Sept.   8
      173
2 years 730
        903

               £.
            420·6833
                   9
           3786·1497
                 903
           113584491
          340753473
73,000 ) 3418,893·1791 ( 46·8341
           498
            608
             249
              303
               111

3418893·1791...Rk. (6)
 1139631
  113963
   11396
 46·83883
      468  Deduct 1/10000th
£46·8341
```

∴ interest = £46. 16. $8\frac{1}{4}$.

Exercise 55.

Find the simple interest and amount of

1. £340. 12. 6 for 4 years at 3 per cent.
2. £96. 15. 5 ... 8 years at 4 per cent.
3. £1057. 7. 6 ... 9 years at 5 per cent.
4. £450. 10. 0 ... 7 years at $3\frac{1}{4}$ per cent.
5. £715. 1. 8 ... 1 year at $2\frac{3}{4}$ per cent.

6. £33. 13. 4 for 15 years at $4\frac{5}{8}$ per cent.
7. £55. 16. 8 ... 1 year at $3\frac{1}{3}$ per cent.
8. £654. 2. 6 ... $4\frac{1}{2}$ years at $4\frac{1}{2}$ per cent.
9. £75. 8. 9 ... $\frac{3}{4}$ year at $3\frac{1}{2}$ per cent.
10. £1725. 18. 6 ... $3\frac{1}{2}$ years at $4\frac{1}{4}$ per cent.
11. £476. 18. 6 ... $4\frac{3}{4}$ years at $3\frac{1}{4}$ per cent.
12. £385. 9. 10 ... $\frac{1}{4}$ year at $5\frac{3}{4}$ per cent.
13. £543. 17. 6 ... $2\frac{1}{4}$ years $3\frac{1}{8}$ per cent.
14. £3450. 12. 7 ... $8\frac{1}{2}$ years at $4\frac{5}{8}$ per cent.
15. £45. 15. 6 ... $1\frac{3}{4}$ years at $2\frac{3}{4}$ per cent.

16. £964. 18. 9 at $4\frac{3}{4}$ per cent. for 1 year 219 days.
17. £317. 10. $2\frac{3}{4}$ at $3\frac{5}{8}$ per cent. for 3 years 73 days.
18. £27. 16. 9 at £3. 12. 6 per cent. for 4 years 7 months.
19. 1000 guineas at $4\frac{3}{8}$ per cent. for 1 year 5 months.
20. £460. 3. 6 at $4\frac{3}{4}$ per cent. for 3 years $8\frac{3}{4}$ months.
21. £550. 14. 8 at $4\frac{3}{4}$ per cent. for 2 years 9 months 25 days.
22. £5260. 10. 2 at $2\frac{7}{8}$ per cent. for 6 years 5 months 21 days.
23. $500\frac{1}{2}$ guineas at $2\frac{1}{4}$ per cent. for 1 year 7 months 18 days.
24. £386. 14. $4\frac{1}{2}$ at $5\frac{3}{8}$ per cent. for 5 months 16 days.
25. £266. 13. 4 at $3\frac{1}{2}$ per cent. for 19 weeks.
26. £987. 15. $8\frac{3}{4}$ at £4. 13. 4 per cent. for 37 weeks.
27. £228. 11. $5\frac{1}{4}$ at $4\frac{3}{4}$ per cent. for 1 year 23 weeks.

28. £787. 0. $7\frac{1}{2}$ at $4\frac{1}{2}$ per cent. for 92 days.
29. £1108. 13. 9 at $5\frac{1}{4}$ per cent. for 191 days.
30. £184. 3. 9 at $5\frac{1}{2}$ per cent. from July 17th to Dec. 5th.
31. £853. 0. 10 at $3\frac{1}{4}$ per cent. from June 18th to Sept. 25th.
32. £3057. 14. 7 at $4\frac{3}{4}$ per cent. from April 6th to Oct. 27th.
33. £1253. 8. 5 at $3\frac{5}{8}$ per cent. from Jan. 16th to Mar. 23rd (leap year).
34. £473. 3. 6 at $3\frac{1}{2}$ per cent. from April 14th to July 6th.
35. £164. 15. 11 at $5\frac{3}{4}$ per cent. from 9 Nov. 1867, to 3 Mar. 1868.
36. £1327. 3. 8 at $5\frac{3}{8}$ per cent. from Oct. 18th, 1869, to May 27th, 1871.

332. CASE II. *Having given the interest or amount, rate per cent., and time, to find the principal.*

1st. Let the *interest* be given. Find the interest of £100 at the given rate per cent. for the given time; then, since the interest is directly proportional to the principal producing it, we say

this interest : given interest = £100 : principal reqd.

2nd. Let the *amount* be given. Find the amount of £100 at the given rate per cent. for the given time; then, since the amount is directly proportional to the principal (330), we say

this amount : given amount = £100 : principal reqd.

Ex. 1. What sum of money must be lent, that its interest may come to £29. 6. 8 at $2\frac{1}{2}$ per cent. in 2 years 7 months?

	£.	s.	d.	
	2 .	10 .	0	= int. on £100 for 1 year.
			2	
	5 .	0 .	0	
6 mo. | $\frac{1}{4}$	1 .	5 .	0	
1 mo. | $\frac{1}{6}$	0 .	4 .	2	
	6 .	9 .	2	= int. on £100 for 2 years 7 months;

∴ £6. 9. 2 : £29. 6. 8. = £100 : principal reqd;

∴ principal reqd $= £100 \times \frac{29\frac{1}{3}}{6\frac{11}{24}} = £454.\ 3.\ 10\frac{1}{4}\frac{25}{31}$.

Ex. 2. What principal will amount to £452. 16. 8 in 2 years 8 months at $4\frac{3}{4}$ per cent.?

Int. of £100 for 2 yrs. 8 mo. $= £4\frac{3}{4} \times 2\frac{2}{3} = £12\frac{2}{3}$;

∴ amount .... $= £112\frac{2}{3}$;

and ∴ $£112\frac{2}{3} : £452\frac{5}{6} = £100$: principal reqd;

∴ principal reqd $= \frac{452\frac{5}{6}}{112\frac{2}{3}} \times £100 = £\frac{5225}{13} = £401.\ 18.\ 5\frac{1}{2}\frac{2}{13}$.

Ex. 3. What sum must have been lent to have amounted to £465. 13. 10 in 127 days at $4\frac{3}{4}$ per cent.?

£.
950
127
―――
6650
11400
―――
120650
40216
4021
402
―――
165289
Deduct... 16
―――
101·65273) 46569·166 (485·120 = £438. 2. 5.
5908 074
825 437
12 215
2 050
17

950 is 100 × double the rate p. c. The interest of £100 is 1·65273, and therefore we have

101·65273 : 465·6916 = £100 : sum lent.

333. CASE III. *Having given the principal, time, interest or amount, to find the rate per cent.*

Find the interest on the given principal for the given time at 1 per cent.; then, since the rate per cent. is directly proportional to the interest, say

this interest : given interest = 1 : rate per cent. reqd;

that is,—rate per cent. required is found *by dividing the given interest by the interest at 1 per cent.*

Ex. 1. At what rate per cent. will £142. 10*s*. amount to £163. 13. 11¼ in 4¼ years?

Given Int. = £163. 13. 11¼ − £142. 10 = £21. 3. 11¼.

Int. on £142. 10 at 1 per cent. for 4¼ years = £6. 1. 1½;

$\therefore$ Rate per cent. $= \dfrac{£21.\ 3.\ 11\frac{1}{4}}{£6.\ 1.\ 1\frac{1}{2}} = 3\frac{1}{2}.$

Ex. 2. At what rate per cent. will the interest on £345. 15*s*. become £192. 17. 6 in 8¾ years?

```
         £.
       345·75
            9
       ------
       311175
¼ |      8643
       ------
      30·2532 ) 192·8750 ( 6·37535      ∴ Rate per cent. = 6·37535
                 11 3558                                  = 6⅜ very nearly.
                  2 2798
                    1621
                     108
                      17
                       2
```

Ex. 3. At what rate per cent. will a sum of money double itself in 12½ years?

In 12½ years the interest is equal to the principal,

$\therefore$ interest on £100 for 12½ years = £100;

but interest on £100 at 1 per cent. for 12½ years = £12½,

$\therefore$ rate per cent. $= \dfrac{£100}{£12\frac{1}{2}} = 8.$

334. CASE IV. *Having given the principal, rate per cent., and interest or amount, to find the time.*

Find the interest on the given principal for one year; then, since the time is directly proportional to the interest, say

one year's int. : given int. = 1 : no. of years reqd;

that is,—the number of years is found *by dividing the given interest by the interest for 1 year.*

If it be manifest that the time is less than a year, then, as before, —the number of *days* is found *by dividing the given interest by the interest for 1 day.*

Ex. 1. In what time will £425 amount to £635. 7. 6 at $5\frac{1}{2}$ per cent.?

Given interest = £635. 7. 6 − £425 = 210. 7. 6.

$$\text{Int. on } £425 \text{ for 1 year} = \frac{£425 \times 5\frac{1}{2}}{100};$$

$$\therefore \text{No. of years} = \frac{£210\frac{3}{8} \times 100}{£425 \times 5\frac{1}{2}} = 9.$$

Ex. 2. In what time will a sum of money treble itself at 8 per cent.?

The time will be the same whatever sum of money be taken as the principal: suppose, then, the principal to be £100,

$$\therefore \text{given interest} = 2 \times \text{principal} = £200,$$

$$\text{and interest on } £100 \text{ for 1 year} = £8;$$

$$\therefore \text{No. of years} = \frac{£200}{£8} = 25.$$

Ex. 3. In how many days will the interest on £498. 16. 8 amount to £10. 9. $3\frac{1}{4}$ at $6\frac{1}{8}$ per cent.?

```
       £
     498·833        (331, Rk. 6)
          12
     -------
     5985996
 ¼ |  124708
   ---------
     6110·7
     2036·9
      203·7
       20·4
   --------
  ·083,7,1 ) 10·463 ( 125
              2092
               418
                 0   ∴ No. days is 125.
```

EXERCISE 56.

What principal will amount to

1. £762. 16. $4\frac{5}{8}$ in 5 years at $3\frac{1}{2}$ per cent.?
2. £235. 13. 4 in $1\frac{3}{4}$ years at $3\frac{3}{4}$ per cent.?
3. £843. 13. 6 in $15\frac{1}{2}$ years at $2\frac{3}{4}$ per cent.?

What principal will produce

4. £36. 12. 6 interest in $2\frac{1}{2}$ years at $3\frac{1}{8}$ per cent.?
5. £23. 18. 10 interest in $1\frac{1}{4}$ years at $4\frac{7}{8}$ per cent.?
6. £13. 15. $9\frac{3}{4}$ interest in $\frac{3}{4}$ year at $3\frac{3}{4}$ per cent.?

What principal will amount to

7. £1357. 14. 3 in 2 years 7 months at $4\frac{1}{2}$ per cent.?
8. £725. 12. 6 in 2 years 9 months 18 days at $2\frac{1}{2}$ per cent.?
9. Find the principal whose interest amounts to £57. 16. 8 in 1 year 9 months 24 days at $3\frac{3}{4}$ per cent.?
10. Find to the nearest penny the sum that must be invested at $3\frac{3}{4}$ per cent. for 21 years to amount to £1000.

What principal will amount to

11. £346. 13. 4 at $3\frac{1}{3}$ per cent. in 240 days?
12. £58. 4. 6 at $5\frac{1}{2}$ per cent. from June 16th to Nov. 7th?
13. £73. 5. 0 at $4\frac{3}{4}$ per cent. from April 22nd to July 25th?
14. £5253. 8. 9 at $4\frac{5}{8}$ per cent. from Feb. 18th to Sept. 16th?

At what rate per cent. will

15. £170. 6. 3 amount to £190. 15*s*. in 3 years?
16. £33. 6. 8 amount to £38. 4. 2 in $4\frac{1}{2}$ years?
17. £136. 17. 6 amount to £164. 5. 0 in $6\frac{1}{4}$ years?

At what rate per cent. will the interest on

18. £35. 15. 0 amount to £4. 0. $5\frac{1}{4}$ in $4\frac{1}{2}$ years?
19. 500 guineas amount to £103. 9. $4\frac{1}{2}$ in 3 years 7 months?
20. £239. 16. 6 amount to £22. 13. 0 in 2 years 10 months?
21. £750. 12. 6 amount to £158. 14. $6\frac{1}{4}$ in 5 years 7 months 20 days?
22. The interest of a sum of money at the end of $6\frac{1}{4}$ years is three-eighths of the sum itself; what rate per cent. was charged?
23. What must be the rate per cent. that the interest at the end of 16 years 8 months may be equal to seven-eighths of the sum lent?
24. £7. 19. 6 was charged for the loan of £743. 10. 0 for 87 days; what was the rate per cent.?

25. What is the rate per cent. when the interest on £185. 14*s.* for 125 days amounts to £3. 0. 5?

26. At what rate per cent. will the interest on £1368. 15*s.* become £14. 4. 7½ from July 5th to Nov. 10th?

---

27. In how many years will £340. 12. 6 amount to £381. 10*s.* at 4 p.c.?

28. In how many years will the interest on £35. 15*s.* amount to £4. 0. 5¼ at 2⅓ per cent.?

29. In how many years will £1451. 6. 6 amount to £1667. 4. 2¼ at 4½ per cent.?

30. In how many years and months will the interest on 1000 guineas amount to £103. 9. 4½ at 2⅝ per cent.?

31. In how many years will £2860. 16. 9¾ amount to £3529. 11. 3 at 5⅓ per cent.?

32. In how many years, months and days will £2251. 17. 6 amount to £2728. 1. 0¾ at 3¾ per cent.?

33. In how many years will a sum of money amount to half as much again as itself at 7½ per cent.?

34. In how many years will a sum of money double itself at 6¼ p.c.?

35. In how many days will the interest on £243. 6. 8 amount to £2. 0. 5 at 3⅛ per cent.?

36. In how many days will £556. 17. 6 amount to £565. 18. 9 at 4⅜ per cent.?

37. In how many days will the interest on £371. 15*s.* amount to £3. 19. 9 at 4½ per cent.?

38. On Jan. 1st, 1870, a person borrowed £4835 at 3⅔ per cent., promising to return it as soon as it amounted to £5000: on what day did the loan expire?

---

39. A sum of money amounts in 10 years at 3⅓ per cent. simple interest to £506. 15. 1½; in how many years will it amount to £703. 16. 6¾?

40. A person lent another a sum of money for 72 days at 3 per cent. per annum. At the end of that time he received £293. 12. 0½: what was the sum lent?

41. The sum of £327 is borrowed at the beginning of the year at a certain rate of interest, and after 9 months £400 more is borrowed at double the previous rate. At the end of the year the interest on both loans is £13. 3. 6. What is the rate of interest at which the first sum was borrowed?

42. What sum of money laid out at 4 per cent. will give 1*d.* interest a day? and what sum at $3\frac{1}{2}$ per cent. will give a guinea interest per day?

43. The simple interest on £5812. 10*s.* for ·3 of a year is £87. 3. 9; find the interest on £2906. 5*s.* for 1·6 of a year at the same rate. At what rate per cent. is the interest calculated?

44. Find the interest on an Exchequer bill for £3587. 15*s.* for 48 days at the rate of $1\frac{1}{2}$*d.* per cent. per day. What rate per cent. per annum would this rate give for the year 1871?

45. What sum will amount to £425. 19. $4\frac{3}{4}\frac{1}{8}$ in 10 years at $3\frac{1}{2}$ per cent.; and in how many years more will it amount to £453. 11. 7?

PRESENT WORTH AND DISCOUNT.

335. The PRESENT WORTH or PRESENT VALUE of a sum of money due at the end of a given time is that sum which with its interest for the given time amounts to the sum due. Thus, if £350 in 6 months amounts to £357, it follows that £350 paid now is equivalent to £357 paid at the end of 6 months; that is, the present worth of £357 due at the end of 6 months is £350.

336. DISCOUNT is the abatement made when a sum of money is paid before it is due. But a sum of money due at the end of a given time is discharged *now* by the payment of its present worth; discount therefore is the difference between the sum due and its present worth, and therefore

$$\text{Present worth} + \text{Discount} = \text{Sum due}; \qquad (1)$$

but, by definition (335),

$$\text{Present worth} + \text{int. of Present worth} = \text{Sum due};$$

therefore

$$\text{Discount of Sum due} = \text{interest of its Present worth}. \qquad (2)$$

337. Again, from (1) we have

amount of Present worth + amt. of Discount = amt. of Sum due:

or Sum due (335) + amt. of Discount = Sum due + int. of Sum due;

therefore amount of Discount = interest of Sum due: (3)

and therefore the difference between the *discount* and the *interest* of the sum due, is the *interest of the discount.*

338. To find the *present worth* of a sum of money due at the end of a given time is to find what sum (principal) will for the given time at a given rate amount to the sum due, and is therefore equivalent to Case II. in Simple Interest (332). We proceed thus—

Find what £100 amounts to for the given time at the given rate; the present worth of this sum (amount) due at the end of the given time is £100; therefore we say

∴ this sum : given sum due = £100 : present worth reqd.

To find the *discount*—(1) Find the present worth of the sum due, and subtract it from this sum: or

(2) Since the discount is directly proportional to the sum due, proceed thus:—Find what £100 amounts to at the given rate for the given time; the discount on this sum due at the end of the given time is the difference between this sum and £100; therefore

this sum : its discount = sum due : discount reqd.

Ex. 1. Find the present worth and discount of £275. 6. 8 at the end of 18 months at $4\frac{1}{2}$ per cent.

$$\text{Amount of £100 for 18 months} = 100 + \frac{9}{2} \times \frac{3}{2} = 106\tfrac{3}{4};$$

$$\therefore \text{Present worth of £}106\tfrac{3}{4} \text{ due at the end of 18 months} = \text{£}100;$$

$$\text{and therefore £}106\tfrac{3}{4} : \text{£}275\tfrac{1}{3} = \text{£}100 : \text{present worth reqd};$$

$$\therefore \text{Present worth reqd} = \frac{826}{3} \times \frac{4}{427} \times \text{£}100$$

$$= \text{£}257.\ 18.\ 5\tfrac{1}{2}\tfrac{34}{61}.$$

$$\begin{array}{rl} \text{And £}275\ .\ \ 6\ .\ 8 & = \text{sum due,} \\ \text{£}257\ .\ 18\ .\ 5\tfrac{1}{2}\tfrac{34}{61} & = \text{present worth;} \\ \hline \therefore \text{£}17\ .\ \ 8\ .\ 2\tfrac{1}{4}\tfrac{27}{61} & = \text{discount.} \end{array}$$

Ex. 2. Find the discount of £85. 12. 6 due 90 days hence at $5\frac{1}{2}$ per cent.

$$\text{Amount of £100 in 90 days} = 100 + \frac{11}{2} \times \frac{90}{365} = 100 + \frac{99}{73};$$

$$\therefore \text{Discount of } 100 + \frac{99}{73} \text{ due 90 days hence} = \text{£}\frac{99}{73};$$

and $\therefore\ 100+\frac{99}{73} : \frac{99}{73} = £85.\ 12.\ 6$: discount reqd;

or $7399 : 99 = 85{\cdot}625$: discount reqd;

```
            99
        770625
       770625
7399 ) 8476·875 ( 1·145
       1077 8
        337 9
         42 0
          5 0
```

$\therefore$ Discount reqd $= £1.\ 2.\ 10\frac{3}{4}$.

Ex. 3. The discount of a sum of money due 90 days hence at $5\frac{1}{2}$ per cent. is $£1.\ 2.\ 10\frac{3}{4}$; find the sum due, also its present worth.

By Ex. 2, $\frac{99}{73}$ is the discount of $100+\frac{99}{73}$; hence

$$\frac{99}{73} : 100+\frac{99}{73} = £1.\ 2.\ 10\tfrac{3}{4} : \text{sum due};$$

$$\text{or } 99 : 7399 = £1{\cdot}145 : £85.\ 12.\ 6 \text{ sum due.}$$

$$\text{Also } \frac{99}{73} : 100 = £1{\cdot}145 : \text{present worth of sum due.}$$

339. When the *sum due, its present worth* or *discount*, and the *time* are given, to find the *rate per cent.* allowed, we proceed precisely as in Interest (333); and so too when the other quantities are given, to find the time (334).

Ex. 4. The discount of £295. 15*s.* due at the end of 2 years 8 months is found to be £33. 5*s.*; at what rate per cent. is interest allowed?

The present worth is £262. 10*s.*; we have therefore to find—At what *rate per cent.* will £262. 10*s.* amount to £295. 15*s.* in 2 years and 8 months?

$$\text{Now given interest} = £33.\ 5s.,$$

$$\text{and interest on } £262.\ 10s. \text{ at 1 per cent.} = \frac{£262\frac{1}{2}\times 2\frac{2}{3}}{100};$$

$$\therefore \text{Rate per cent. reqd} = \frac{33\frac{1}{4}\times 100}{262\frac{1}{2}\times 2\frac{2}{3}} = \frac{133}{4}\times 100\times\frac{2}{525}\times\frac{3}{8}$$

$$=4\tfrac{3}{4}.$$

EXERCISE 57.

Find the present worth of

1. £926. 10. 0 due 2 years hence at 4½ per cent.
2. £605. 10. 6 .. 3 years ....£4. 15s. ...
3. £1079. 2. 5 ... 1 yr. 6 mo. .... 5 ....
4. £132. 3s. ... 2¼ years .... 4½ ....
5. £748. 11. 8 ... 3 years .... 5 ....
6. £1250. 10s. ... 2½ years .... 3½ ....
7. 25 guineas ... 18 months .... £3. 12. 6...
8. £245. 13. 4 ... 3⅓ years .... 4½ ....
9. £46. 16. 8 ... 9 months .... 3¾ ....
10. £843. 12. 6 ... 15½ years .... 2¾ ....
11. £1243. 2. 6 ... 3 yrs. 5 mo. .... £3. 2. 6. ...
12. £1144. 8. 1 ... 4 yrs. 90 days............ 2¼ ....

Find the discount of

13. £4120. 8. 7 due 9 months hence at 4 per cent.
14. £12382. 4s. ... 4 months .... 3¾ ....
15. £55447 ... 2½ years .... 4½ ....
16. £520. 17. 6 ... 3½ years .... 4½ ....
17. £461. 15. 10½ ... 3 months .... 7½ ....
18. £753. 12. 6 ... 5 months .... 3¾ ....
19. £450 ... 2 yrs. 9 mo. .... 4½ ....
20. 1000 guineas ... 1 yr. 115 days .... 3½ ....
21. £251. 12. 6 ... 3 yrs. 9 mo. 18 da. ... 6¼ ....
22. £376. 10. 6 ... 60 days .... 6¾ ....
23. £8765. 18. 9 ... 272 days .... 4⅝ ....
24. £862. 10s. ... 55 days .... 4¾ ....
25. £3245. 15s. ... 136 days .... 5¼ ....
26. What is the present value of £1 due 1 year hence at 1 per cent.?
27. If the present worth of £328. 13. 5 due 3 months hence be £325. 8. 4, what rate per cent. is allowed?
28. If the discount of £13735 at 3¾ per cent. be £335, how long was the sum paid before it was due?
29. On what sum of money due at the end of 1 year and 4 months does the discount at 4¾ per cent. amount to £48. 9s.?

30. A tradesman on being paid ready money deducts 19*s*. $3\frac{3}{4}$*d*. from an account of £20. 5. $6\frac{3}{4}$ due at the end of 12 months; what rate of interest does he allow?

31. *A* offers for an estate £37800, and *B* offers £45400 to be paid at the end of 4 years. Which is *now* the better offer and by how much, allowing 5 per cent. interest?

32. Find the difference between the discount on £196. 4. $4\frac{1}{2}$ due 6 months hence at 8 per cent., and the interest on the same sum for the same time at the same rate.

33. If the discount on £78. 9. 9 due 8 months hence be £3. 0. $4\frac{1}{2}$, at what rate per cent. is the discount calculated?

34. The discount on a sum of money due $3\frac{1}{4}$ years hence at $5\frac{3}{4}$ per cent. is £16. 14. 9; find the sum.

35. How many years hence is £589. 6. 3 due, when its present value at $3\frac{1}{2}$ per cent. is 500 guineas?

36. Find the difference between the amount of £494. 10 for 2 years, and the present worth of the same sum due at the end of 2 years, at $3\frac{3}{4}$ per cent.

37. Find the difference between the interest and discount of £1114. 11. 8, the time being $1\frac{3}{4}$ years, and the rate 4 per cent. per annum.

38. Find the discount on £170. 18. 5 due 52 days hence at $2\frac{1}{2}$*d*. per cent. per day.

39. A farmer buys 75 sheep for £120 payable at the end of a twelvemonth, and the same day sells them at 35*s*. a head ready money; what did he gain by the transaction, reckoning interest at 5 per cent. per annum?

40. Find the difference between the interest on £246. 13. 4 for $2\frac{3}{4}$ years at $5\frac{1}{2}$ per cent., and the discount on £283. 19. 6 due $2\frac{3}{4}$ years hence at the same rate. Explain the result (336, 2).

41. If a person's salary be paid at the beginning instead of at the end of the month, what part of the month's salary ought to be abated, reckoning $4\frac{1}{2}$ per cent. per annum?

42. A tradesman marks his goods with two prices, one for ready money and the other for credit of 6 months: what ratio should the two prices bear to each other, allowing interest at $7\frac{1}{2}$ per cent. per annum? If the credit price of an article be £33. 4*s*., what is the cash price?

43. The discount on £275 for a certain length of time is £25; what is the discount on the same sum (1) for twice that length of time, and (2) for half that length of time?

44. The interest on £522. 10*s*. for a certain time is £34. 7. 6; find the discount on the same sum for the same time.

45. If the discount on a sum of money due at the end of 8 months at $6\frac{2}{3}$ per cent. be £43. 15. $9\frac{3}{4}$; find the present worth of the sum.

46. The interest on a certain sum of money for 2 years is £71. 16. $7\frac{1}{2}$, and the discount for the same time is £63. 17*s.* Find the rate per cent. per annum and the sum (337, 3).

47. The difference between the interest and the discount on a certain sum of money at $4\frac{3}{4}$ per cent. for $3\frac{1}{2}$ years is £2. 12. $7\frac{3}{4}$; find the discount on the sum (337), and the sum itself.

48. A man bought a horse for 30 guineas and sold him immediately for £38. 10*s.* payable at the end of 6 months. If the use of the money be reckoned at $6\frac{1}{4}$ per cent. per annum, what is now his gain per cent.?

49. I purchase a piece of land for £3500 and sell it the same day for 4000 guineas, to be paid in two equal instalments at the end of 3 and 6 months respectively: how much do I make by my bargain, the use of money being worth 6 per cent.?

50. A person's salary of 1000 guineas is paid in four quarterly payments at the end of each quarter: what sum at the beginning of the year is equivalent to these quarterly payments, reckoning interest at 5 per cent.?

51. What sum must be paid now in order that a person may receive £250 at the end of every year for the next three years, the rate of interest being $3\frac{1}{2}$ per cent.?

52. £125 is due at the end of 3 months and £90 at the end of 7 months; what sum at the present time is equivalent to both these sums, calculating interest at $4\frac{1}{2}$ per cent.? In what time will the result amount to £125 + £90 at the same rate of interest?

DISCOUNTING BILLS.

340. A bill of exchange is a written instrument in which one person orders another to pay to him, or to some other person, a sum of money at a specified time. Thus:

£500. *London, 1st January,* 1881.

Two months after date pay C. D. or order Five hundred pounds, value received.

Accepted, E. F.

To E. F.,
Park Street, Liverpool. A. B.

Here E. F. engages to pay to C. D. or his order £500 at the end of two months from 1st Jan. 1881.

A promissory note, or note of hand, is a written instrument in which one person promises to pay another a sum of money at a specified time. Thus:

£600. *London, 1st January, 1881.*

Three months after date, I promise to pay C. D. or order Six hundred pounds, value received.

A. B.

Here A. B. engages to pay to C. D. or to his order £600 at the end of 3 months from 1st Jan. 1881.

341. A bill of exchange or a promissory note always runs 3 days beyond the time specified, and these three days are called *days of grace*. Thus a bill drawn on 1st Jan. at 2 months is nominally due on 1st March, but really on 4th March. Moreover, calendar months are always reckoned, so that a bill at 3 months, whether drawn on 30th or 31st Jan., is nominally due on the 30th April, and really on 3rd May.

If now the holder of a bill wishes to realize it, he presents it to a banker or bill-discounter, and if the banker or bill-discounter be satisfied of the credit of the parties to the bill, he discounts it; that is, he pays the sum specified on the bill, deducting discount for the time it has still to run. But with bankers and bill-discounters, discount is the *interest* of the sum specified, whereas, properly speaking, it is the interest of the present worth of that sum (336, 2). And as the present worth of a sum due at a future time is less than the sum itself, the *true* discount is less than the banker's or *mercantile* discount; and therefore the banker obtains a small advantage.

Ex. A bill for £343. 15. 6 is drawn on 15th April at 4 months, and discounted on 8th May at $4\frac{3}{4}$ per cent.; how much did the holder receive? (See Art. 331, Remarks 5 & 6.)

May 23	
June 30	
July 31	
Aug. 18	
102	

The bill is really due on 18 Aug., and therefore when it was discounted it had to run from 8 May to 18 Aug., or 102 days: the discount, therefore, on this bill is the *interest* of £343. 15. 6 at $4\frac{3}{4}$ per cent. for 102 days.

```
              £.
           343·775
                 9
          ________
          3093·975
  ½ |      171·887
          ________
          3265·862
               102
          ________
          6531 724
         326586 2
          ________
73000 ) 333117·924
         111039
          11104
           1110
        ________
        4·56370 = £4. 11. 3¼ discount,
```

and 339·2113 = £339. 4. 2¾ paid to holder of bill.

Exercise 58.

1. A bill is drawn for £33. 15*s.* on July 17th at 2 months, and discounted Aug. 11th at 3⅝ per cent.; how much did the holder receive?

2. Find the discount on a bill for £843. 12. 6 drawn Dec. 18th at 6 months and discounted Jan. 26th at 5½ per cent.

3. What does a bill-discounter give as the present worth of a bill for £562. 2. 6 drawn Sept. 4th at 5 months and discounted the same day at 6½ per cent.? How much is the result less than the *true* present worth?

4. A bill is drawn for £321. 4. 3 on Dec. 31st at 2 months and discounted Jan. 14th at 4⅝ per cent.: how much is charged for discount? By how much does the discount charged exceed the *true* discount?

5. What deduction does a banker make in discounting a bill for £7716. 6. 9 drawn Oct. 10th at 9 months and discounted March 15th at 6¼ per cent.? Find also the true discount.

6. How much does a banker give as the present worth of a bill for £5253. 8. 6 drawn Nov. 6th at 10 months and discounted by him on Feb. 21st at 4½ per cent.? Find also the *true* present worth.

7. Find the discount charged in discounting a bill for £58. 4. 3 drawn April 9th at 7 months and discounted June 19th at 5¼ per cent. Find also the true discount.

8. Find the present worth of a bill for £657. 2. 6 drawn Sept. 24th at 4 months and discounted Dec. 12th at 6¼ per cent. How much is this less than the true present worth?

9. A bill is drawn for £1687. 5s. on March 31st at 3 months and discounted same day at $5\frac{1}{4}$ per cent.; how much is received, and what is the true present worth?

10. On the 31st Oct. a bill is drawn at 6 months for £309. 15. 4 and discounted Jan. 27th at 7 per cent.; what was charged for discount, and how much does this charge exceed the true discount?

COMPOUND INTEREST.

342. In COMPOUND INTEREST, the interest of each period is added to its principal, and the amount forms a new principal for the next period.

The period is always understood to be a year, unless the contrary is stated; but it may be half a year, or a quarter, or a month,...

CASE I. *Having given the principal, rate per cent., and time (number of periods), to find the interest or amount at compound interest.*

From the definition of compound interest we proceed thus:—

Find the amount of the given principal for one period at simple interest; this amount is the principal for the second period. Find the amount of this second principal in the same manner; and continue the process till the amount for the last period has been found. This last amount is the amount required; and if we subtract from it the given principal we obtain the interest.

Remark 1. If there are more than 2 periods, employ decimals; and as we only wish the result to be correct up to the third place (275), we need not retain more than 4 places, or, if there are many periods, 5.

Remark 2. We may multiply by 3, and divide by 100 at the same time, if we set down the figures of the product two places farther to the right; thus

$$\begin{array}{r} 85{\cdot}7463 \\ \underline{2{\cdot}5724} \end{array}$$

where we neglect the figures *beyond* the fourth place, but before we set down *in* the fourth place, we add the nearest ten (152) from the preceding figure.

Ex. 1. Find the compound interest of £450. 16. 9 for 3 years at $4\frac{1}{4}$ per cent.

$$
\begin{array}{r|rl}
 & \pounds. & \\
 & 450\cdot8375 & \\
 & 18\cdot0335 & \rbrace \text{ 1st year's interest} \\
\frac{1}{4} & 1\cdot1270 & \\
\hline
 & 469\cdot9980 & \text{amount in 1 year.} \\
 & 18\cdot7999 & \\
\frac{1}{4} & 1\cdot1749 & \\
\hline
 & 489\cdot9728 & \text{amount in 2 years.} \\
 & 19\cdot5989 & \\
\frac{1}{4} & 1\cdot2249 & \\
\hline
 & 510\cdot7966 & \text{amount in 3 years.} \\
 & 450\cdot8375 & \text{principal.} \\
\hline
 & 59\cdot9591 & = \pounds59.\ 19.\ 2\tfrac{1}{4} \text{ interest.}
\end{array}
$$

Ex. 2. Find the amount at compound interest of £87. 3. 6 for $1\frac{1}{2}$ years at $4\frac{3}{4}$ per cent. per annum, payable half-yearly.

Here there are 3 periods of half-a-year each, and the rate per cent. *per period* is $\frac{1}{2}$ of $4\frac{3}{4}$ or $2\frac{3}{8}$.

$$
\begin{array}{r|rl}
 & \pounds. & \\
 & 87\cdot1750 & \\
 & 1\cdot7435 & \\
\frac{3}{4} \mid \frac{1}{4} & \cdot2179 & \rbrace \text{ interest for 1st period.} \\
\frac{1}{4} \mid \frac{1}{2} & \cdot1089 & \\
\hline
 & 89\cdot2453 & \text{amount in 1 period.} \\
 & 1\cdot7849 & \\
 & \cdot2231 & \\
 & \cdot1115 & \\
\hline
 & 91\cdot3648 & \text{amount in 2 periods.} \\
 & 1\cdot8273 & \\
 & \cdot2284 & \\
 & \cdot1142 & \\
\hline
 & 93\cdot5347 & \text{amount in 3 periods or } 1\tfrac{1}{2} \text{ years.}
\end{array}
$$

∴ amount reqd = £93. 10. $8\frac{1}{4}$.

Ex. 3. Find the compound interest of £45. 12. 6 for $3\frac{1}{4}$ years at $3\frac{3}{4}$ per cent. per annum, payable yearly.

Having found the amount for 3 periods of 1 year each, we have still to find the interest of the remaining $\frac{1}{4}$ year; this is done either

by considering the ½ year as a new period and finding its interest at 1¾ per cent., or by finding the interest of the next full period and taking ½ of it :—we take the first method.

	£.	
	45·6250	
	1·3688	
$\frac{1}{6}$	·2281	
	47·2219	
	1·4166	
$\frac{1}{6}$	·2361	
	48·8746	
	1·4662	
$\frac{1}{6}$	·2443	
	50·5851	amount in 3 years.
	·5058	int. for last ½ year.
½	·2529	
½	·1264	
	51·4702	amount in 3½ years.
	45·6250	principal.
	5·8452	interest reqd.

∴ interest reqd = £5. 16. 10¾.

Ex. 4. Find the amount at compound interest of £75. 14. 9 for 2 years 7 months, 21 days at 4¼ per cent.

Having found the amount for 2 years, we shall find the interest for the remaining 7 months 21 days, by finding the interest for the 3rd year, and taking aliquot parts; thus—

	£.	
	75·7375	
	3·0295	
$\frac{1}{16}$	·1893	
	78·9563	
	3·1582	
$\frac{1}{16}$	·1973	
	82·3118	amount in 2 yrs.
	2·2445	int. for 7 mo. 21 da.
	84·5563	amount reqd.

		£.	
		82·3118	prin. for 3rd yr.
		3·2924	
	$\frac{1}{16}$	·2057	
		3·4981	int. for 3rd yr.
6 mo.	½	1·7490	
1 mo.	$\frac{1}{6}$	·2915	
15 da.	½	·1457	
6 da.	$\frac{2}{5}$	·0583	
		2·2445	

∴ amount reqd = £84. 11. 1½.

Ex. 5. Find the difference between the simple and compound interest of £549. 17. 6 at 5½ per cent. in 3 years.

	£.	
	549·8750	
	27·4938	} = 30·2431 simple interest for 1 year.
$\frac{1}{10}$	2·7493	3
	580·1181	90·7293 .... 3 years.
	29·0059	
	2·9006	
	612·0246	
	30·6012	
	3·0601	
	645·6859	
	549·875	
	95·8109	compound interest.
	90·7293	simple ....
	5·0816	difference = £5. 1. $7\frac{1}{2}$.

343. CASE II. *Having given the interest or amount, rate per cent., and time, to find the principal.*

1st. Let the interest be given. Find the interest of £1 at the given rate for the given time; then, since the interest is directly proportional to the principal, we say

its interest : given interest = £1 : principal reqd;

therefore the principal is found *by dividing the given interest by the interest of £1 at the given rate for the given time.*

2nd. Let the amount be given. In like manner, the principal is found *by dividing the given amount by the amount of £1 at the given rate for the given time.*

PRESENT WORTH AND DISCOUNT.

344. We find the present worth and discount at compound interest thus:—

Find what £1 amounts to at the given rate for the given time; the present worth of this sum (amount) due at the end of the given time is £1; and since the present worth is directly proportional to the sum due, we say

this sum : sum due = £1 : present worth reqd;

therefore the present worth is found *by dividing the sum due by the*

amount of £1 at the given rate for *the given time.* The discount is found by subtracting the present worth from the sum due.

Ex. 1. What sum of money must be put out at compound interest for 3 years at 4 per cent. for the interest to come to £32. 15*s.*?

```
£.
1·00                     ·1,2,4,8,6,4 ) 32·7500 ( 262·285
   4                                   7·77720
1·04                                    28536
  416                                    3563
1·0816                                   1066
  43264                                    68
1·124864                                    6
1
·124864 int. on £1.       ∴ principal reqd = £262. 5. 8½.
```

Ex. 2. Find the discount on £324. 16. 9 due 4 years hence at 3½ per cent. compound interest.

```
    £.
    1·000            1·1,4,7,5,2,2 ) 324·8375 ( 283·0774 present w.
      30                             95 3331
½      5                              3 5313
   1·03500                              887
     3105                                84
     5175                                 4
   1·071225                               0
     32137
     5356             £324 . 16 . 9  sum due.
   1·108718             283 .  1 . 6½ present worth.
     33261               41 . 15 . 2½ discount.
     5543
   1·147522 amount of £1.
```

EXERCISE 59.

Find the amount at compound interest of

1. £225. 10. 0 for 2 years at 3 per cent.
2. £7853. 16. 8 ... 3 .... 5 ....
3. £45. 18. 9 ... 4 .... 7 ....
4. £756. 3. 4 ... 2 .... 3¼ ....
5. £653. 5. 0 ... 3 .... 2½ ....
6. £2554. 12. 9 ... 3 .... 6¾ ....

Find the compound interest of

7. £325. 4. 0 for 3 years at 4 per cent.
8. 19 guineas ... 5 .... 8 ....
9. £97. 10. 6 ... 4 .... $4\frac{1}{8}$ ....
10. £87. 13. $8\frac{1}{2}$ ... 4 .... $2\frac{3}{4}$ ....
11. £1627. 15. 6 ... 3 .... $6\frac{5}{8}$ ....
12. £157. 14. 8 ... 6 .... $3\frac{7}{8}$ ....
13. £186. 14. 9 for $2\frac{1}{4}$ years at 6 per cent. payable half-yearly.
14. £646. 18. $2\frac{1}{2}$ ... $1\frac{3}{4}$ .... 8 .... quarterly.
15. £850. 2. $6\frac{3}{4}$ ... 3 .... $4\frac{3}{4}$ .... half-yearly.
16. £2350. 5. 9 ... 2 .... $3\frac{1}{4}$ .... quarterly.
17. £7256. 3. 4 for $2\frac{1}{3}$ years at $3\frac{1}{2}$ per cent. per annum.
18. £917. 5. 9 for $2\frac{3}{4}$ years at $4\frac{3}{4}$ per cent. per annum.
19. £439. 18. 4 for 4 years 5 months at $5\frac{3}{4}$ per cent.
20. £2352. 14. 8 for 2 years 10 mo. 15 da. at $6\frac{1}{4}$ per cent.

21. Find the difference between the simple and compound interest of £1750. 10*s.* for 3 years at $5\frac{1}{4}$ per cent.

22. *A* and *B* each lend £787. 15*s.* for 5 years at $7\frac{1}{2}$ per cent., the former at simple and the latter at compound interest: find the difference between the amounts they will receive at the end of the given time.

23. What is the difference between the simple interest of 1000 guineas for 4 years at $3\frac{3}{4}$ per cent., and the compound interest of the same sum for the same time at $3\frac{1}{2}$ per cent.?

24. Find the difference between the simple and compound interest of £3333. 6. 8 for $3\frac{1}{2}$ years at $3\frac{1}{2}$ per cent.

25. The difference between the simple and compound interest of a certain sum of money for 3 years at $4\frac{1}{2}$ per cent. is £8. 13. $7\frac{3}{8}$: find the sum.

26. A person at the beginning of each year lays aside £280, and employs the money at $3\frac{1}{2}$ per cent. compound interest: how much will he be worth at the end of 5 years?

27. The population of a city is 765240 and its annual increase is at the rate of 2·7 per cent.: what will be the number of its inhabitants at the end of 5 years?

28. The population of England and Wales in April, 1871, was 22704108, and the annual increase was 1·24 per cent.; what would be the population, estimated at this rate, in April, 1875?

29. What is the difference between the simple and compound interest of £517. 17. 6 for 2 years 9 mo. 25 days at $4\frac{3}{8}$ per cent.?

30. A banker borrows money at $3\frac{1}{2}$ per cent. per annum, and pays the interest at the end of the year; he lends it out at 5 per cent. per annum payable quarterly, and receives the interest at the end of the year; by this means he gains £200 a year: how much money does he borrow?

31. What sum will amount to £405. 3. $4\frac{1}{2}$ in 4 years at 5 per cent. compound interest?

32. What sum of money must be paid now in order to receive £360. 10*s*. two years hence, allowing $3\frac{1}{2}$ per cent. compound interest?

33. *A* offers £80000 for an estate; *B* offers £95000 to be paid at the end of 4 years. Which is now the better offer, and by how much, allowing $4\frac{1}{2}$ per cent. compound interest?

34. What principal put out at compound interest for 3 years at $4\frac{1}{6}$ per cent. will amount to £647. 15*s*.?

35. Find the discount on £1250 due 4 years hence at $4\frac{1}{2}$ per cent. compound interest.

36. What is the present worth of a legacy of £2350 to be paid to a person on his coming of age, and who is now 18, reckoning $3\frac{3}{4}$ per cent. compound interest?

37. What sum of money will in $1\frac{1}{4}$ years amount to £245. 5*s*. at $5\frac{1}{2}$ per cent. compound interest, payable quarterly?

38. Find the discount on £2450. 18. 9 due $3\frac{1}{2}$ years hence at $3\frac{3}{4}$ per cent. compound interest.

39. £10000 is due at the end of 4 years: find the difference between its present worth calculated at $5\frac{1}{2}$ per cent. simple, and $5\frac{1}{2}$ per cent. compound interest.

40. What sum of money ought to be paid now in order to receive £365 at the end of each year for the next 3 years, allowing compound interest at the rate of $4\frac{1}{2}$ per cent.?

STOCKS.

345. When the English Government wishes to raise a sum of money which cannot be met by the annual revenue, it usually sells Annuities of £3, in sufficient quantities to realize the sum required. These Annuities are payable half-yearly, and can be

transferred, wholly or in part, from one person to another: the Government reserving to itself the right of redeeming them at £100 each whenever it pleases, but allowing no corresponding right of redemption to the annuitant. Strictly speaking therefore, the Government does not *borrow* money, nor raise a *loan:* what it does, is to incur the liability of paying regularly every half-year the amount of these Annuities. The price such an Annuity will realize depends on a great variety of circumstances; it has ranged between £47⅝ and £107, and at the present time, May 1881, is about £102.

346. When Government first issued Annuities, they were for *limited* periods, and a duty or tax was set apart as a *fund* for their complete discharge, and thus *the Funds* meant the duties or taxes set apart for the payment and redemption of Government Annuities. But when Government began to issue Annuities for *unlimited* periods, charging their payment upon the annual revenue, and setting aside no fund for their redemption, *the Funds* came to mean the sum of money which Government would require to redeem these Annuities at £100 each, although no fund whatever had been set apart for their redemption, and to invest money in the Funds, to purchase any portion of that sum.

347. The capital of public companies, as of the Bank of England, the East India Company, &c., is called *Stock*, and the division of profits at the end of each half-year is called the *dividend.* In imitation of this language, the capital required to redeem the various Government Annuities is called *the Stocks*, and the half-yearly payment of the Annuities, although it is invariable, is called a *dividend.*

Hence every Annuity represents £100 Stock, or £100 in the Funds: and so for any number of Annuities and portions of an Annuity.

348. When the price of £100 Stock, or of an Annuity, is exactly £100, it is said to be at *par*, if below £100 at a *discount*, and if above £100 at a *premium.*

349. The most important of the Stocks are the following:

(1) Consolidated Annuities or Consols, so called from the consolidation of the stock of various Annuities into a joint 3 per cent. stock. They amount to about £395,000,000 stock.

(2) Reduced Annuities, so called because they have been reduced to a £3 Annuity from a higher Annuity: they amount to about £105,000,000 stock.

(3) New three per cent. Annuities, which have originated from the conversion of a higher Annuity to a 3 per cent. Annuity: they amount to about £196,000,000 stock.

Besides these, there is a small amount of new $2\frac{1}{2}$ per cent. Annuities, and still smaller amounts of new $3\frac{1}{2}$ per cent., and new 5 per cent. Annuities.

350. Consols are paid half-yearly on 5 Jan. and 5 July: Reduced Annuities and New Three per cent. Annuities on 5 April and 5 Oct.: the price of the two latter ought therefore to be always the same, but ought to differ from that of Consols either in excess or defect by $\frac{1}{4}$ of £3, or £$\frac{3}{4}$: but owing to the comparative scarcity of Consols on the Stock Exchange, their price is adventitiously higher than that of New or Reduced.

351. Consols and other Stocks are usually quoted on the Exchange between two prices, for example,

3 per Cent. Consols	$101\frac{1}{2}$ to $101\frac{5}{8}$,
Consolidated Bank	$3\frac{1}{8}$ ... $3\frac{3}{8}$ prem.,
London and North-Western Railway	$159\frac{7}{8}$ ... 160,

and these figures express that buyers are bidding at the first price, and that sellers are offering at the second price: and the sale is effected according to the temper of the market at one of the two prices or at an intermediate one.

The purchase and sale of Government Stock is made through a broker, who charges at the rate of £$\frac{1}{8}$ or 2*s.* 6*d.* per cent. upon the amount of Stock he buys or sells: hence, if the broker buys Consols at $92\frac{5}{8}$ the actual buyer will give $92\frac{5}{8}+\frac{1}{8}$ or $92\frac{3}{4}$, and if the broker sells at $92\frac{1}{2}$ the actual seller will receive

$92\frac{1}{2} - \frac{1}{8}$ or $92\frac{3}{8}$. The broker's charge on transactions in Foreign, Railway and other Stock is not uniform: but is usually a percentage on the proceeds.

Remark. When in any question we wish the broker's charge to be allowed, it will be followed by the letter B.

352. All questions in Stocks may be solved by Reduction to the Unit, or by Rule of Three; as may be seen from the following Examples.

Ex. 1. When the 3 per cent. Consols are offered at $90\frac{5}{8}$, how much stock can be bought with £825? B.

Here $90\frac{5}{8}+\frac{1}{8}$ or $90\frac{3}{4}$ will purchase £100 stock, at the same rate how much stock will £825 purchase? therefore we have

$$£90\tfrac{3}{4} : £825 = £100 \text{ stock} : \text{stock reqd};$$

$$\therefore \text{stock reqd} = £100 \times 825 \times \frac{4}{363} = £909.\ 1.\ 9\tfrac{9}{11}.$$

Ex. 2. How much must be given for £1750 stock in the $3\frac{1}{2}$ per cents. when the price is $96\frac{1}{4}$? B.

Here £100 stock can be bought for $96\frac{1}{4}+\frac{1}{8}$ or $96\frac{3}{8}$, at the same rate what can £1750 stock be bought for?

$$\therefore £100 \text{ stock} : £1750 \text{ stock} = £96\tfrac{3}{8} : \text{cost reqd};$$

$$\therefore \text{cost reqd} = £\frac{771}{8} \times \frac{1750}{100} = £1686.\ 11.\ 3.$$

Ex. 3. How much will be received from the sale of £2450. 10 Stock, when the quotation is £$96\frac{1}{2}$ for sale? B.

Here £100 stock realizes $96\frac{1}{2}-\frac{1}{8}$ or £$96\frac{3}{8}$, at the same rate what will £2450. 10 stock realize?

$$\therefore £100 \text{ stock} : £2450.\ 10 \text{ stock} = £96\tfrac{3}{8} : \text{money realized}.$$

Ex. 4. What amount of India Five per Cent. Stock at $111\frac{5}{8}$ must be sold to realize £1776. 15*s.*?

Here £100 stock realizes £$111\frac{5}{8}$, at the same rate how much stock will be required to realize £1776. 15?

$$\therefore £111\tfrac{5}{8} : £1776\tfrac{3}{4} = £100 \text{ stock} : \text{stock reqd}.$$

Ex. 5. What rate per cent. is obtained on money, invested in the Two-and-a-half per Cents. at $74\frac{5}{8}$? B.

Here $74\frac{5}{8}+\frac{1}{8}$ or £$74\frac{3}{4}$ produces £$2\frac{1}{2}$ per annum, at the same rate what will £100 produce?

$\therefore$ £$74\frac{3}{4}$: £100 = £$2\frac{1}{2}$: rate per cent. reqd.

Ex. 6. At what price would a person have to purchase Three-and-a-half per Cents. to get 4 per cent. for his money? B.

Here, by investing £100, he wishes to get £4 per annum, at the same rate how much must he invest to get $3\frac{1}{2}$?

$\therefore$ £4 : £$3\frac{1}{2}$ = £100 : £$87\frac{1}{2}$ price reqd of $3\frac{1}{2}$ per cent. stock.

But £$87\frac{1}{2}$ includes the broker's charge of £$\frac{1}{8}$, therefore the quotation must be £$87\frac{3}{8}$.

Ex. 7. What income is derived from £3775. 12. 6 Stock in the New Three-and-a-half per Cents.?

$$\begin{array}{r|r} & £. \\ & 3775{\cdot}625 \\ & 3 \\ \hline & 11326875 \\ \frac{1}{6} & 1887812 \\ \hline & 132{\cdot}14687 \end{array}$$

$\therefore$ Income = £132. 2. $11\frac{1}{4}$.

Ex. 8. A person invests £1545 in India Four-per-Cent. Stock at $100\frac{3}{4}$; find the amount of his half-yearly dividend.

Here £$100\frac{7}{8}$ produced a yearly Dividend of £4, or a half-yearly Dividend of £2, at the same rate what Dividend will £1545 produce?

$\therefore$ £$100\frac{7}{8}$: £1545 = £2 : half-yearly Dividend reqd.

Ex. 9. How much money must a person invest in the Three per Cents. at $92\frac{1}{4}$ to obtain an annual income of £187. 10? B.

To obtain an income of £3, he must invest £$92\frac{1}{4}$ + £$\frac{1}{8}$ or £$92\frac{3}{8}$, at the same rate how much must he invest to obtain £187. 10?

$\therefore$ £3 : £$187\frac{1}{2}$ = £$92\frac{3}{8}$: money reqd.

Ex. 10. A person invests £850 in Consols when they are at $89\frac{1}{2}$, and sells out when they are at $93\frac{3}{8}$: what is his gain? B.

Here an Annuity which costs £$89\frac{5}{8}$ is sold for £$93\frac{1}{4}$; therefore on £$89\frac{5}{8}$ there is a gain of £$3\frac{5}{8}$; at the same rate what will be the gain on £850?

$\therefore$ £$89\frac{5}{8}$: £850 = £$3\frac{5}{8}$: gain reqd.

Ex. 11. A person buys Railway Stock at 89¾, and sells out at 103½, and clears £385: how much money did he invest?

Here what cost him £89¾ he sells for £103½, and therefore, on investing £89¾, he gains £13¾;

$$\therefore £13\tfrac{3}{4} : £385 = £89\tfrac{3}{4} : \text{money req}^{\text{d}} \text{ to be invested.}$$

Ex. 12. Whether is it better to invest in the Three per Cents. at 92⅞, or in the Four per Cents. at 108⅜? B.

Here £93 invested in the 3 per cents. produces yearly £3: how much would £93 invested in the 4 per cents. at 108½ produce? The answer is got from the following statement:

$$108\tfrac{1}{2} : 93 = £4 : \text{what } £93 \text{ produces in 4 per cents.};$$

$$\therefore \text{what } £93 \text{ produces in 4 per cents.} = £4 \times 93 \times \tfrac{2}{217} = £3\tfrac{3}{7};$$

$$\therefore \text{the 4 per cents. is the better investment.}$$

Ex. 13. Find the difference per cent. in income between investing in the Three per Cents. at 92⅞, and in the Four per Cents. at 108⅜? B.

$$£93 \text{ invested in the 3 per cents. produces yearly } £3;$$

$$\therefore £1 \quad \dots\dots\dots\dots \quad £\frac{3}{93} \text{ or } £\frac{1}{31}.$$

$$\text{Similarly } £1 \quad \dots \text{ 4 per cents. } \dots \quad £\frac{4}{108\frac{1}{2}} \text{ or } £\frac{8}{217}.$$

$$\therefore \text{difference in income on } £1 = £\frac{8}{217} - £\frac{1}{31} = £\frac{1}{217},$$

$$\text{and difference in income per cent.} = £\frac{100}{217} = £0.\ 9.\ 2\tfrac{1}{2}\tfrac{86}{217}.$$

Ex. 14. A person transfers £3622. 10 Stock from the Three per Cents. at 92¼ to the Four per Cents. at 100⅝: find how much of the latter Stock he will hold, and the alteration in his income.

The quantity of Stock held is *inversely proportional* to the price;

$$\therefore £100\tfrac{5}{8} : £92\tfrac{1}{4} = £3622\tfrac{1}{2} : \text{Stock in 4 per cents.};$$

$$\therefore \text{Stock in 4 per cents.} = £\frac{7245}{2} \times \frac{369}{4} \times \frac{8}{805} = £3321.$$

$$\text{Now income from 3 per cents.} = £3622\tfrac{1}{2} \times \frac{3}{100} = £108.\ 13.\ 6,$$

$$\text{and} \quad \dots \text{ 4 per cents.} = £3321 \times \frac{4}{100} = £132.\ 16.\ 9\tfrac{1}{2}\tfrac{2}{5};$$

$$\therefore \text{alteration in income} = £24.\ 3.\ 3\tfrac{1}{2}\tfrac{2}{5}.$$

Ex. 15. The Three per Cent. Consols are paid on 5 Jan.; what rate per cent. is obtained in buying Consols on 23 April at $93\frac{5}{8}$?

From 5 Jan. to 23 April is 108 days: and

Int. on £100 for 108 days at 3 per cent. is 17*s.* 9*d.*

Now	£93. 15. 0	Price of Consols including broker's charge;
and	0. 17. 9	Growing Dividend;
∴	92. 17. 3	Net Price.

Hence £92. 17. 3 : £100 = £3 : Rate per cent.

or Rate per cent. = £3. 4. $7\frac{1}{4}$.

Exercise 60.

How much Stock can be purchased for

1. £2317. 4. 9 in the Three per Cents. at $95\frac{5}{8}$? B.
2. £12546. 13. 8 in India Stock at 252?
3. £1756. 9. 6 in Italian Five per Cents. at $65\frac{5}{8}$?
4. £6451. 3. 6 in Bank Stock at $217\frac{1}{2}$?
5. £630. 17. 6 in L. Railway Stock at $27\frac{1}{4}$ below par?
6. £550. 13. 4 in M. Railway Stock at $8\frac{1}{2}$ above par?

How much money can be obtained from the sale of

7. £9763. 6. 8 Consolidated Annuities at 97? B.
8. £7925. 8. 4 Reduced Annuities at $98\frac{3}{8}$? B.
9. £756. 18. 9 New South Wales Stock at $106\frac{3}{8}$?
10. £1250. 10. 6 Stock in the Dutch Four per Cents. at $64\frac{1}{2}$?

How much money must be given for the purchase of

11. £673. 6. 8 Stock in the Russian Five per Cents. at $90\frac{1}{2}$?
12. £5550 Railway Stock at $97\frac{5}{8}$?
13. £3257. 15. 6 Consols at $91\frac{3}{8}$? B.
14. £10000 Bank Stock at $211\frac{3}{8}$?

15. What amount of Consols must be sold to realise £3525. 2. 9 when the sale price is $91\frac{3}{8}$? B.

16. What amount of Railway Stock must be sold, when the quotation is $12\frac{3}{8}$ above par, to realize £5653. 10. 6?

What interest per cent. per annum is obtained from investing money in

17. Three per Cent. Consols at $91\frac{3}{4}$? B.

18. Dominion of Canada Five per Cent. Stock at $105\frac{5}{8}$?

19. L. Railway Stock at $117\frac{1}{4}$, whose dividends are at the rate of $5\frac{1}{2}$ per cent. per annum?

20. M. Railway Stock at $88\frac{3}{8}$, whose half-yearly dividends are at the rate of $2\frac{1}{4}$ per cent.?

At what price must a person purchase

21. Three per Cent. Consols to obtain $3\frac{3}{4}$ per cent. for his money?

22. Italian Five per Cents. to get $8\frac{1}{3}$ per cent. per annum on the money he invests?

23. Two-and-a-half per Cent. Stock to obtain $3\frac{5}{8}$ per cent. on any investment he may make in them?

24. Bank Stock paying half-yearly dividends at $4\frac{1}{4}$ per cent. to get £5. 6. 3 per cent. per annum for his money?

25. A person holds £3852. 12. 6 Stock in the L. and Y. Railway: find his half-yearly dividend calculated at the rate of $7\frac{3}{4}$ per cent. per annum.

26. What annual income does a person derive from £17835. 18. 9 Stock in the Three per Cents. and £3247. 8. 11 Stock in the New Three-and-a-Half per Cents.?

What income per annum will a person obtain from investing

27. £2350 in the Three-and-a-Half per Cents. at $97\frac{5}{8}$? B.

28. £27643. 17. 6 in the Three per Cent. Consols at $91\frac{5}{8}$? B.

29. £3923. 15*s*. in the Argentine Six per Cents. at $64\frac{1}{2}$?

30. £2175. 10*s*. in G. Railway Stock at $89\frac{1}{2}$, whose dividends are at the rate of $4\frac{1}{2}$ per cent. per annum?

31. £3614. 19. 6 in L. and Y. Railway Stock at $159\frac{1}{4}$, whose dividends are at the rate of $7\frac{3}{4}$ per cent. per annum?

32. 78765 fr. 85 c. in Three per Cent. French Rentes at 63 fr. 35 c.?

How much money must a person lay out in

33. Three per Cent. Consols at $90\frac{1}{8}$ to secure an annual income of £146. 10*s*.? B.

34. Three-and-a-Half per Cent. Reduced Annuities at $96\frac{5}{8}$ to obtain an income of 100 guineas per annum? B.

35. L. Railway Stock at $115\frac{1}{2}$, paying half-yearly dividends at the rate of $2\frac{3}{8}$ per cent., to obtain an income of £155. 13. 4 a year?

36. Egyptian Seven per Cents. at $78\frac{3}{4}$ to get an annual income of £728. 8. 9?

37. Bank Stock at $250\frac{3}{4}$, paying annual dividends at the rate of 10 per cent., to derive an income of £545. 8. 9 a year?

38. If I lay out £1359. 15*s*. in the purchase of Consols at $92\frac{3}{8}$ and afterwards sell at $94\frac{1}{4}$, what profit shall I make? B.

39. A person expended £2653. 10*s*. in the purchase of New Three-and-a-Half per Cents. at $97\frac{3}{8}$, and after a time sold out at $96\frac{1}{8}$: find his loss. B.

40. A person laid out £2550 in a Three-and-a-Half per Cent. Stock at 91, and after receiving the half-year's dividend he sold out at $90\frac{7}{8}$: how much did he gain?

41. A person held £3559. 10*s*. Stock in the L. Railway, having bought in at $117\frac{1}{2}$: he received the half-year's dividend at the rate of $5\frac{1}{2}$ per cent. per annum, and then sold out at $119\frac{1}{8}$: how much money has he made?

42. A person bought Dutch Four per Cent. Stock at $65\frac{1}{2}$, and sold it when the price had risen to $69\frac{1}{8}$, thereby clearing £125. 12. 6: how much money did he lay out?

43. A person bought M. Railway Stock at $88\frac{3}{4}$, and after receiving the half-year's dividend at the rate of $4\frac{1}{2}$ per cent. per annum sold out at $93\frac{5}{8}$ and made a profit of £142. 10*s*.: how much Stock did he buy?

44. If a person invest £12654 in South Australian Stock at $99\frac{3}{4}$, at what price must he sell to gain £1581. 15*s*.?

45. If a person invest £25350 in the Three per Cent. Consols at $92\frac{5}{8}$, at what price must he sell out after receiving the dividend to make a profit of £250?

46. Whether is it better to invest in the Three per Cents at $89\frac{1}{2}$, or in the Three-and-a-Half per Cents. at 95? B.

47. Whether is it better to invest in the Two-and-a-Half per Cents. at $65\frac{5}{8}$, or in the Four-and-a-Half per Cents. at $103\frac{3}{8}$?

48. Compare the two investments: Three-and-a-Quarter per Cent. Stock at $92\frac{5}{8}$ and Three per Cent. Stock at $87\frac{7}{8}$.

49. Whether would it be better to invest in L. Railway Stock at $117\frac{5}{8}$ whose dividends are $5\frac{1}{2}$ per cent. per annum, or in M. Railway Stock at $89\frac{5}{8}$ whose dividends are $4\frac{1}{2}$ per cent. per annum? and compare the two investments.

50. Which is the better investment per cent. per annum, and by how much: Bank Stock at $221\frac{5}{8}$ paying dividends at the rate of 8 per cent., or Three per Cent. Consols at $93\frac{7}{8}$?

51. A person holds £3545 Consolidated Annuities: if he sells out at $92\frac{1}{8}$ and invests the proceeds in the Two-and-a-Half per Cents. at $75\frac{5}{8}$, how much of the latter Stock will he hold?

52. A person held £6820. 17. 6 Stock in the Three-and-a-Half per Cents.; he sold out at $92\frac{1}{8}$ and transferred the proceeds to Railway Stock at $117\frac{1}{2}$: how much Railway Stock does he hold?

53. A person invested £5330 in the Three per Cents. at 91, and when they had risen $1\frac{3}{4}$ per cent. he sold out and invested the money in the Stock of the Dominion of Canada at $102\frac{1}{4}$: how much Canadian Stock does he hold?

54. A person invested £4950 in the Four per Cents. at $99\frac{3}{8}$, and when they had risen to par he sold out and invested the money in Consols at $6\frac{1}{4}$ discount: what amount of Consols does he hold?

55. A person laid out £749. 5*s*. in the purchase of Five per Cent. Stock at par, and after receiving the half-yearly dividend he sells out at 4 premium and invests the proceeds in C. Railway shares at $87\frac{7}{8}$: how much Railway Stock does he hold?

56. If a person transfer £3000 Stock in the Three per Cents. at $89\frac{3}{8}$ to the Three-and-a-Half per Cents. at $98\frac{1}{4}$, find what amount of the latter Stock he will hold, and the alteration in his income.

57. A person transfers £1708. 7. 6 Stock in the Four per Cents. at $102\frac{5}{8}$ to the Three per Cents. at $88\frac{1}{4}$: find the alteration in his income.

58. A person expended £3565 in the purchase of Two-and-a-Half per Cents. at $62\frac{3}{8}$, and when they had fallen to 58 he sold out and invested the money in the Four per Cents. at $96\frac{1}{4}$: find his gain or loss in income.

59. A person derived an income of £226. 19. 2 from Stock in the Four per Cents.: this Stock he sold out at par and invested the proceeds in M. Railway Stock at $148\frac{5}{8}$, whose annual dividends are at the rate of 7 per cent.: find the alteration in his income.

60. A person has an annual income of £191. 5*s*. from Stock in the Three per Cents.: if he were to sell out at $92\frac{7}{8}$ and invest the money in Five per Cent. Stock of New Zealand at 105, how much of the latter Stock would he hold, and what would be the alteration in his income?

61. What rate per cent. is obtained in buying Three per Cent. Reduced Annuities on 17 Feb. at $92\frac{1}{8}$? B. *The Dividends are paid on 5 April and 5 Oct.*

62. A man invests £8063 in the Three per Cents. at $94\frac{5}{8}$: what will be his clear income after an income-tax of 10*d*. in the pound has been deducted? B.

63. What sum must a man invest in the Three per Cents. at 9$\frac{5}{8}$, in order to have a clear income of £230 after paying an income-tax of 10*d.* in the pound?

64. What must be the price of the Three per Cents. so that by investing £32850, a man may have a clear income of £1030. 10*s.* after an income-tax of 11*d.* in the pound has been deducted?

65. When the Two-and-a-Half per Cents. are at 83$\frac{3}{4}$, what ought to be the price of the Three-and-a-Half per Cents. to give the same rate of interest?

66. A gentleman in Australia has been receiving 12 per cent. on his capital in the colony; he brings his capital home, invests it in the Three per Cents. at 94$\frac{3}{8}$, and his income in England is £2400 a year: what was his income in Australia?

67. The income derived by a legatee from money invested in his behalf in the Three per Cents. at 93$\frac{1}{2}$ is £68. 3. 6. What was the amount of the legacy?

68. A person holds £4675 Stock in the Five per Cents.: what sum must he lay out in the purchase of Four-and-a-Half per Cents. at 102$\frac{1}{4}$ so that his income from both sources may be £843. 10*s.*?

69. A person invests £5187. 10*s.* in the Three per Cents. at 83, and when the funds have risen to 84 he transfers three-fifths of his capital to the Four per Cents. at 96: find the alteration in his income.

70. A person sells out of the Three-and-a-Half per Cents. at 92$\frac{3}{4}$ and realizes £18550. If he invests two-fifths of the produce in the Four per Cents. at 96, and the remainder in the Three per Cents. at 90, find the alteration in his income.

71. A man invests £4297. 10*s.* in the Three per Cents. at 95$\frac{1}{2}$. He sells out one-third when the funds have fallen to 94, £1600 Stock when they have risen to 96$\frac{1}{4}$, and the remainder at par. What sum does he gain? If he invests the proceeds in the French Three per Cents. at 67·50, what would be the difference in his income?

72. *A* and *B* are two railway companies that pay respectively 4$\frac{1}{2}$ per cent. and 1$\frac{5}{8}$ per cent. per annum on their £100 shares. When the price of a share in *A* is 101$\frac{1}{4}$ and in *B* 32$\frac{1}{4}$, in which company is it more advantageous to invest? and what is the difference of income that would arise from the investment of £17415 in one rather than in the other?

73. Which is the better investment,—Three per Cent. Stock at 87$\frac{7}{8}$, or shares at £233 each, on each of which a dividend of £7. 13. 4 is paid annually? How much more money must be invested in one rather than in the other to produce an annual income of £460?

74. A person possesses £3200 Three per Cents. which he sells at $99\frac{7}{8}$: he invests the proceeds in railway shares at £56 a share, which shares pay 5 per cent. interest on £45, the amount paid on each share. How much is his income altered by the transaction?

75. What amount of Stock must be sold out of the Three per Cents. at $87\frac{1}{2}$ to pay the present worth of £1645. 17. 6 due 10 months hence at $3\frac{3}{4}$ per cent.?

76. If the Three per Cents. be at $92\frac{3}{4}$, and the Four per Cents. at $123\frac{1}{4}$, in which should one invest? and how much is one investing in each when the difference in income is half a crown?

77. If the French Three per Cents. be at 60 when the English are at 95, the exchange between the two countries being 25 francs to the pound, how much French Stock in francs can be bought by selling £6000 Stock out of the English funds?

78. A person derived an income of £434. 15*s.* from the Three per Cent. Consols: he sold out at $92\frac{7}{8}$ and invested the proceeds in the Four-and-a-Half per Cent. French Rentes at 98·70, the rate of exchange being 25 fr. 60 c. for £1: what income does he derive from the Rentes in francs?

79. A person having to pay £1085 at the end of 2 years invested a certain sum of money in the Three per Cent. Consols, allowing the dividends to accumulate until the payment of the debt, and also an equal sum the next year; supposing the investments to be made and the debt to be paid when Consols are at 73, what must be the sum invested on each occasion that there may be just sufficient to pay the debt at the proper time?

80. If a person invest in the Three per Cents. so as to receive 3 per cent. clear on his investment when there is an income-tax of 9*d.* in the pound, what percentage clear does he receive (1) when the income-tax is reduced to 5*d.* in the pound, and (2) when it is raised to 1*s.* in the pound?

81. In the Three per Cents. what fraction of a given amount of Stock is paid for annual interest, (1) without any deduction, and (2) after a deduction of 9*d.* in the pound for income-tax?

What is the amount of Stock for which £115. 10*s.* is paid as annual interest after 9*d.* in the £1 has been so deducted?

82. A proprietor of Three per Cent. Consols receives his half-yearly dividend and lays it out in the purchase of more Consols at 90. His next half-year's dividend is £457. 10*s.*: how much does this dividend exceed the former?

83. A person after paying an income-tax of $7d.$ in the pound has a clear income of £262. 2. 6 derived from Stock in the Four-and-a-Half per Cents.; he sells out two-thirds of this Stock at $93\frac{3}{4}$ and invests the money in L. Railway Stock at $112\frac{1}{2}$, which pays $5\frac{1}{4}$ per cent. per annum: what is now his clear income after paying the income-tax as before?

84. If the Three per Cents. be at 95, and the Government offer to receive tenders for a loan of £5,000,000, the lender to receive £5,000,000 Stock in the Three per Cents. together with a certain sum in the Three-and-a-Quarter per Cents., what sum in the Three-and-a-Quarter per Cents. ought the lender to accept?

EXCHANGE.

353. EXCHANGE, or FOREIGN EXCHANGES, is concerned with the payment of a sum of money in the currency of one country by means either directly or indirectly of an equivalent sum in the currency of another country. Thus, if A in London owes B in Paris 6754*fr.* 40*c.*, the first question is how must he go to work to discharge the debt, and the second what rate of exchange must be used in the discharge, or in other words, what relation must exist between the sovereign and the franc, the units of money of the countries, in making the payment. On both these points, the *mode of payment* and the *rate of exchange*, we shall now offer some short explanations.

354. The payment of a sum of money due abroad may be made by sending

(1) specie, that is, coined money;

(2) bullion, that is, gold or silver in bars;

(3) a bill of exchange.

If payment of a sum of money, due in France, be made by sending specie there, we must ascertain how many francs will be given for a sovereign at the French mint; and this is deduced by making a comparison between the weight and fineness of the metals composing the sovereign and the Napoleon. The result is found to be 1·2585 Napoleons or 25·170*f.* (Ex. 3 seq.), and is called the *par* of exchange: and generally—the relation between the

standard coins of two countries, as determined from their intrinsic value, is called the ***nominal rate of exchange***, or ***par of exchange***.

The value of any weight of French gold coin is $15\frac{1}{2}$ times the value of the same weight of silver coin; but this relation between gold and silver does not always obtain in the open market: hence we often see that gold is at so many millièmes, or at so much per mille premium; and in sending specie to France this premium must be taken into account. Thus, at the par of exchange £1 is equal to $25{\cdot}170$*f.*, but when gold is at 4 per mille premium £1 is equal to $25{\cdot}170$*f.* + $\cdot 101$*f.*, or to $25{\cdot}271$*f.*

In England the gold coin and the metal composing it are of the same value, and therefore a person only consults his convenience whether he exports gold in specie or in bar; but it is otherwise with silver. Coined silver with us is worth 66*s.* per lb., or 5*s.* 6*d.* an oz.: whereas its marketable value is only about 5*s.* an oz., and therefore, when silver is exported, it is always exported in bar, and never in specie.

It is only, however, under exceptional circumstances that a debt, due abroad, is paid by sending specie or bullion: the usual method is by a Bill of Exchange. When a merchant ships goods from one port to another, as from London to New York, he draws a Bill for their amount, payable in London, on the merchant in New York, who accepts the Bill. If now the exports from London to New York be equal in value to those from New York to London, the amount of bills due in London by New York will be equal to the amount due in New York by London: or the claims and the liabilities of New York in London will be equal, and the former may, by a suitable arrangement, be employed in satisfying the latter: and the same may be done with regard to the claims and liabilities of London in New York, and thus the transmission of bullion from either place to the other may be entirely avoided. The following is the usual mode of proceeding: supposing I owe 2000$ to a merchant in New York, I go to a banker who specially undertakes such transactions, and buy a bill for the given amount, payable in New York, at a rate of exchange

agreed upon between us; I transmit the bill to my creditor, who presents it to the person on whom it is drawn in New York and receives the amount.

355. As we supposed the exports from London and New York to be equal, creditors in London will be as anxious to sell bills on New York as debtors to buy them, and the *real* rate of exchange will deviate but slightly from the *par* of exchange. But suppose the exports from New York are in excess of those from London, or the *balance of trade* is in favour of New York and against London, the claims of New York in London will be in excess of its liabilities, and the London importers will give more than the par value of such bills as may be got, to avoid the cost of transmitting bullion; and on the other hand, for the same reason, the exporters in New York, not finding sufficient purchasers for all their bills on London, will sell them at less than their par value. Now—the real rate of exchange between two countries depending on the balance of trade, is called the *course of exchange*: and the course of exchange is at a *premium* or a *discount* according as it is *above* or *below* the par of exchange. Of course no merchant will give a premium or submit to a discount greater than would cover the cost of transmitting specie or bullion.

356. But suppose that the balance of trade is against London as regards New York, but in favour of London as against Hamburgh; the London merchant may find it advantageous to remit to Hamburgh, and then for Hamburgh to remit to New York: and this method is adopted when the course of exchange by this circuitous route is less than the direct course of exchange. But—the finding the course of exchange between two places, calculated from a comparison of the courses of exchange between them and one or more intervening places, is called the *Arbitration of Exchange*. If one place only intervene the Arbitration is *Simple*, and when more than one *Compound*.

357. The rates of exchange at which bills have been negociated on foreign places are published in London every Tuesday and Friday in the following form:—

COURSE OF EXCHANGE.

Paris	Short	25 $12\frac{1}{2}$ to 25 $22\frac{1}{2}$
Do.	3 ms.	25 30 ,, 25 35
Berlin	,,	20 60 ,, 20 64
Petersburg	,,	28 ,, $27\frac{3}{4}$
Vienna	,,	13 0 ,, 13 10
Lisbon	90 days	$52\frac{1}{8}$,, 52

The meaning of this abbreviated Course is:—Bills of Paris at short sight have been negociated at prices varying from 25*f.* $12\frac{1}{2}$*c.* to 25*f.* $22\frac{1}{2}$*c.* for £1, and Bills at 3 months from 25*f.* 30*c.* to 25*f.* 35*c.* for £1: Bills on Berlin at 3 months at prices from 20*m.* 60*pf.* to 20*m.* 64*pf.* for £1: Bills on St. Petersburg at 3 months at 28*d.* to $27\frac{3}{4}$*d.* for 1 rouble: Bills on Vienna at 3 months at 13*fl.* to 13*fl.* 10*kr.* for £1: and Bills on Lisbon at 90 days at $52\frac{1}{8}$*d.* to 52*d.* for 1 milreis.

In the exchange on Paris £1 is called the *fixed* price, and the varying number of francs and centimes the *variable* price; on Lisbon 1 milreis is the *fixed* price, and the number of pence the *variable* price.

MONEY TABLE.

France, Belgium, Switzerland	1 franc	= 100 centimes	= 0 . $9\frac{5}{8}$ nearly.
Italy	1 lira	= 100 centesimi	
Spain	1 peseta	= 100 centesimos	
Greece	1 drachmê	= 100 lepta	
German Empire. .	1 marc	= 100 pfennige	= 0 . $11\frac{3}{4}$ ...
Austria	1 florin	= 100 kreuzers	= 1 . $11\frac{3}{8}$ ...
Russia	1 rouble	= 100 copeks	= 3 . $1\frac{1}{2}$ ...
Denmark	1 kröne	= 100 öre	= 1 . $1\frac{1}{3}$ ...
Holland	1 florin	= 100 cents	= 1 . 8 ...
Portugal	1 milreis	= 1000 reis	= 4 . 6 ...
United States . .	1 dollar	= 100 cents	= 4 . $1\frac{1}{16}$ ...
India	1 rupee	= 16 annas	= 1 . $10\frac{3}{4}$ ...

Francs, lire, pesetas, and drachmai are declared by the French Monetary Convention to be of equal value, and to be interchangeable with one another in those countries where these coins are in circulation.

Ex. 1. Find the par of exchange between the U. S. gold eagle, weighing 258 grains $\frac{9}{10}$ fine, and the sovereign of which 1869 weigh 40 lbs. of gold $\frac{11}{12}$ fine.

$$\begin{aligned} \pounds\text{s reqd} &= 1 \text{ eagle,} \\ 1 &= 258 \text{ grs. \$ standard,} \\ 10 &= 9 \text{ grs. fine,} \\ 11 &= 12 \text{ British standard,} \\ 5760 &= 1 \text{ lb.} \\ 40 &= \pounds 1869; \end{aligned}$$

$$\therefore \pounds\text{s reqd} = \frac{258 \times 9 \times 12 \times 1869}{10 \times 11 \times 5760 \times 40} = 2{\cdot}054838,$$

i.e. 1 eagle $= \pounds 2{\cdot}054838...$

$\therefore$ 1 dollar gold $= \pounds{\cdot}2054838... = 4{\cdot}109676s.$
$= 4s.\ 1{\cdot}316d. \quad = 4s.\ 1\frac{3}{10}d.$ nearly,

and $\pounds 1 = \frac{10000000}{2054838}\ \$ = 4{\cdot}8665...\ \$.$

REMARK. Formerly the rate of exchange of New York on London was a *variable* number of dollars for £22. 10s., this fixed price being adopted because it was equal to 100 dollars at 4s. 6d. each; but now the rate of exchange is a *variable* number of dollars and cents for £1. The *par* of exchange is, as we have just found, 4·8665 \$ for £1.

Ex. 2. Find the relation between the sovereign and the Napoleon, as determined from the intrinsic value of the two coins :—(1) 40 lbs. British standard gold, $\frac{11}{12}$ fine, is coined into 1869 sovereigns; (2) 1 kilog. French standard gold, $\frac{9}{10}$ fine, is coined into 155 Napoleons; (3) 1 kilog. is equal to 15432 grains.

$$\begin{aligned} \text{Napoleons reqd} &= \pounds 1, \\ 1869 &= 40 \text{ lbs. British standard,} \\ 12 &= 11 \text{ lbs. fine,} \\ 1 &= 5760 \text{ grains......} \\ 15432 &= 1 \text{ kilog......} \\ 9 &= 10 \text{ kilog. French standard,} \\ 1 &= 155 \text{ Napoleons;} \end{aligned}$$

$$\therefore \text{Napoleons reqd} = \frac{40 \times 11 \times 5760 \times 10 \times 155}{1869 \times 12 \times 15432 \times 9} = 1{\cdot}261106,$$

i.e. $\pounds 1 = 1{\cdot}261106$ Napoleons $= 25{\cdot}22212$ francs
$= 25f.\ 22c.$

Ex. 3. What is the value of a sovereign in France, or at the French mint?

A person taking a kilogramme of standard gold, $\frac{9}{10}$ fine, to the French Mint will receive 155 Napoleons, or 3100 francs; but then he must pay a mintage of 6f. 70c., so that in reality he only receives 3093 f. 30 c.: hence we have

Francs reqd = 1 sovereign,
1869 = 40 lbs. British standard gold,
12 = 11 lbs. fine gold,
1 = 5760 grains......
15432 = 1 kilog......
9 = 10 kilog. French standard,
1 = 3093·3 francs;

$$\therefore \text{ francs reqd} = \frac{40 \times 11 \times 5760 \times 10 \times 3093{\cdot}3}{1869 \times 12 \times 15432 \times 9} = 25{\cdot}1\dot{6}8,$$

i.e. £1 = 25 *f.* 17 *c.*

Ex. 4. The price of standard gold is £3. 17. 10½ per oz., and in Paris it is at 4¾ per mille premium: find the rate of exchange in Paris; and if the short exchange be 25·60, find how much per cent. gold is dearer in London than in Paris.

By short exchange is meant exchange on bills at sight, or at short sight, which is usually 3 days.

Since £3. 17. 10½ per oz. is equal to £1869 per 40 lbs., we have from the last example:

£1 in Paris = 25·170 f.
4¾ per mille prem. = ·120;
∴ rate of exchange = 25·290.
But short exchange = 25·60;
∴ diff. = ·31.

Hence £1 in London will purchase 25·60 f., but in Paris only 25·29 f., therefore it is ·31 f. dearer in London than in Paris:

but 25·60 : 100 = ·31 : 1·21,

or gold is nearly 1¼ per cent. dearer in London than in Paris.

Ex. 5. Exchange £676. 17. 6 for francs, when the rate of exchange is 25*f.* 17½ *c.* for £1.

$$\begin{array}{r l}
 & \text{francs.} \\
 & 25{\cdot}175 \\
 & \underline{677} \\
 & 176225 \\
 & 176225 \\
 & \underline{151050} \\
 & 17043{\cdot}475 \\
2s.\ 6d. \text{ is } \tfrac{1}{8} & \underline{3{\cdot}146875} \\
 & 17040{\cdot}328125 = 17040f.\ 33c.
\end{array}$$

Ex. 6. Find the arbitrated rate of exchange between London and Paris when the course of exchange between London and Amsterdam is 12 florins 3¼ stivers for £1, and between Amsterdam and Paris 209¼ francs for 100 florins. 1 stiver = 5 cents.

$$\begin{aligned}
\text{Francs reqd} &= £1, \\
£1 &= 12{\cdot}1625 \text{ florins}, \\
100 &= 209\tfrac{1}{4} \text{ francs};
\end{aligned}$$

$$\therefore \text{francs reqd} = \frac{12{\cdot}1625 \times 209\frac{1}{4}}{100} = 25{\cdot}45 \text{ francs},$$

or London gives 25*f.* 45*c.* for £1.

Ex. 7. A New York merchant remits 27940 florins to Amsterdam by way of London and Paris, at a time when the exchange of New York on London is 4·88½ \$ for £1, of London on Paris is 25*fr.* 40 *c.* for £1, and of Paris on Amsterdam is 212 francs for 100 florins; ⅛ per cent. brokerage being paid in London and in Paris.

$$\begin{aligned}
\text{Dollars reqd} &= 27940 \text{ florins}, \\
100 &= 212 \text{ fr.}, \\
100 &= 100\tfrac{1}{8} \text{ fr. with brokerage}, \\
2540 &= 100\tfrac{1}{8}\ £\text{s} \quad . \quad . \quad . \quad . \quad . \\
1 &= 4{\cdot}88\tfrac{1}{2}\ \$;
\end{aligned}$$

$$\begin{aligned}
\therefore \text{Dollars reqd} &= \frac{27940 \times 212 \times 801 \times 801 \times 4885}{100 \times 100 \times 2540 \times 8 \times 8 \times 1000} \\
&= 11420{\cdot}317 \quad . \quad . \\
&= 11420\ \$\ 32c.
\end{aligned}$$

EXERCISE 61.

1. How many francs will be given in Paris for £688. 14. 8 when the course of exchange is 25 f. 42½ c. for £1?

2. How many dollars must be given for a letter of credit on London for £2346. 10s., when the exchange is 489 c. for £1?

3. Exchange £598. 16. 9 into money of the German Empire, the course of exchange being 20 m. 54 pf. for £1.

4. Exchange £1270. 13. 9 for florins and kreuzers of Vienna, the rate of exchange being 12 fl. 85 kr. for £1.

5. Reduce £1857. 14. 3 to rupees, &c. at the rate of 1s. 11⅞d. for 1 rupee.

6. How many florins and cents at Amsterdam must be given for a bill on London for £745. 3. 6, at the rate of 12 fl. 7½ c. for £1?

7. What sum of money in London must be given for a bill of 15643·25 $ on New York, when the rate of exchange is reckoned at 4·85 $ for £1?

8. Reduce 37847 lire 60 c. to Hamburgh currency, the rate of exchange being 124⅜ lire for 100 marcs.

9. A traveller goes to Paris with £57. 10s., which he exchanges for French money at the rate of 25 f. 35 c. for £1. During his stay in France he spends 830 f. 50 c., and on leaving exchanges the remainder of his French money for English at the rate of 25 f. 20 c. for £1. What sum will he receive?

10. A person on leaving England exchanged his money for French money at the rate of 25 francs for £1; and on arriving at Vienna receives 135 (paper) florins for 15 20-franc pieces: what was his loss in English money, supposing a florin to be worth 1s. 8½d.? and what was his loss in French money?

11. Some years ago to pay 18 kreuzers I gave a thaler, and received back 22 kreuzers 10 silber-groschen and half a gulden;—1 thaler was 30 silber-groschen, and 1 gulden was 60 kreuzers: how many gulden were worth 4 thalers?

12. When £1 is equivalent to 25 f. 35 c. to 20 m. 64 pf. and to 13 fl. 5 kr., what is the value of 36980 marcs in English, in French, and in Austrian money?

13. What is the arbitrated rate of exchange between Hamburgh and Paris in francs per 100 marcs, when the course of exchange between London and Paris is 25·45 francs for £1, and between London and Hamburgh 2048 pfennige for £1?

14. When the exchange between London and Paris is 25 fr. 60 c. for £1 and between Lisbon and Paris $557\frac{1}{2}$ centimes for 1 milreis, what is the value of the milreis in pence?

15. The rate of exchange between London and Petersburg is $31\frac{3}{8}d.$ for one rouble, between Vienna and Petersburg is $95\frac{5}{8}$ florins for 60 roubles, and between Paris and Vienna is $93\frac{3}{4}$ florins for 200 francs: find the arbitrated rate between London and Paris in francs for £1.

16. Find the arbitrated rate of exchange between Vienna and London in florins and cents for £1, when the exchange between Paris and Vienna is $212\frac{1}{2}$ francs for 100 florins, between Paris and Berlin $124\frac{1}{4}$ francs for 100 marcs, and between Berlin and London 20 marcs 50 pf. for £1.

17. When the exchange between London and Lisbon is $52\frac{3}{8}d.$ for 1 milreis, Lisbon and Paris 552 francs for 100 milreis, Paris and Hamburgh $124\frac{1}{4}$ francs for 100 reichmarcs, Hamburgh and Amsterdam 166 marcs 40 pf. for 100 florins; what is the corresponding exchange between Amsterdam and London in florins and stivers for £1?

18. A merchant in London owed another in Petersburg 2460 roubles 50 copeks, which he remitted through Paris when the exchange between London and Paris was 25 fr. 35 c. for £1, and between Paris and Petersburg 339 centimes for 1 rouble. Shortly afterwards the exchange between London and Paris was 25 fr. $62\frac{1}{2}$ c. for £1, and between Paris and Petersburg 337 c. for one rouble. How much would he have gained by the delay?

19. Find the value of £1 in marcs and pfennige of North Germany, having given that 1 kilogramme of fine gold is coined into $139\frac{1}{2}$ 20-marc pieces, that 1 lb. of standard gold is coined into $46\frac{29}{40}$ sovereigns, that standard gold is $\frac{11}{12}$ fine, and that 1 kilogramme is 15432 grains.

*20. Calculate the par of exchange between the dollar and the shilling when British standard silver is valued at $5s.\ 0\frac{5}{8}d.$ per oz., having given that

1 dollar weighs $412\frac{1}{2}$ grains, and is $\frac{9}{10}$ fine: and

1 lb. Troy standard silver, $\frac{37}{40}$ fine, is coined into 66 shillings.

21. When British standard silver is valued at $5s.\ 1\frac{5}{8}d.$ an ounce, find how many francs are equal to 20s., having given that

1 lb. Troy standard silver, $\frac{37}{40}$ fine, is coined into 66 shillings,

1 kilog. French st. silver, $\frac{9}{10}$ fine, is coined into 200 francs; and

1 kilog. is equal to 15432 grains.

22. Having given the same elements as in the last question, find what must be the price of British standard silver, an ounce, in order that the par of exchange as determined from the silver coinage of the two countries shall be the same as that determined from the gold coinage,—that is, that £1 shall be equal to 25 f. 22 c. (See Ex. 2, p. 347.)

23. How many ounces of bar gold $21\frac{1}{2}$ carats fine will be required to be sent to France to pay a debt of 27654 f. 40 c.? (See Ex. 3, p. 348.)

24. The exchange of London on Amsterdam at 3 months is 12 fl. $13\frac{3}{8}$ stiv.: what is the short price, allowing 5 per cent. per annum?

25. The short exchange of London on Paris is 25 f. $32\frac{1}{2}$ c.: find the exchange at 3 months, reckoning $4\frac{1}{2}$ per cent. per annum.

26. The exchange of London on Paris at 3 months is 25 f. 70 c., and of Paris on London is 25 f. 25 c.: find the difference between the rates of the short exchanges, reckoning $3\frac{1}{2}$ per cent. per annum.

27. At how much per mille premium is gold quoted at Paris when the exchange there is 25·27 francs per £1? (See Ex. 3, p. 348.)

28. When British standard gold is quoted at Paris at 3 per mille premium and is $\frac{8}{10}$ per cent. dearer there than in London, what is the short exchange on London?

29. Gold is quoted at Paris at 30 per mille premium and the short exchange on London is 26·50 francs for £1: find how much per cent. gold is higher in London than in Paris. (See Ex. 4, p. 348.)

30. A merchant in Hamburgh delivers goods to a merchant in London at 5 m. 16 pf. per pfund, a pfund being equal to 1·068 lbs. The London merchant remits payment at 2052 pf. per £1, and sells the goods at £27 for 100 lbs. What is his gain or loss per cent.?

31. A Lyons merchant could sell silk at home at 7 f. 10 c. per metre, gaining thereby $6\frac{1}{2}$ per cent.: but at Vienna he could sell it at 10 fl. 25 kr. for 3 metres net, and gain thereby $8\frac{1}{4}$ per cent. What rate of exchange is hereby established between Austria and France in florins for 200 francs?

32. The sum of £1000 is laid out in London in bills on Vienna at 11 fl. 35 c. for £1. The bills are sold in Hamburgh at 54 fl. 90 c. for 100 marcs, less 1 month's discount at 4 per cent. per annum, commission on sales $\frac{1}{2}$ per cent., and brokerage for sales and returns $\frac{1}{4}$ per cent. The returns are made in bills on Madrid at 3 m. 92 pf. for 1 peso, and are sold in London at 47*d*. for 1 peso, less brokerage for purchase and sales $\frac{1}{4}$ per cent. Find the profit on the original outlay.

CHAPTER XIV.

SQUARE AND CUBIC MEASURE. DUODECIMALS.

358. A PARALLELOGRAM is a quadrilateral figure whose opposite sides are parallel: they are also equal.

A RECTANGLE is a parallelogram which has all its angles right angles.

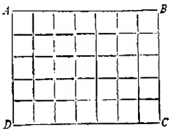
A SQUARE is a rectangle which has all its sides equal.

359. A PARALLELOPIPED is a solid figure bounded by six quadrilateral figures of which every opposite two are parallel.

A RECTANGULAR PARALLELOPIPED or RECTANGULAR SOLID is a parallelopiped bounded by six rectangles: as a *brick*.

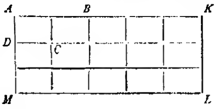
A CUBE is a rectangular solid bounded by six squares: as a *die*.

360. *To find the area of a rectangle.*



Let $ABCD$ be a rectangle of which the length AB is 7 feet and the breadth AD is 5 feet. Divide AB into 7 parts each equal to 1 foot, and AD into 5 parts each equal to 1 foot; through the points of division draw straight lines parallel to AB and AD; then the rectangle will be divided into 7×5 squares, each side of which is 1 foot in length: that is, the area of the rectangle is 7×5 square feet (203).

Again, suppose the number of feet in the length and in the breadth to be mixed numbers or fractions: for example, let the length AB be $7\frac{4}{5}$ feet and the breadth AD be $5\frac{2}{3}$ feet. Produce AB to K so that AK is five times AB, and AD to M so that AM is 3 times AD; and draw straight lines parallel to AK and AM, as shewn in the figure.



Now AK is $7\frac{4}{5} \times 5$ feet, or 39 feet, and AM is $5\frac{2}{3} \times 3$ feet, or 17 feet; therefore the area of the rectangle is 39×17 square feet. But the rectangle $AKLM$ is divided into 5×3 rectangles, each equal to the rectangle $ABCD$; therefore the area of the rectangle $ABCD$ is $\frac{1}{15}$ of the area of the rectangle $AKLM$, or is equal to

$$\frac{39 \times 17}{5 \times 3} \text{ sq. feet, or } \frac{39}{5} \times \frac{17}{3} \text{ sq. feet, or } 7\tfrac{4}{5} \times 5\tfrac{2}{3} \text{ sq. feet.}$$

Hence, whether the number of feet in the length and breadth be integral or fractional, their product is the number of square feet in the area.

In like manner, if the numbers giving the length and breadth be given in inches, or in yards, or in miles, their product will give the area in square inches, or in square yards, or in square miles.

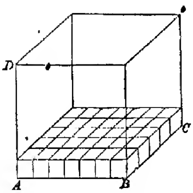
We have then this Rule,—

Express the length and breadth in units of the same denomination; the product will give the area in square units of that denomination.

COR. *If we divide the number of square units in the area by the number of units in* either *side, we shall find the number of units in the* other *side.*

361. *To find the volume of a rectangular parallelopiped, or rectangular solid.*

Let the figure represent a rectangular solid, whose length AB is 7 feet, breadth BC 5 feet, and height AD 6 feet. Divide AB into 7 parts, BC into 5 parts, and AD into 6 parts, each equal to 1 foot; and through the points of division draw planes parallel to the sides; then the figure will be divided into $7 \times 5 \times 6$ cubes, each of the sides of which is 1 foot; that is, the volume of the rectangular solid is $7 \times 5 \times 6$ cubic feet.



In the same way as in Art. 360, it may be shewn that if the number of feet in the length, and breadth, and height be mixed numbers or fractions, still their product will give the number of cubic feet in the volume. Also, if the numbers giving the three dimensions be given in inches, or in yards, or in miles, their product will give the volume in cubic inches, or in cubic yards, or in cubic miles.

We have then the following Rule,—

Express the length, breadth and height in units of the same denomination: their product will give the volume in cubic units of that denomination.

COR. *If we divide the number of cubic units in the volume by the product of the number of units in any two of its dimensions, we shall find the number of units in the third dimension; and if we divide the number of cubic units in the volume by the number of units in any one dimension, we shall find the product of the number of units in the other two dimensions, or the number of square units in that face of the solid whose sides are those two dimensions.*

Ex. 1. Find the area of a rectangular floor 23 ft. 8 in. long and 15 ft. 10 in. wide.

$$
\begin{array}{rrl}
\text{Length} = & 284 & \text{in.} \\
\text{Breadth} = & \underline{190} & \text{in.} \\
 & 25560 & \\
 & \underline{284} & \\
144\left\{\begin{array}{r} 12 \\ 12 \end{array}\right. & \begin{array}{r})\,\underline{53960} \\ )\,\underline{4496} \end{array} & \begin{array}{l} \text{sq. in.} \\ -8 \end{array} \\
 & 9\,)\,374 & ..\ 104 \text{ in.} \\
 & 41 & \text{sq. yds. } 5 \text{ ft. } 104 \text{ in.}
\end{array}
$$

$$
\begin{aligned}
\text{Area} &= 23\tfrac{2}{3} \times 15\tfrac{5}{6} \text{ sq. ft.} \\
&= \frac{71}{3} \times \frac{95}{6} \text{ sq. ft.} \\
&= \frac{6745}{18} \text{ sq. ft.} \\
&= 374\tfrac{17}{18} \text{ sq. ft.} \\
&= 41 \text{ sq. yds. } 5 \text{ ft. } 104 \text{ in.}
\end{aligned}
$$

Ex. 2. Find the area of the walls of a rect. room 23 ft. 8 in. long, 15 ft. 10 in. wide, and 11 ft. 11 in. high.

The four walls form together a rectangle whose length is the *circuit* of the room, and width the height of the room.

$$
\begin{array}{rrl}
\text{Length} = & 284 & \text{in.} \\
\text{Width} = & \underline{190} & \text{in.} \\
 & 474 & \\
 & \underline{2} & \\
\therefore \text{Circuit} = & 948 & \text{in.} \\
\text{Height} = & \underline{143} & \text{in.} \\
 & 2844 & \text{in.} \\
 & 3792 & \\
 & \underline{948} & \\
144\left\{\begin{array}{r} 12 \\ 12 \end{array}\right. & \begin{array}{r})\,\underline{135564} \\ )\,\underline{11297} \end{array} & \begin{array}{l} \text{sq. in.} \\ \end{array} \\
 & 9\,)\,941 & ..\ 60 \text{ in.} \\
 & 104 & \text{sq. yds. } 5 \text{ ft. } 60 \text{ in.}
\end{array}
$$

$$
\begin{aligned}
\text{Circuit} &= 2 \times (23 \text{ ft. } 8 \text{ in.} + 15 \text{ ft. } 10 \text{ in.}) \\
&= 2 \times 39 \text{ ft. } 6 \text{ in.} \\
&= 79 \text{ ft.}; \\
\therefore \text{Area of walls} &= 79 \times 11\tfrac{11}{12} \text{ sq. ft.} \\
&= \frac{11297}{12} \text{ sq. ft.} \\
&= 941 \text{ sq. ft. } 60 \text{ in.} \\
&= 104 \text{ sq. yds. } 5 \text{ ft. } 60 \text{ in.}
\end{aligned}
$$

Ex. 3. From a rectangular plot of land 55 yards 2 ft. 9 in. wide, what length must be cut off to make a garden of one acre?

$$55 \text{ yds. } 2 \text{ ft. } 9 \text{ in.} = 55\frac{2\frac{3}{4}}{3} \text{ yds.} = 55\tfrac{11}{12} \text{ yds.}$$

$$\text{and } 1 \text{ acre} = 4840 \text{ sq. yds.};$$

$$
\begin{aligned}
\therefore \text{Length of garden} &= \frac{4840}{55\frac{11}{12}} \text{ yds.} = \frac{4840 \times 12}{671} \\
&= \frac{5280}{61} \text{ yds.} \\
&= 86 \text{ yds. } 1 \text{ ft. } 8\tfrac{4}{61} \text{ in.}
\end{aligned}
$$

Ex. 4. Find the cost of carpeting a room 23 ft. 8 in. long and 15 ft. 10 in. broad, with carpet 27 in. wide, at 5*s.* 3*d.* a yard.

Area of floor $= 23\frac{2}{3} \times 15\frac{5}{6}$ sq. feet;

$\therefore$ length of carpet $= \dfrac{23\frac{2}{3} \times 15\frac{5}{6}}{2\frac{1}{4}}$ ft.

$= \dfrac{23\frac{2}{3} \times 15\frac{5}{6}}{2\frac{1}{4} \times 3}$ yds.

$\therefore$ cost of carpet $= \dfrac{23\frac{2}{3} \times 15\frac{5}{6} \times 5\frac{1}{4}}{2\frac{1}{4} \times 3}\,s.$

$= \dfrac{71}{3} \times \dfrac{95}{6} \times \dfrac{21}{4} \times \dfrac{4}{9} \times \dfrac{1}{3}\,s.$

$= \dfrac{47215}{162}\,s.$

$= £14.\ 11.\ 5\frac{11}{27}.$

$$
\begin{array}{rl}
 & 284 \text{ in.} \\
 & \underline{190} \text{ in.} \\
 & 25560 \\
 & \underline{284} \\
27 \left\{ \begin{array}{r} 9\,) \\ 3\,) \end{array} \right. & \begin{array}{l} \underline{53960} \text{ sq. in. in area.} \\ \underline{5995{\cdot}5} \end{array} \\
12\,) & \underline{1998{\cdot}5185} \text{ in. in length.} \\
3\,) & \underline{166{\cdot}5432} \text{ ft. } \ldots\ldots\ldots \\
 & 55{\cdot}5144 \text{ yds } \ldots\ldots\ldots \\
5s. = \frac{1}{4} & 13{\cdot}8786 \\
3d. = \frac{1}{80} & \underline{\cdot6939} \\
 & £14{\cdot}5725 \\
 & = £14.\ 11.\ 5\frac{1}{4}.
\end{array}
$$

Ex. 5. A room is to be papered whose length is 23 ft. 8 in., breadth 15 ft. 10 in., and height 11 ft. 9 in.: in it there are two windows each 9 ft. 6 in. high by 5 ft. wide, a fireplace 4 ft. 6 in. high by 6 ft. wide, and a door 7 ft. 6 in. high by 3 ft. 6 in. wide. Find the cost of the paper required at 17*s.* 6*d.* per piece of 12 yards, and the width of the paper 26 inches.

Circuit of room $= 2 \times (23$ ft. 8 in. $+ 15$ ft. 10 in.$) = 2 \times 39$ ft. 6 in. $= 79$ ft.;

$\therefore$ area of walls $= 79 \times 11\frac{3}{4}$ sq. ft. $= 928\frac{1}{4}$ sq. ft.

But area of windows $= 2 \times 9\frac{1}{2} \times 5$ sq. ft. $= 95$ sq. ft.,

...... fireplace $= 4\frac{1}{2} \times 6$ sq. ft. $= 27$ sq. ft.,

...... door $= 7\frac{1}{2} \times 3\frac{1}{2}$ sq. ft. $= 26\frac{1}{4}$ sq. ft.

Area to be papered $= 928\frac{1}{4}$ sq. ft. $- 148\frac{1}{4}$ sq. ft. $= 780$ sq. ft.;

$\therefore$ length of paper $= \dfrac{780}{2\frac{1}{6}}$ ft. $= 780 \times \dfrac{6}{13}$ ft. $= 360$ ft. $= 120$ yds.;

and $\therefore$ cost of paper $= 120 \times \dfrac{17\frac{1}{2}}{12}\,s. = 10 \times 17\frac{1}{2}s. = 175s.$

$= £8.\ 15.$

Ex. 6. Find the solid content of a block of marble in the form of a rectangular solid whose length is 10 ft. 4 in., breadth 5 ft. 3 in., and thickness 4 ft. 7 in.

$$\text{Solid content} = 10\tfrac{1}{3} \times 5\tfrac{1}{4} \times 4\tfrac{7}{12} \text{ cu. ft.}$$

$$= \frac{31}{3} \times \frac{\overset{7}{\cancel{21}}}{4} \times \frac{55}{12} \text{ cu. ft.}$$

$$= 248\tfrac{31}{48} \text{ cu. ft.}$$

$$= 248 \text{ cu. ft. } 1116 \text{ in.}$$

$$\begin{array}{lr}
\text{Length} = & 124 \text{ in.} \\
\text{Breadth} = & \underline{63} \\
 & 372 \\
 & \underline{744} \\
 & 7812 \\
\text{Thickness} = & \underline{55} \\
 & 39060 \\
 & \underline{39060} \\
\end{array}$$

$$1728 \left\{ \begin{array}{l}
12\,\underline{)\ 429660} \text{ cu. in.} \\
12\,\underline{)\ 35805} \\
12\,\underline{)\ 2983} - 108 \\
\end{array} \right.$$

$$248 \text{ cu. ft. } 1116 \text{ in.}$$

Ex. 7. The length and breadth of a dormitory are respectively 56 ft. and 23 ft. 6 in.: what must its height be so that it shall be capable of accommodating 24 boys, allowing 750 cubic feet to each boy?

$$\text{Volume of dormitory} = 24 \times 750 \text{ cubic feet;}$$

$$\therefore \text{ height of dormitory} = \frac{24 \times 750}{56 \times 23\frac{1}{2}} \text{ feet} = \frac{4500}{329} \text{ ft.}$$

$$= 13 \text{ ft. } 8\tfrac{44}{329} \text{ in.}$$

Ex. 8. What must be the surface of the bottom of a cistern whose volume is 56 cubic feet and height 2 ft. 4 in.?

$$\text{Area of bottom} = \frac{56}{2\frac{1}{3}} \text{ sq. ft.} = 56 \times \frac{3}{7} \text{ sq. ft.} = 24 \text{ sq. ft.}$$

DUODECIMALS.

362. In Duodecimals, the submultiples of the foot—whether linear, square, or cubic—follow the scale of 12, so that 1 foot is equal to 12 primes ('), 1 prime to 12 seconds ("), 1 second to 12 thirds ('''), 1 third to 12 fourths (''''), &c.; hence

$$\left.\begin{array}{l} 1 \text{ linear foot} \\ 1 \text{ square foot} \\ 1 \text{ cubic foot} \end{array}\right\} = 12' = 144'' = 1728''' = \ldots\ldots = \ldots\ldots;$$

therefore in linear measure the inch is the same as the prime, in square measure as the second, and in cubic measure as the third.

We can therefore readily pass from quantities expressed in duodecimals to those expressed in feet and inches, and conversely; thus,

Ex. (1). 8 ft. 2' 3'' = 8 ft. 2' $\frac{3}{12}$ = 8 ft. $2\frac{1}{4}$ in.

Ex. (2). 15 sq. ft. 10' 11'' 8''' = 15 sq. ft. 131'' $\frac{8}{12}$ = 15 sq. ft. $131\frac{2}{3}$ in.

Ex. (3). 24 cu. ft. 6' 7'' 8''' 9^{iv} 4^{v} = 24 cu. ft. 956''' $\frac{112}{144}$ = 24 cu. ft. $956\frac{7}{9}$ in.

6' 7'' 8'''
79''
956'''

Conversely,

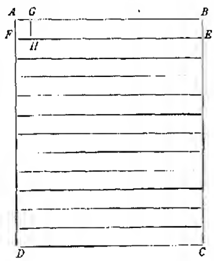
Ex. (4). 5 yds. 2 ft. $5\frac{3}{4}$ in. = 17 ft. 5' $\frac{3}{4}$ = 17 ft. 5' 9''.

Ex. (5). 19 sq. ft. $118\frac{2}{3}$ in. = 19 sq. ft. 118'' $\frac{2}{3}$ = 19 sq. ft. 9' 10'' 8'''.

Ex. (6). 35 cu. ft. $1267\frac{5}{9}$ in.
= 35 cu. ft. 1267''' $\frac{5}{9}$
= 35 cu. ft. 8' 9'' 7''' 6^{iv} 8^{v}.

12) 1267'''
12) 105 - 7'''
8 . 9'' . 7'''

363. Let the figure $ABCD$ represent a square foot. Divide AD into 12 equal parts, then each part as AF is 1'; through the points of division draw straight lines parallel to AB, dividing the sq. foot into 12 equal rectangles, and therefore each of them is a superficial prime. But each of these rectangles is 1 ft. long and 1' broad; therefore a rectangle 1 ft. long and 1' broad is 1' in area: hence a rectangle 7 ft. long and 1' broad is 7' in area, and a rectangle 7 ft. long and 5' broad is 7 × 5 or 35' in area; and this result is usually expressed by saying that



Feet into primes give primes.

Again, divide AF into 12 equal parts, each part will be 1''; through the points of division draw straight lines parallel to AB, dividing the rectangle $ABEF$, which is a superficial prime, into 12 equal rectangles; therefore each of these rectangles is a superficial

second. But each of them is 1 ft. long and 1″ broad; therefore a rectangle 1 ft. long and 1″ broad is 1″ in area; and, as before, a rectangle 7 ft. long and 5″ broad is 7×5 or 35″ in area; that is,

Feet into seconds give seconds.

In like manner, *Feet into thirds give thirds,*

Feet into fourths give fourths, &c.

Again, divide *AB* into 12 equal parts, and through the points of division draw straight lines parallel to *AF*, dividing the rectangle *ABFE* into 12 equal rectangles; therefore each of them is $\frac{1}{12}$ of a superficial prime, or is a superficial second. But each of them, as *AGHF*, is 1′ long and 1′ broad; therefore a rectangle 1′ long and 1′ broad is 1″ in area; and, as before, a rectangle 7′ long and 5′ broad is 35″ in area: hence

Primes into primes give seconds.

And if we divide *AF* into 12 equal parts, each will be 1″; and if through the points of division we draw straight lines parallel to *AG*, we may shew that

Primes into seconds give thirds;

and, as before, *Primes into thirds give fourths,* &c.

Also, *Seconds into seconds give fourths,*

Seconds into thirds give fifths, &c.;

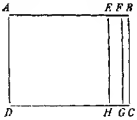
that is, the number of the dashes expressive of the denomination of the product is the sum of the dashes of its factors.

364. By pursuing a similar method we may shew that

Feet into supl feet *give solid feet,*
Feet into supl primes .... *primes,*
Feet into supl seconds .... *seconds,* &c.
Primes into supl primes .... *seconds,*
Primes into supl seconds......... *thirds,* &c.
Seconds into supl seconds ...... *fourths,* &c.;

that is, the number of dashes expressive of the denomination of the product is the sum of the dashes of its factors. Hence in duodecimals the rule given for multiplying a length by a breadth is equally applicable in multiplying a surface by a length.

365. Let us now find the area of a rectangle 9 ft. 4′ 10″ long and 7 ft. broad.



Let *ABCD* be this rect., and let *AE* be 9 ft., *EF* be 4′ and *FB* be 10″, and *AD* be 7 ft.

Now the whole rect. *ABCD* is the sum of the 3 rects. *AH*, *EG*, and *FC*.

But the

rect. *FC* is 7 ft. by 10″, ∴ its area = 70″ or = 5′ 10″,

rect. *EG* is 7 ft. by 4′, ∴ its area = 28′ or = 2 sq. ft. 4′,

and rect. *AH* is 7 ft. by 9 ft., ∴ its area = 63 sq. ft.,

∴ area of the whole rect. = 65 sq. ft. 9′ 10″.

Instead of finding the area of each rect. separately and adding the results, we shall find it more convenient to reduce the first result at once, and to carry to the second, as in Compound Multiplication: thus

$$\begin{array}{rcccc} & \text{ft.} & & ' & & '' \\ & 9 & . & 4 & . & 10 \\ & 7 & & & & \\ \hline \text{sq. ft.} & 65 & . & 9 & . & 10 \end{array}$$

where we say 7 ft. × 10″, or simply 7 × 10 = 70 and 70″ = 5′ 10″—set down 10″ and carry 5′. 7 × 4 is 28, and 5 (carried) is 33: 33′ = 2 sq. ft. 9′—set down 9′ and carry 2. Lastly, 7 × 9 = 63, and 2 (carried) is 65—set down 65 sq. ft.

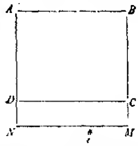
Again,—find the area of a rectangle 9 ft. 4′ 10″ long, and 8′ broad.

As in the last example, the whole rectangle may be made up of 3 rects. whose respective lengths are 9 ft., 4′ and 10″, and whose breadth in each case is 8′: making therefore the same arrangement, we proceed thus:

$$\begin{array}{rccccccc} & \text{ft.} & & ' & & '' & & \\ & 9 & . & 4 & . & 10 & & \\ & 0 & . & 8 & & & & \\ \hline \text{sq. ft.} & 6 & . & 3 & . & 2 & . & 8''' \end{array}$$

saying 8′ × 10″ = 80‴ (the area of the first rect.): but 80‴ = 6″ 8‴—set down 8‴ and carry 6″. 8 × 4 = 32, and 6 (carried) is 38; but 38″, &c.

Lastly. Find the area of a rect. 9 ft. 4′ 10″ long and 7 ft. 8′ broad.



Let $ABMN$ represent this rect. where AD is 7 ft. and DN is 8′: draw DC parallel to AB. The whole rect. is made up of the two rects. AC and DM.

But the rect. AC is 9 ft. 4′ 10″ by 7 ft., and its area has been found to be 65 sq. ft. 9′ 10″, and the rect. DM is 9 ft. 4′ 10″ by 8′, and its area has been found to be 6 sq. ft. 3′ 2″ 8‴; adding these results, the area of the whole rect. is 72 sq. ft. 1′ 0″ 8‴.

Or, combining the work of the last two Examples, the whole will stand thus :—

	ft.	′	″		
	9	4	10		
	7	8			
	65	9	10		= area of rect. AC.
	6	3	2	8‴	= .... DM.
sq. ft.	72	1	0	8	= .... $ABMN$.

We have then the following Rule :—

Write the breadth under the length so that units of the same denomination may be under one another. Multiply the length by the number of feet in the breadth, as in Compound Multiplication; then multiply by the number of primes in the breadth, setting down one place farther to the right; then by the seconds, setting down one place farther again to the right, &c.; add these partial products, and their sum will give the area of the rectangle in sq. feet, primes, &c.

Ex. 1. Find the area of a rectangular floor whose length is 23 ft. 7′ 9″ and breadth 15 ft. 10′ 7″.

	ft.	′	″		
	23	7	9		
	15	10	7		
	354	8	3		
	19	8	5	6‴	
	1	1	9	6	3⁗
sq. ft.	375	6	6	0	3

12) 236   12) 165
19 . 8   13 . 9

or 41 sq. yds. 6 ft. 78$\frac{1}{48}$ in.

Ex. 2. Find the area of a parallelogram whose base is 5 yds. 2 ft. 8¾ in. and altitude 3 yds. 2 ft. 5 in.

$$\begin{array}{rcccccccl}
 & \text{ft.} & & ' & & '' & & & \\
 & 17 & . & 8 & . & 9 & & & \\
 & 11 & . & 5 & & & & & \\
\hline
 & 195 & . & 0 & . & 3 & & & \\
 & 7 & . & 4 & . & 7 & . & 9''' & \\
\hline
\text{sq. ft.} & 202 & . & 4 & . & 10 & . & 9 & = 22 \text{ sq. yds. } 4 \text{ ft. } 58\tfrac{3}{4} \text{ in.}
\end{array}$$

Ex. 3. Find the volume of a water-tank whose length is 20 ft. 8 in., breadth 11 ft. 10 in., and depth 8 ft. 5¾ in.

$$\begin{array}{rcccccccccc}
 & \text{ft.} & & ' & & & & & & & \\
 & 20 & . & 8 & & & & & & & \\
 & 11 & . & 10 & & & & & & & \\
\hline
 & 227 & . & 4 & & & & & & & \\
 & 17 & . & 2 & . & 8'' & & & & & \\
\hline
 & 244 & . & 6 & . & 8 & & & & & \\
 & 8 & . & 5 & . & 9 & & & & & \\
\hline
 & 1956 & . & 5 & . & 4 & & & & & \\
 & 101 & . & 10 & . & 9 & . & 4''' & & & \\
 & 15 & . & 3 & . & 5 & . & 0 & . & 0 & \\
\hline
\left\{ 9 \right) & 2073 & . & 7 & . & 6 & . & 4 & & & \\
3 \,) & 230 - 3 & & & & & & & & & \\
 & 76 \,..\, 21 & & & & & & & & &
\end{array}$$

∴ Volume = 2073 cu. ft. 7′ 6″ 4‴ = 76 cu. yds. 21 ft. 1084 in.

Exercise 62.

1. Find the area of a rectangle 4 ft. 11 in. long and 2 ft. 4 in. broad.

2. Find the area of a floor 19 ft. 4 in. long and 16 ft. 8 in. wide.

3. How many sq. feet and inches are there in a sheet of glass 3 ft. 9 in. in length and 2 ft. 7½ in. in width?

4. How many sq. feet and inches remain out of 313 sq. feet of matting, after covering a floor 16 ft. 9 in. long by 12 ft. 11 in. broad?

5. How many sq. yds., &c. of carpet will cover a floor whose length is 22 ft. 8¾ in. and breadth 16 ft. 7½ in.?

6. Find the area of a square whose side is 7 ft. 10 in.

7. How many acres, &c. are there in a rectangular field 3265 links in length and 384 links in breadth?

8. How many acres, &c. of land will be required to form a street 510 yards long and 37 ft. 7 in. wide? How many sq. yards of flagging will be required to form a pavement 5 ft. 9 in. wide down one side of the street?

9. A rectangular garden is 2 chains 9 links wide and 300 yards long: find its area as a decimal of an acre.

10. Find the area of a rectangular court-yard 13 yards 2 ft. 7 in. long and 23 ft. 10 in. broad.

11. How many sq. yards of ground are covered by a plank 17·64 ft. long and 7¾ inches wide?

12. A garden roller is 3 ft. 7½ in. wide and 5 ft. 10⅜ in. in circumference: how many sq. feet and inches of ground does it pass over in making one complete revolution?

13. How many sq. yards of brickwork are there in the face of a wall surrounding a circular reservoir, the perimeter of the wall being 333⅓ yards and its height 6 ft. 10½ in.?

14. A school-room is 35 ft. 8 in. long, 23 ft. 4 in. wide and 15½ feet high: what is the area of its walls?

15. A piece of canvas of uniform width is 7 ft. 3¾ in. long, and it covers 2 sq. yards 103⅞ in.: what is its width?

16. The floor of a room is 15¾ feet wide, and its area is 23 sq. yards: find its length.

17. The area of a rectangle is 1532 sq. ft. 117 in., and one side is 81 ft. 9 in.: find the other side.

18. The surface of a rectangle 8 inches wide is the fifth part of a sq. yard: what is its length?

19. The area of a sq. garden is 1 ac. 1 p. 29 yds. 6⅔ ft.: find the length of its side in yards.

20. A sheet of glass is 3 ft. 9 in. long and 2 ft. 7½ in. wide: by how much must the width be narrowed to leave a surface of one square yard?

21. A piece of cloth 2 yards 3 qrs. 2½ nl. in length covers 21 sq. ft. 10¼ in.: find its width.

22. A square space containing 140 sq. yds. 36 in. is to be lengthened by 4 ft. 3 in. in one of its dimensions and shortened by 3 ft. 4 in. in the other: what will then be its area?

23. A rectangular piece of ground is 60 yards long and contains half an acre. It consists of a walk 8 feet wide surrounding a grass-plot. Find the area of the plot.

24. How many boards 18 ft. 6 in. long and 7 in. wide will be required to floor a room 10 yards 1 ft. 9 in. long and 8 yards 6 in. wide?

25. How many tiles 7 in. square will be required for the floor of a kitchen 19 ft. 3 in. long by 13 ft. 5 in. wide?

26. The length of a room is double its width, and the area of the floor is 136 sq. yds. 1 ft. 18 in.: find its length.

27. The breadth of a rectangle is one-third of its length, and its area is 635 sq. yards 5 ft. 48 in.: find its length in feet.

28. The area of a rectangular field whose length is 3 times its breadth is 6 ac. 960 yds.: find its perimeter.

29. A rectangular field is 1050 links longer in length than in breadth, and its perimeter is 3400 links: find its area in acres, &c.

30. How many cubic feet are contained in a beam 20 ft. 4 in. long, 1 ft. 5 in. broad, and 10 in. thick?

31. What is the cubical content of a cistern 6 ft. 8 in. long, 5 ft. 10 in. high, and 3 ft. 5 in. wide?

32. How many cubic feet and inches are there in a block of marble, each of its three dimensions being 4 ft. 9 in.?

33. Find the solid content and also the surface of a cube whose edge is $4\frac{1}{2}$ feet.

34. How many cubic feet of air are contained in a room 40 ft. $10\frac{1}{2}$ in. long, 25 ft. 8 in. broad, and 16 ft. 9 in. high?

35. How many cubical packages each having $4\frac{1}{2}$ inches in an edge will fill a rectangular box whose length, breadth, and depth are respectively 4 ft. 4 in., 3 ft. 3 in., and 2 ft. 2 in.?

36. Water is flowing into a cistern whose base measures 4840 sq. inches: how many cubic feet will have been supplied when the depth of water is $3\frac{1}{4}$ feet?

37. Find the length of an edge and the area of a face of a cube of which the solid content is 29 cu. feet. 541 in.

38. The depth of water in a cistern whose base contains 1344 sq. inches is 2 ft. 10 in. Find the depth of the same quantity of water in another cistern whose base contains 1088 sq. inches.

39. A rectangular cistern whose length is $13\frac{3}{4}$ ft. and breadth 6 ft. contains $294\frac{1}{4}$ cu. feet of water: what is the depth of the water? and what is its weight in tons when a cubic inch of water weighs 252·5 grains?

40. A cubic foot of gold is extended by hammering so as to cover an area of 6 acres. Find within one ten-millionth of an inch the thickness of the gold as a decimal of an inch.

41. Express in yards, feet, and inches

(1) 23 ft. 11′ 10″ 8‴.
(2) 71 sq. ft. 9′ 8″ 4‴ 6^iv.
(3) 83 cu. ft. 6′ 5″ 10‴ 8^iv 8^v.
(4) 143 ft. 6′ 9″ 9‴.
(5) 197 sq. ft. 0′ 1″ 3‴ 4^iv 5^v.
(6) 1763 cu. ft. 8′ 10″ 11‴ 0^iv 1^v 2^vi.
(7) 844 sq. ft. 5′ 6″ 0‴ 8^iv 9^v.
(8) 2341 cu. ft. 5′ 6″ 8‴ 8^iv 9^v 10^vi.

Find, *by Duodecimals*, the area of a rectangle whose sides are

42. 17 ft. 4 in. and 9 ft. 11 in.

43. 25 ft. 6 in. 7 parts (or seconds) and 11 ft. 9 in.

44. 7 yds. 1 ft. $6\frac{3}{8}$ in. and 13 ft. $5\frac{1}{4}$ in.

45. 31 ft. 4 in. 6″ and 17 ft. 10 in. 8″.

46. 15 yds. 2 ft. $4\frac{1}{2}$ in. and 9 yds. 2 ft. $4\frac{2}{3}$ in.

47. 451 ft. $6\frac{1}{2}$ in. and 71 ft. $3\frac{3}{4}$ in.

48. 207 ft. $4\frac{1}{4}$ in. and 95 ft. $7\frac{1}{11}$ in.

49. 17 yds. 2 ft. 9 in. 7 pts. and 11 ft. 9 in. 10 pts.

50. 31 yds. $6\frac{5}{8}$ in. and 5 yds. 1 ft. $2\frac{1}{4}$ in.

51. 19 yds. 2 ft. $6\frac{5}{8}$ in. and 7 yds. 1 ft. $3\frac{7}{8}$ in.

52. 15 yds. 2 ft. 4 in. 10 pts. and 7 yds. 1 ft. 11 in. 7 pts.

53. Find *by Duodecimals* the area of a square whose side is
2 yds. 1 ft. $3\frac{3}{8}$ in.; 17 ft. 4 in. $6\frac{2}{3}$ pts.; 123 ft. $6\frac{7}{8}$ in.

Find *by Duodecimals* the volume of a rectangular solid whose dimensions are

54. 13 ft. 8′ 11″, 14 ft. 9 in., and 15 ft. 1′ 2″.

55. 18 ft. 7 in. 4 pts., 17 ft. 3 in. 9 pts., and 11 ft. 11 pts.

56. Find *by Duodecimals* the volume of a cube whose edge is
11 ft. 6 in. 5 pts.; 5 yds. 2 ft. 7 in. 10 pts.; 3 yds. 1 ft. $7\frac{3}{8}$ in.

57. Find the cost of a marble slab whose length is 5 ft. 7 in. and breadth 1 ft. 10 in. at 8*s.* 3*d.* a sq. foot.

58. What will be the expense of painting a wall 22 ft. 6 in. long by 10 ft. 8 in. high at 1*s.* $10\frac{1}{2}$*d.* a sq. yard?

59. Find the cost of carpeting a floor 23 ft. 4 in. long and 16 ft. 9 in. wide with carpet at 6*s.* 3*d.* a sq. yard.

60. Find the side of a square court-yard the expense of paving which at 3*s.* 9*d.* per sq. yard is £38. 10. 5.

61. An upholsterer covers a floor 21 ft. 8 in. by 16 ft. 6 in. with carpet 27 inches wide: find the cost of the carpet at 3*s.* $4\frac{1}{2}$*d.* per yard in length.

62. Find the cost of painting the four walls of a room whose length, breadth, and height are respectively 24 ft. 3 in., 16 ft. 8 in., and $11\frac{1}{2}$ ft., at 3*s*. $4\frac{1}{2}$*d*. per sq. yard.

63. How much paper, $\frac{3}{4}$ of a yard wide, will be sufficient to paper a room 22 ft. 5 in. long, 12 ft. 1 in. broad, and 11 ft. 3 in. high? and how much will it cost at $4\frac{1}{2}$*d*. a yard?

64. A room is 20 ft. 6 in. long by 15 ft. 6 in. wide, and is 16 ft. high. It has two doors, each 8 ft. high by 3 ft. 9 in. wide, and three windows, one 5 ft. by 7 ft., the other two each 5 ft. by 4 ft. How much paper a yard wide will be required to paper it?

65. A box with a lid is to be made of inch-and-a-half plank; the external dimensions to be 3 ft. 6 in., 2 ft. 6 in., and 1 ft. 9 in. How many square feet of plank will be used in the construction?

66. The floor of a hall is 260 feet long and 93 feet wide. Find the cost of supplying it with carpet at 8*s*. 3*d*. per square yard, and oil-cloth at 3*s*. 9*d*.; the oil-cloth to be laid along the sides and ends a yard wide, and the carpet to extend 6 inches over the oil-cloth everywhere.

67. A rectangular piece of land is 1345 links in length and 860 in breadth: how many square feet would be occupied on paper, by a plan of the land drawn from a scale of an inch and a half to the chain?

68. How many pounds of gunpowder, of which each cubic foot weighs 932 oz., will fill a box whose height is 2 ft. 5 in., breadth 1 ft. 7 in., and length 5 ft. 9 in.?

69. If 67 gallons of water be equal to $10\frac{3}{4}$ cubic feet, how many gallons can be contained in a cistern 5 ft. 10 in. long, 3 ft. 7 in. wide, and 2 ft. 8 in. deep?

70. Find the cost of digging a cellar 18 ft. 4 in. long, 12 ft. 9 in. broad, and 14 ft. 3 in. deep, at 6*d*. per cubic yard. (By Duodecimals.)

71. A pond whose area is 4 acres, is frozen over with ice uniformly 6 inches thick. If a cubic foot of ice weigh 896 oz., find the whole weight of the ice in tons.

72. An iron bar is $1\frac{3}{4}$ of an inch broad, $\frac{5}{8}$ of an inch thick, and $4\frac{1}{2}$ feet long. Find its weight at $4\frac{1}{2}$ ounces per cubic inch.

73. If 56 cu. ft. 912 in. be the content of an open cistern, 6 ft. 2 in. long, and 3 ft. 4 in. wide, what will be the cost of lining the inside of it with lead at 10*s*. $1\frac{1}{2}$*d*. per square yard?

74. What would be the cost of paving a hall, 50 yards long and 50 feet broad, with marble slabs 1 foot long and 9 inches broad, the price of the slabs being £5 per dozen?

75. A canal, 10 miles long, has an average width of 7 yards and is 5 feet deep. How soon would the excavation of it be completed by 100 men, each removing, on an average, 15 cubic yards per day?

76. To what uniform depth must a piece of ground, 414 yards long and 37 yards wide, be excavated, that the earth taken out may form an embankment of 25530 cubic yards, supposing the earth to be increased one-ninth in volume by removal?

77. A side of *one* square is 9·49 inches; and if this square were made 6·24 inches longer in one of its dimensions, and 6·24 inches shorter in its other dimension, the area of the rectangle thus formed would be equal to that of *another* square. Find a side of the latter square.

78. The depth of a box is equal to a third of its length and the width is a third as much again as the depth. The content is 32 cubic feet: find the dimensions.

79. A postage stamp is $\frac{7}{8}$ of an inch long and $\frac{3}{4}$ of an inch broad: how many postage stamps will be required to paper a room 14 ft. 9 in. long, 9 ft. 3 in. wide, and 10 ft. 6 in. high, the room containing two windows each $5\frac{1}{2}$ ft. by 4 ft., and three doors each 6 ft. by 3 ft.?

80. A room whose height is 11 feet and length twice its breadth, takes 143 yards of paper, 2 feet wide, for its four walls: how much carpet will it require?

81. The carpeting of a room, twice as long as broad, at 5*s.* per square yard, cost £6. 2. 6; and the painting of the walls at 9*d.* per square yard cost £2. 12. 6. What was the height of the room?

82. The breadth of a room is twice its height, and half its length; and the volume of air in the room is 4096 cubic feet. Find its length.

83. The content of a cistern is the sum of two cubes whose edges are 10 inches and 2 inches; and the area of its base is the difference of two squares whose sides are $1\frac{1}{4}$ ft. and $1\frac{3}{8}$ ft. Find its depth.

84. A factory has 120 windows, 80 of which severally contain 16 panes, each 15 inches by 12 inches; and the remainder severally contain 12 panes, each 1 foot square. Find the cost of glazing the whole at 1*s.* 6*d.* per square foot.

85. The area of the Yorkshire coal-field is $937\frac{1}{2}$ square miles and the average thickness of the coal is 70 feet. If a cubic yard of coal weigh a ton, and the annual consumption of coal in Great Britain be 70,000,000 tons, find the number of years for which this coal-field alone would supply the country with coal at the present rate of consumption.

If the coal thus annually consumed were piled up into a rectangular stack the length of whose base is half a mile and the breadth a quarter of a mile, what would be its height in yards?

EXAMINATION PAPERS.

I. ARMY EXAMINATION. FOR DIRECT COMMISSIONS.

July, 1870.

1. FIND the number of sovereigns which are equal in value to two millions three thousand and forty pence.

2. If 8 tons 4 lbs. of sugar are sold for £466. 15. 5, what is the price of a pound?

3. Find the number of square feet in 3 acres 1 rood 2 perches.

4. If 12 horses are fed for 17 days at the cost of £11. 1*s*., how many days can 4 horses be fed for £11. 14*s*., the price of food and the rate of consumption being the same in both cases?

5. Find the cost of $5\frac{14}{17}$ lbs. at £1. 16. 10 the pound.

6. From the sum of $\frac{1}{18}$ and $\frac{3}{4}$ of $1\frac{1}{3}$ subtract the sum of $\frac{1}{12}$ and $\cdot 3$ of $3\frac{1}{3}$.

7. Express 89 gallons 1 quart 1 pint as the decimal of 572 gallons.

8. Find the number of grains in $\cdot 35$ of 3 lbs. 1 oz. Troy.

9. What is the simple interest on £429. 3. 4 in 4 years at $2\frac{1}{3}$ per cent. per annum?

10. Extract the square root of 3312400.

II. *For Direct Commissions. May*, 1870.

1. FIND the number of pounds Troy in three hundred millions, three thousands, eight hundred and forty grains.

2. If 257 pounds of tea cost £34. 16. $0\frac{1}{2}$, what is the price of a pound?

3. If a peck of corn weighs 7 lbs. 5 oz. Avoirdupois, what is the weight at that rate of 73 quarters 1 bushel?

4. A rectangular cistern 9 feet long, 5 feet 4 inches wide, and 2 feet 3 inches deep, is filled with liquid which weighs 2,520 pounds. How deep must a rectangular cistern be which will hold 3,850 pounds of the same liquid, its length being 8 feet, and its width 5 feet 6 inches?

5. Divide $\frac{7}{8}$ of $\frac{9}{11}$ by $2\frac{3}{5}$ of $1\frac{1}{4}$. How many square yards are there in the fraction of an acre which the result represents?

6. Multiply 28·8 by ·0595, and divide the product by 9520.

7. Find, as a fraction in its lowest terms, the difference between $\frac{3}{1100}$ and $\frac{1}{275}$, and then express that difference by a recurring decimal.

8. Find the value of ·00625 of a sovereign.

9. What is the principal sum to be placed at simple interest at the rate of $4\frac{1}{2}$ per cent. per annum, that in 16 months it may amount to £39. 12. 6?

10. Extract the square root of 452929.

III. *Control Department. April,* 1872.

1. In 164,723 pints how many quarters, bushels, pecks, &c.?

2. If 15 men can reap 20 acres of corn in 6 days working 14 hours a day, how many men must be employed to assist 10 other men to reap 6 acres in $1\frac{3}{4}$ days of 8 hours a day?

3. Find (by Practice) the dividend on £721. 13. 6 at 14*s.* 2*d.* in the pound.

4. Find the amount of £8900 in three years at $2\frac{1}{2}$ per cent. compound interest (neglecting fractions of a penny).

5. Add together $\frac{14}{17}$, $3\frac{1}{6}$, $1\frac{5}{21}$, and $\frac{7}{17}$.

6. Subtract $5\frac{4}{17}$ from $6\frac{1}{8}$.

7. Multiply $4\frac{1}{5}$, $\frac{15}{17}$, $1\frac{3}{11}$, and $\frac{1}{3}$.

8. Divide $3\frac{3}{8}$ by $\frac{4}{5}$.

9. Add together 84·6912, ·001567, 1·0056 and 549·2.

10. Subtract 23·69428 from 50·012.

11. Multiply 40·061 by ·0054.

12. Divide ·055757592 by ·009207.

13. Reduce 33 yards to the decimal of a mile.

---

14. Extract the cube root of 9555119848.

15. Reduce ·2671875 to a vulgar fraction; and if the unit be £6, reduce the fraction to shillings, pence and decimals of a penny.

16. When the 3 per cents. are at $87\frac{1}{4}$, and shares paying 5 per cent. are at $130\frac{1}{2}$, which is the more profitable investment? and what sum does a person invest, when the difference of the incomes resulting from the two investments is £561?

IV. UNIVERSITY OF OXFORD. LOCAL EXAMINATIONS.

Senior Candidates. May, 1870.

1. WRITE out Troy weight. How many grains are there in a pound Avoirdupois? How many pounds avoirdupois are equivalent to one hundred and nine thousand three hundred and seventy-five pounds Troy?

2. Divide two hundred and eight pounds two shillings and sixpence farthing by twenty-three; and multiply one mile seven furlongs thirty-nine poles by forty-one.

3. Obtain, by Practice, the cost of three hundred and fifty-five things at one pound sixteen shillings and eight pence each; and calculate a person's wages for five months three weeks and six days at one pound seven shillings and five pence per month.

4. Simplify $\frac{315}{216}$, and $\frac{5}{12}$ of $\frac{9}{16}$ of $2\frac{3}{11}$. Also add the results.

5. Add together $\frac{3}{5}$ of £1. 6. $6\frac{3}{4}$, $\frac{11}{336}$ of a guinea, $\frac{5}{84}$ of a sovereign, ·4375 of a shilling, and ·1375 of half a sovereign.

6. Divide ·045 by ·0015, 4·5 by 150, and ·45 by ·15.

7. What part of a sovereign is three pence three farthings? and what decimal fraction of a pole is an inch?

8. Extract the square root of 893830609, and raise 1·05 to the fourth power.

9. Compare the simple and compound interest on £21. 10*s.*, at the end of four years, reckoning money at 5 per cent. per annum.

10. If a farthing be the interest on a shilling for a calendar month, what is the rate per cent. per annum?

11. If twenty-seven hundred-weight twenty-one pounds cost three hundred and seventy-nine pounds two shillings and three pence three farthings, what will be the cost of three hundred-weight three quarters fifteen pounds?

12. If three persons are boarded four weeks for seven pounds, how many can be boarded thirteen weeks five days for one hundred and twelve pounds?

V. *Senior Candidates. May,* 1871.

1. How many persons can be paid 11*s.* 6*d.* each out of a sum of £170. 10. 2? And, if the balance be also distributed equally among them, how much more will each receive?

2. Write out the tables of Solid Measure and Measure of Capacity.

If a gallon contains 277·274 cubic inches, how many cubic yards are there in 100 bushels?

3. Find the cost of 7 lbs. 2 oz. 5 dwts. 4 grs. at 3*s*. $7\frac{1}{2}$*d*. per dwt.

4. To $3\frac{1}{2}$ of $5\frac{1}{6}$ add $\frac{2}{3}$ of $(6\frac{1}{4} - 1\frac{1}{2})$; and from their sum subtract $\frac{2\frac{1}{6}}{\frac{3}{4}}$.

5. Divide 2·375 by 250, by 2·25, and by ·00005.

6. Reduce $\frac{3}{160}$ and $\frac{1}{32}$ to decimals; and ·2361 to a vulgar fraction.

7. Find the value of 2·5 of 1 ton 6 cwt. + 3·125 of 2 qrs. 16 lbs. + 3·75 of 448 oz.

8. Extract the square roots of 22099401 and $210\frac{1}{4}$.

9. How many hours a day must 24 men work to accomplish as much in 5 days as 25 men could do in 4 days if they worked 6 hours a day?

10. What will a debt of £4250 amount to, if it is left standing for $2\frac{1}{2}$ years at 5 per cent. per annum compound interest?

11. If a school-room is 25 ft. long and 20 ft. wide, how many children will it accommodate, allowing for each of them 8 superficial feet at the least? And if the room is 10 ft. 4 in. high, what cubical space is there for each child?

12. Find the price of Three per Cent. Stock, when an investment of £434. 12. 6 produces an income of £14. 5*s*.

VI. UNIVERSITY OF CAMBRIDGE. LOCAL EXAMINATIONS.

Senior Students. Dec. 1869.

1. DIVIDE 69 miles 7 f. 39 po. 2 ft. by 492.

2. Add together $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{6}{7}$; and subtract the result from $100\frac{1}{70}$. Reduce to their simplest forms

$$\frac{1}{2+\cfrac{1}{3+\cfrac{1}{4}}};\quad \frac{1}{2} \text{ of } \frac{1}{3} \times \frac{3}{4} \text{ of } \frac{4}{3} \div \left(\frac{7}{6} + \frac{1}{3} \text{ of } 20\right);\quad \frac{3\frac{1}{6} - 2\frac{1}{4}}{5\frac{3}{7} + 1\frac{1}{3}}.$$

3. Add together 536·421, 53642·1, 5·36421; and subtract the result from 100000.

Prove that $\frac{\cdot 21}{\cdot 412} = \frac{2331}{2321}$.

4. State your rule for the division of decimals; divide 2 by ·2, ·002 by ·02, 2·2 by 2·1.

5. Find the value of

$\frac{1}{4}$ of 17*s.* 8*d.* + 2·625 of 1*s.* − $\frac{3}{4}$ of $\frac{5}{8}$ of 5*s.* 4*d.* + ·263 of 25*s.*;

and reduce the result to the decimal of £5.

6. Extract the square root of 35·672 and of ·4 to 4 places of decimals.

7. What would be the cost of paving a hall 50 yards long by 50 feet broad with marble slabs 1 foot long and 9 inches broad, the price of the slabs being £5 per dozen?

8. Find the difference between the simple interest and the discount on £100 for 5 years at 5 per cent.

If the present worth of £218 due two years hence be £200, what is the present worth of £1000 due six years hence at the same rate?

9. Of the boys in a school one third are over 15 years of age, one third between 10 and 15. A legacy of £100 can be exactly divided amongst them by giving 10*s.* to each boy over 15, 6*s.* 8*d.* to each between 10 and 15, and 3*s.* 4*d.* to each of the rest. How many boys are there in the school?

10. If the price of candles $8\frac{1}{2}$ inches long be 9*d.* per half-dozen, and that of candles of the same thickness and quality $10\frac{1}{4}$ inches long be 11*d.* per half-dozen, which kind do you advise a person to buy?

What would be the saving per cent. should your advice be followed?

VII. *Senior Candidates. Dec.* 1871.

1. WHAT is the meaning of 25, $2\frac{1}{2}$, and 2·3?

In subtraction how do you get over the difficulty of taking a greater digit from a less? Illustrate your answer by taking 874 from 953.

2. Divide 15997 by 21 by short division. Give a reason for your method of determining the remainder.

3. Supposing unity to be represented by $3\frac{7}{8}d.$ find the value of fifty millions five thousand and six.

How many tons, cwt. &c. are there in as many ounces as there are seconds in a week?

4. Add together $\frac{7}{15}$, $\frac{9}{17}$, $\frac{13}{35}$, $\frac{23}{24}$.

Simplify $8\frac{1}{2} - 4\frac{2}{3} + \frac{3}{5} - 1\frac{1}{4}$ of $\frac{2}{5}$: and $1 + \cfrac{1}{2 + \cfrac{1}{3 + \cfrac{1}{4}}}$.

5. Find, by Practice, the value of $2037\frac{5}{8}$ cwt. of soap at £1. 19. $4\frac{1}{2}$ a cwt.

6. Multiply ·0204 by 40·2: divide ·04 by 20, ·4 by ·002, ·0004 by ·0002 and 400 by ·02. Prove the truth of two of your results by means of vulgar fractions.

7. Find a decimal which shall be within $\frac{1}{10000}$ of $\frac{4}{17}$.
Find the value of $\dot{5}4$ of ·3027 of 1 m. 6 fur. 12 po.

8. A closed vessel formed of metal 1 inch thick whose external dimensions are 8 ft. 3 in., 7 ft. 5 in. and 4 ft. 3 in. weighs 3 cwt. 1 qr. 8 lbs. What would be the weight of a solid mass of metal of the same dimensions?

9. On a piece of work 3 men and 5 boys are employed who do half of it in 6 days. After this one more man and one more boy are put on and $\frac{1}{8}$ more is done in 3 days; how many more men must be put on that the whole may be completed in one day more?

10. If 9 oz. of gold, 10 carats fine, and 5 oz., 11 carats fine, be mixed with 6 oz. of unknown fineness and the fineness of the resulting mixture be 12 carats, what was the unknown fineness?

11. If Three per Cent. Consols be at $90\frac{3}{8}$, what sum must I invest in order to secure from them a yearly income of £470 after paying an income-tax of 5*d*. in the pound, brokerage being at $\frac{1}{8}$ per cent?

12. Find the square root of 13 to five decimal places.
A cubical block contains 39 cub. ft. 1529 cu. in.; find the number of sq. yds. &c. in its surface.

VIII. FOR COMMISSIONS IN THE ROYAL MARINES.

N.B. Decimals that do not terminate or repeat are to be worked to 5 places.

1. How much is $\frac{5}{8}$ of £27. 13. 7?

2. Divide $\frac{4}{5}$ of $\frac{3}{4}$ of $9\frac{1}{3}$ by $\frac{2}{3}$ of $\frac{7}{8}$ of $5\frac{1}{4}$.

3. Reduce £36. 17. $8\frac{3}{4}$ to the decimal of £5.

4. Reduce $3\frac{11}{17}$ to a decimal; and ·45 and ·27899 to vulgar fractions.

5. Find the square root of 536·9.

6. What is the cost of 137 tons 8 cwt. 60 lbs. of coal at £1. 2. 8 per ton?

7. If a man walking at the rate of 4 miles an hour can travel a certain distance in 3 hours 25 m., in what time could he run the distance at the rate of 7 miles an hour?

8. What is the interest on £564. 10s. for 11 months at 6⅜ per cent. per annum?

9. If the weight of a cubic foot of water is 62·355 pounds avoirdupois, what is the error in calculating the weight of 1000 cubic feet on the supposition that a cubic fathom weighs 6 tons?

10. A publican mixes 4 gallons of gin which is worth 15*s.* a gallon, with 4 gallons of water and a gallon of base spirit worth 10*s.*; what will he gain per cent. on his outlay by selling the mixture at 2*s.* 10*d.* per bottle of six to the gallon?

11. A man having £550 in cash, invests it in the Three-and-a-half per Cents. when they are at 88⅞; afterwards when they are at 93 he sells out, and invests his money in a mortgage which brings him in 5¼ per cent. What difference does the transaction make in his income?

IX. INDIA FOREST DEPARTMENT.

Jan. 1870.

1. Shew that the $\sqrt{2}$ lies between $\frac{17}{12}$ and $\frac{41}{29}$.

2. Extract the cube root of 669·921875 cubic feet, and reduce the result to inches.

3. What is the present worth of £1,842. 15*s.*, payable a quarter of a year hence, at 5 per cent.?

4. What length of paper 22½ inches wide would be required to paper the walls of a room 18 ft. 9 in. long, 13 ft. 3 in. broad, and 14 ft. 6 in. high?

5. Express 4 acres 2 roods 16 perches as the decimal of a square mile.

6. Multiply by duodecimals 7 ft. 3 in. 5 pts. by 5 ft. 7 in. 4 pts. and the product by 4 ft. 2 in. What does the product become when expressed in cubic feet and inches?

7. A man purchases £700 Stock in the Three per Cent. Consols at 94½, and also invests £585 in the purchase of Russian Five per Cent. Stock at 97½. How much Stock has he standing in his name? If he sells out of the Three per Cents. at 95 and out of the Five per Cents. at 96¼, does he gain or lose by the transaction, and how much?

8. Divide $91{\cdot}86\dot{3}$ by $87{\cdot}\dot{5}\dot{6}$.

9. A tradesman's stock in trade is valued on January 1st, 1868, at £8,000, he has also £350 in cash and owes £1,870; during the year his personal expenses, £300, are paid out of the proceeds of his business, and on January 1st, 1869, his stock is valued at £7,950, he has £570 in cash and owes £1,510. What is the whole profit on the year's transactions after deducting 5 per cent. interest on the capital with which he began the year?

10. Two clocks point to 2 o'clock at the same instant on the afternoon of Christmas day; one loses 8 seconds, and the other gains 9 seconds in 24 hours: when will one be half an hour before the other, and what time will each clock then shew?

X. FOR ADMISSION INTO THE R. M. ACADEMY, WOOLWICH.

July, 1870.

1. If building-ground be bought for 15*s.* 9*d.* a square yard, what will be the cost of half an acre of such ground?

The purchaser of the half-acre builds a house upon it and lays out the ground at a further cost of £2,094. 5*s.*, what rent per annum must he obtain so as to realize 9 per cent. on his whole outlay?

2. If 20 English navvies, each earning 3*s.* 6*d.* a day, can do the same piece of work in 15 days that it takes 28 foreign workmen, each earning three francs a day, to complete in 20 days; taking the value of the franc at 10*d.*, determine which class of workmen it is most profitable to employ. If a piece of work done by navvies cost £3,000, what would be the cost of the same work done by foreign workmen?

3. Reduce $\dfrac{[illegible] + [illegible]}{[illegible] - [illegible]}$.

Divide £2·4 by ·00625; and without using the common rule for the extraction of the square root, prove that 1·83 is the square root of 3·361.

4. What is meant by the "course of exchange" between two countries?

A merchant in New York wishes to remit to London 5,110 dollars, a dollar being equal to 4*s.* 6*d.* English, for what sum in English money must he draw his bill when bills *on* London are at a premium of $9\frac{1}{2}$ per cent.?

XI. *Royal Military Academy. Dec.* 1870.

1. DIVIDE £26. 9. 7 by 82.

2. Find the difference in yards and fractional parts of a yard between 10 chains 5 links and 1 furlong 2 rods.

3. An account after a discount of $2\frac{1}{2}$ per cent. is taken off becomes £16. 14. 9. What was the sum due before the discount was subtracted?

4. If a litre is ·22 gallons, find to the nearest penny in English money the value of a pint of liquid which is worth 10 francs the litre, 1200 francs being equivalent to £49.

5. Express ·101 of 1 lb. 5 oz. as a decimal of 1 cwt.

6. A person borrows £100, and at the end of each year pays £25 to reduce the principal and to pay interest at 4 per cent. on the sum which has been standing against him through that year. How much will remain of the debt at the end of 3 years?

XII. *Royal Military Academy. July,* 1871.

1. FIND the number of inches in the length 1 mile 1 furlong 3 poles.

2. Find the value of 17 quarters 1 bushel of corn at the price of 4*s*. $8\frac{3}{4}$*d*. the bushel.

3. What is the least weight which can be expressed either by a number of Troy pennyweights or by a number of Avoirdupois ounces? Give the answer in Troy weight.

4. A vessel holds $2\frac{1}{4}$ pints. How many times can it be filled from a cask of 56 gallons, and will there be any remainder?

5. Find the value of $\frac{\cdot 06812}{\cdot 131}$ of £1. 0. $1\frac{1}{4}$.

6. A school of boys and girls contains 453 children, and the boys are ·525252... of the girls. How many boys are there?

7. If a metric system of area were adopted wherein 1 acre 1 rood 3 perches is represented by 5·12, express the unit of measurement in square yards and decimal parts of a square yard.

8. At what rate per cent. is the deduction made when 19*s*. $10\frac{1}{2}$*d*. is taken from an account of £39. 15*s*. in consideration of immediate payment?

9. What is the compound interest to the nearest penny on £83. 14. 7 in 7 years at the rate of 3 per cent. per annum? How much does this compound interest exceed the simple interest in the same time on this principal at the same rate per cent.?

XIII. EXAMINATION FOR ADMISSION INTO THE INDIA CIVIL ENGINEERING COLLEGE.

June, 1871.

1. CONVERT [illegible] and [illegible] into decimal fractions: divide the second result by the first, and convert the quotient into a vulgar fraction in its lowest terms.

2. Find the length of the side of a square which is equal in area to the rectangle, the sides of which are 513 yards 1 foot 11 inches, and 1,628 yards 11 inches.

3. Find the length of the edge of a cube which contains 450 feet 1,088 inches.

4. The external length, breadth, and height of a rectangular wooden closed box are 18 inches, 10 inches, and 6 inches respectively, and the thickness of the wood is half an inch. When the box is empty it weighs 15 lbs., and when filled with sand 100 lbs. Compare the weight of equal bulk of wood and sand.

5. A cask weighing 2 cwt. 12 lbs. 4 oz. floats in a square cistern of water, whose side is 2 ft. 6 in.; on the removal of the cask, find how much the water will sink in the cistern, supposing a cubic foot of water to weigh 63 lbs.

6. The interest on a certain sum of money for two years is £71. 16. $7\frac{1}{2}$, and the discount on the same sum for the same time is £63. 17*s*., simple interest being reckoned in both cases. Find the rate per cent. per annum, and the sum.

XIV. UNIVERSITY OF LONDON. MATRICULATION EXAMINATION.

Jan. 1870.

1. FIND the value of [illegible], and divide [illegible] by the result.

Divide ·0075 by 25·6, and state the principle upon which you fix the position of the decimal point in the quotient.

2. Reduce nine inches and nine tenths to the decimal of a mile; and find the value of ·0625 of 1 ton 2 cwt. 3 qrs. 12 lbs.

3. *A* sells goods to *B* for £115. 19. 2, and gains 10 per cent. on the price he originally paid for them. *B* sells the same goods again, and loses 10 per cent. on the price at which he bought them. At what price did *A* buy the goods, and at what price did *B* sell them?

4. What annual income will be produced by £13,000 invested in a Three-and-half per Cent. Stock at 91? and by the same sum invested in a Four per Cent. Stock at 96?

5. Extract the square root of 10,074,538,384; and find the value of $\dfrac{\sqrt{2}-1}{\sqrt{2}+1}$ to four places of decimals.

Prove that no square number can end with one of the digits 2, 3, 7, 8.

XV. *Matriculation Examination. Jan.* 1871.

1. SIMPLIFY $$\frac{\dfrac{2\frac{1}{2}}{54}+\dfrac{1}{8}-\dfrac{1}{216}+\dfrac{1}{12}}{\dfrac{1}{2}+\dfrac{1}{3}-\dfrac{1}{6}},$$

and divide ·000133 by 8·75.

2. State and prove the Rule for finding the Least Common Multiple of two given numbers.

Define a Prime Number. Express 364, 2520 and 5445 as products of powers of prime numbers.

3. Find the value of ·01625 of £204. 3. 4; and reduce 8 lb. 5 oz. 14 drs. to the decimal of a quarter.

4. What fraction when multiplied by itself produces $\dfrac{4880\frac{1}{3}}{5043}$?

What is the length of each side of a square court which contains 43785·5625 sq. feet?

5. Supposing a gallon to contain 277¼ cubic inches, find approximately the number of gallons of water which would cover a square mile of ground to the depth of an inch.

XVI. *Matriculation Examination. June,* 1871.

1. IF the circumference of a coach-wheel measures 17 ft. 7½ in., how often will it turn round in travelling a distance of 8 miles 264 feet?

2. At what rate of simple interest will £325 amount to £379. 3. 4 in 5 years?

What rate of interest for money can be obtained from Three per Cent. Stock, when £100 of Stock can be bought for £85 in cash?

3. Express [illegible], and [illegible] in their lowest terms.

4. Find the value in decimals of $\dfrac{1}{3+\dfrac{1}{7+\dfrac{1}{16}}}$;

and the quotient of the recurring decimal ·2323... divided by the recurring decimal ·28752875...

5. Extract the square root of 324·00005625; and find the value of

$$\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}-\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}},$$

in both cases to 6 places of decimals.

6. If a man can do a piece of work in 77 hours which a boy wants 121 hours for, in how many hours minutes and seconds can they do it conjointly?

XVII. UNIVERSITY OF OXFORD. RESPONSIONS.

Hilary Term, 1871.

1. FIND the present value of £1836 due four years hence at 5 per cent. simple interest.

2. What will be the cost of carpeting a room 25 feet long and 13 ft. 6 in. broad with carpet costing 10*s*. a square yard?

3. What is the rent of a field containing 112 acres 2 roods $29\frac{1}{2}$ poles at £2. 12. 10 an acre?

4. Divide 1368·2394 by 2400·21 and by ·00240021; and add together ·14, ·0116, and ·315.

5. Simplify (1) $\dfrac{1\frac{1}{3} \text{ of } \frac{27}{64}}{\frac{11}{12} \text{ of } 9\frac{9}{11}} \div \dfrac{4\frac{2}{7} \text{ of } \frac{21}{160}}{2\frac{2}{5} \text{ of } \frac{15}{34}}$;

(2) $\left(\frac{1}{3}+\frac{2}{5}+\frac{6}{7}+\frac{4}{15}\right)\div\left(1-\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{4}{5}\right)$.

6. Find the value of

$$\frac{15\frac{3}{8}}{7\frac{3}{8}} \text{ of } £1+\frac{1}{5} \text{ of } £140.\ 10.\ 6+\frac{3}{5} \text{ of a guinea.}$$

7. Find the Least Common Multiple of 5, 7, 16, 28, 48, 76: and the Greatest Common Measure of 1288, 1736, 104.

8. If 120 men can build a house 60 feet high in 15 days, how many will it take to build a house 55 feet high in 10 days?

9. Find the difference between the simple and the compound interest on £955 at 6 per cent., for 4 years.

10. A train 88 yards long overtook a person walking along the line at the rate of 4 miles an hour and passed him completely in 10 seconds: it afterwards overtook another person and passed him in 9 seconds. At what rate per hour was this second person walking?

XVIII. *Responsions. Trinity Term*, 1871.

1. FIND the value of 50,000 boxes of matches at 3*s*. 9*d*. per gross (12 dozen).

2. The floor of a room 19 feet 6 inches long by 13 feet wide is covered by a carpet each strip of which is 3 feet 3 inches in width. The strips of carpet cost 4*s*. 9*d*. per yard. What was the cost of the whole carpet?

3. What vulgar fraction is equivalent to the sum of 1·45 and ·175 divided by 2·5?

4. Simplify

$$\frac{4\frac{1}{6} - \frac{2}{3} \text{ of } \frac{5}{6}}{1\frac{3}{5} + 2\frac{3}{7} - \frac{2}{5}}, \text{ and } \frac{1}{\frac{1}{2} + \frac{1}{3} + \frac{\frac{1}{6}}{\frac{3}{4} - \frac{1}{3}}}.$$

5. Find the value of £2. 6. 8 ÷ $1\frac{3}{4}$ of $\frac{3}{7}$, and of

$\frac{3}{5}$ of 15*s*. + $\frac{5}{8}$ of 4*s*. 6*d*. − $\frac{3}{4}$ of 4*s*. 11*d*.

6. Multiply ·035 by ·00795, and divide 2·5 by ·36.

7. If a tithe-owner receives £104. 15. 1 for a tithe-rent-charge of the nominal amount of £100, what will he receive for a rent-charge of £113. 16. 6?

8. Find the Greatest Common Measure of 7056 and 7092, and the Least Common Multiple of 8, 12, and 20.

9. The Guernsey pound contains 18 ounces Avoirdupois, and the Guernsey shilling contains 13 English pence. If a Guernsey pound of butter costs 1*s*. 6*d*., Guernsey money, what will be the price in English money of $2\frac{1}{2}$ lbs. Avoirdupois?

10. The interest on £250 for 73 days amounts to £18. 15*s*. Find the rate of interest per cent. per annum.

11. If the price of £100 Three per Cent. Stock is $93\frac{1}{4}$ sterling, what would be the income obtained from investing £2000 sterling in that Stock?

XIX. UNIVERSITY OF CAMBRIDGE. PREVIOUS EXAMINATION.

March, 1869.

1. In subtraction how do you evade the difficulty of taking a greater digit from a less? Illustrate your answer by taking 791 from 943.

2. What is a measure?—a common measure?—the Greatest Common Measure? Find the G.C.M. of $13 \times 17 \times 19$, $17 \times 19 \times 21$, $19 \times 21 \times 13$.

3. Divide 24763 by 56 by short division. Explain how you determine the remainder. Justify your method.

4. Add together $23{\cdot}07\dot{6}$, $19{\cdot}24\dot{5}$, $31{\cdot}\dot{2}0\dot{3}$; and multiply $3{\cdot}62015$ by $1{\cdot}00236$.

5. Add $\frac{2}{3}$ of $\frac{3}{7}$ to $\frac{4}{7}$ of $2\frac{1}{3}$, and multiply the result by $\frac{3}{4}$ of $\frac{5}{8} \div \frac{3}{8} + \frac{1}{3}$.

6. What fraction is 8 lbs. 1 oz. 19 dwts. 9 grs. of 13 lbs. 7 oz. 5 dwts. 15 grs.? If 8 lbs. 1 oz. 19 dwts. 9 grs. cost £10. 6. 6, what will 13 lbs. 7 oz. 5 dwts. 15 grs. cost?

7. "15 ft. 4 in. × 14 ft. 6 in." Write the preceding in words, and explain what it may mean with reference to the next question.

8. Find the cost of varnishing the floor of a room 14 ft. 6 in. broad, and 15 ft. 4 in. long, at 6*d*. per square yard.

9. Find by Practice the cost of (i) 1842 articles at £2. 6. $9\frac{3}{4}$ each; (ii) 2 tons 7 cwt. 22 lbs. 5 oz. at £32 per ton.

10. A scuttle of coals is charged 6*d*. when coals are 27*s*. a ton; how much ought the scuttle to hold?

11. Find the difference between the Discount and Interest on £313. 19*s*. for 8 months at 6 per cent.

12. The Interest on £300 from 2 June to 20 Sept. is £2. 5. $2\frac{1}{2}$; at what rate is it calculated?

13. A tradesman's prices are 20 per cent. above cost price, if he allow a customer 10 per cent. on his bill, what profit does he make?

14. On a stream, B is intermediate to and equidistant from A and C; a boat can go from A to B and back in 5 hrs. 15 min., from A to C in 7 hrs. How long would it take to go from C to A?

15. I have a certain sum of money wherewith to buy a certain number of nuts, and I find that if I buy at the rate of 40 a penny I shall spend 5*d*. too much, if 50 a penny 10*d*. too little. How much have I to spend?

XX. *Previous Examination. Dec.* 1869.

1. WHAT is meant by Numeration? Express in figures twenty-five millions, thirty-one thousand, and seven.

2. When is one quantity said to measure another? What is the Greatest Common Measure of two numbers?

Find the G.C.M. of 1019527 and 1231845.

3. Simplify:

$$\frac{6\frac{4}{18}}{6-4\frac{4}{11}}+\frac{7}{9}\times 1\frac{3}{37} \text{ of } \frac{3}{56}-\frac{6+\cfrac{1}{6+\cfrac{1}{6-\frac{1}{6}}}}{6}.$$

4. Multiply 35·603 by 27·61.

Divide 186·4302 by 31·02; 18643·02 by ·3102; and ·1864302 by 3·102.

5. Multiply 43·291 by 6·24, and verify your result.

6. What fraction is (i) 9 poles 25 sq. yds. 4 sq. ft. 81 sq. in. of 29 poles 28 sq. yds. 2 sq. ft. 57 sq. in.? (ii) £9. 1. 3 of £12. 13. 9?

7. Find the expense of building a wall 108 yds. long, 3 ft. 8 in. high, 14 in. thick, at £14. 13. 4 per rod.

N.B. A rod of brickwork consists of 272¼ superficial feet, the work being 14 inches thick.

8. The area of a square garden is 4 roods 1 pole 29 yds. $6\frac{5}{8}$ ft., find the length of its side.

9. Find by practice the value of (i) 6420 articles at 4*s*. 3½*d*. each; (ii) 5 tons 3 qrs. 17 lbs. at £7. 3. 4 per ton.

10. *A* pays £9. 3. 4 more rates than *B*, their incomes being equal: living in different towns they are rated at 2*s*. and 1*s*. 4*d*. in the £ respectively; what is their income?

11. At what rate will the interest on £326 for 15 years amount to £220. 1*s*.?

12. *A* and *B* run a race of ¾ mile on a course ¼ of a mile round: they run in opposite directions and *A* wins by 40 yards; where was *B* when *A* passed the post the first time?

XXI. *Previous Examination. Dec.* 1869.

1. FIND the sum of the products two and two of 229, 373, 584.

Multiply the result by 10000 and express the result in words.

2. Divide eighteen thousand one hundred and five million and thirty-two thousand by nine thousand seven hundred and thirteen; and find [illegible] of the quotient.

3. Find the value of 500 times the difference between an eighty-fourth part of $2\frac{1}{8}$ cwt. and a thirtieth part of 1 cwt. 0 qrs. 3 lbs.

4. Find the value of $\frac{1}{6}$ of $\frac{4}{7}$ of $\frac{8}{15}$ of $2\frac{3}{4}$ of 247 guineas.

5. What will it cost to pave an area 266 ft. 3 in. long and 48 ft. 9 in. broad at $11\frac{1}{4}d.$ per square yard?

6. State and explain the rule for the multiplication of fractions. Multiply together the fractions $4\frac{1}{3}$, $2\frac{3}{4}$, and add the result to $4\frac{2}{3} + 3\frac{1}{3}$.

7. Divide 3833336 by $\cdot 031 \times 2\cdot 05$; and by $8\cdot 99 \times 20\cdot 8$.

8. Find the square of 58·019; and the square root of 18947·5225.

9. Find the value of $\frac{1}{5} \times \cdot\dot{4}\dot{7}$ of £360. 2. 3; and of $\cdot 01 \times \cdot 101$ of £74. 18. 6.

10. Define *interest* and *discount*.

What is the present worth of £5747 due 9 months hence, the rate of interest being $3\frac{1}{2}$ per cent.?

11. A ditch is being dug at the rate of 81 ft. per day by 54 men; after 13 days' work 8 of them are replaced by boys, and the work goes on for 11 days more, at the end of which the whole length dug is 1889 feet. How much work per day do the boys do?

12. A man buys goods at £15. 6. 3, and sells them again at £11. 15. $9\frac{3}{4}$. How much does he lose per cent.?

XXII. *Previous Examination. March*, 1870.

1. MULTIPLY 709514 by 381569, and express the result in words.

2. Multiply the difference between [illegible] and [illegible] by the sum of $4\frac{7}{18}$ and $1\frac{3}{8}$; and the result by the difference between $10\frac{3}{4}$ and $5\frac{3}{8}$.

3. Find the Greatest Common Measure and the Least Common Multiple of 936 and 2925.

4. Find the square roots of the sum and difference of 29347781 and 24983860.

5. The sum of the ages of A and B is now 60 years, and their ages 10 years ago were as 5 to 3. Find their present ages.

6. Reduce to their simplest forms the expressions:

$$\text{(i)}\ \left(\frac{1}{3}+\frac{4}{7}\right)\frac{20\frac{1}{4}}{3\frac{5}{7}+2\frac{1}{4}};\ \text{and (ii)}\ \left(3\frac{4}{5}+5\frac{1}{3}-\frac{1}{45}\right)\left(4\frac{2}{3}-3\frac{1}{4}\right)$$

divided by
$$1\frac{4}{11}+2\frac{1}{3}-\left(2\frac{9}{10}-\frac{1}{8}-\frac{1}{22}\right).$$

7. Divide ·0863547 by ·000713 to four places of decimals: also divide 104 by 7, and shew why the decimal recurs.

8. Find the value of ·414 + ·0352 + 6·101; and of ·90563 of £1.

9. Multiply £14. 6. 3 by 6·9869; and divide £147 by 5·137.

10. Define Simple and Compound Interest; and find the Interest Simple and Compound on £5000 for 3 years at $4\frac{1}{2}$ per cent.

11. A man buys goods at the rate of £24 per cwt.: and sells 3 tons 13 cwts. 1 qr. for £1000. How much has he gained or lost per cent. on his outlay?

12. If 175 men and 240 boys do in 1330 days the same amount of work as 603 men and 1005 boys in 350 days, compare the average daily work done by each man with that done by each boy.

XXIII. *Previous Examination. April,* 1870.

1. EXPRESS in words, 42146675; and, if the quotient obtained by dividing it by a certain number be 743, determine the number.

2. Simplify the fractions:

$$\text{(i)}\ \frac{6\frac{2}{3}-1\frac{6}{11}}{2\frac{1}{3}+1\frac{4}{7}}.\qquad \text{(ii)}\ \left(\frac{5}{7}\text{ of }1\frac{6}{13}\right)\div\frac{2\frac{5}{7}}{3\frac{1}{4}}.\qquad \text{(iii)}\ \frac{1}{4-\cfrac{1}{2-\cfrac{1}{1-\cfrac{5}{13}}}},$$

and to the difference of the first and third of these fractions add the second.

3. Multiply 76·371 by 8·54.

Divide ·37848 by ·456; 3·7848 by ·0456, and 3784·8 by ·00456.

4. Divide 2·9801 by 7·456.

5. What fraction of 2 qrs. 10 lbs. 7 oz. 9 drs. is 1 qr. 7 oz. 13 drs.?

What part is £1. 2 fl. 2 c. 5 m. of £6. 1. 6?

6. Find the expense of turfing a plot of ground, which is 40 yds. long, and 100 feet wide, with turfs each a yard in length and 1 foot in width; the turfs, when laid, costing 6*s*. 9*d*. per hundred.

7. A square block of stone, 2 feet in thickness, is, in cubic content, 5 cub. ft. 24 in.: what is the length of its edge?

8. Find by practice the value (i) of 5362 things at £1. 5. 3¼, (ii) of 3 lbs. 4 oz. 7 dwts. at £4. 2. 6 per lb.

9. *A* gives away in charity $\frac{1}{8}$ of his income, and pays $\frac{1}{10}$ of it in rates and taxes—with these deductions he has £473. 13. 1 left, what is his gross income?

10. A force of police 1921 strong is to be distributed among 4 towns in proportion to the number of inhabitants in each: the population being 4150, 12450, 24900, and 29050 respectively: determine the number of men sent to each.

11. Find the discount on £388. 17. 9 due 18 months hence, interest being reckoned at 4 per cent.

12. Find the alteration in income occasioned by shifting £3200 Stock from Three per Cents. at 86⅝, to Four per Cent. Stock at 114⅝: the brokerage being $\frac{1}{8}$ per cent. on each transaction.

XXIV. *Previous Examination. Dec.* 1870.

1. **SUBTRACT two thousand two hundred and two millions two thousand and two from eight thousand six hundred and sixty-six millions sixty-six thousand and sixty-six. Divide the difference by sixty-four, and express the quotient in words.**

2. Simplify:

(1) $\dfrac{8\frac{3}{8} - 7\frac{3}{4} + 5\frac{3}{8} - 4\frac{1}{8}}{13 - 11\frac{3}{10} + 10\frac{7}{8} - 9\frac{17}{20}} \times \frac{3}{11}$ of 365, (2) $\dfrac{6 + \dfrac{1}{6 - \dfrac{1}{6}}}{4 - \dfrac{1}{4 - \dfrac{1}{4}}} \times 10\frac{1}{4}$.

3. Divide 1·9517 by 673000 and 64000 by ·0008.

Reduce $4\frac{177}{1250}$ and $5\frac{43}{640}$ to decimals and ·9375 and ·4925 to vulgar fractions.

4. Find the value of ·0013671875 of a mile, and reduce 10 lbs. 5 oz. to the decimal of a ton.

5. Sound travels at the rate of 1140 feet a second. If a shot be fired from a ship moving at the rate of 10 miles an hour, how far will the ship have moved before the report is heard at a place $14\frac{1}{4}$ miles off?

6. Divide £29 into an equal number of half-sovereigns, crowns, half-crowns, shillings, sixpences and fourpences.

7. Extract the square root of 32239684, of $17\frac{8}{49}$ and of ·121 to three places of decimals.

8. What is the rent of 10 acres 3 roods 26 poles at £2. 8. $10\frac{3}{4}$ per acre?

If 3 cwts. 3 qrs. 21 lbs. $12\frac{3}{8}$ oz. cost £4. 8. 9, what is the price per cwt.?

9. If the cost of papering a room $8\frac{1}{4}$ yards long and $6\frac{2}{3}$ yards wide with paper 2 feet wide at 4*d*. per yard be £2. 19. 8, find the height of the room.

10. Find the present value of £808. 1. 4 due 3 years and 9 months hence at 4 per cent. per annum simple interest.

11. By selling out £4500 in the India Five per Cent. Stock at $112\frac{1}{2}$ and investing the proceeds in Egyptian Seven per Cent. Stock a person finds his income increased by £168. 15*s*. What is the price of the latter Stock?

12. If 3 per cent. more be gained by selling a horse for £83. 5*s*. than by selling him for £81, what must his original price have been?

13. At what distance from London will a train which leaves London for Rugby at 2·45 P.M. and goes at the rate of 41 miles an hour meet a train which leaves Rugby for London at 1·45 P.M. and goes at the rate of 25 miles an hour, the distance between Rugby and London being 80 miles?

XXV. *Previous Examination. Dec.* 1870.

1. MULTIPLY three hundred and forty-three millions four thousand nine hundred and seven by two hundred and sixteen millions three thousand six hundred and six.

2. Divide 830728 by 231 by short division and explain the process.

3. Simplify

$$(1)\quad \frac{\frac{1}{23} \text{ of } 6\frac{4}{17} \text{ of } 24\frac{[illegible]}{[illegible]} - 4\frac{[illegible]}{[illegible]} \times 3\frac{[illegible]}{[illegible]} \div 3\frac{[illegible]}{[illegible]}}{8\frac{[illegible]}{[illegible]} \times 5\frac{[illegible]}{[illegible]} \div 4\frac{[illegible]}{[illegible]} - 7\frac{[illegible]}{[illegible]} \times 5\frac{[illegible]}{[illegible]} \div 14\frac{[illegible]}{[illegible]}} \times 4\frac{[illegible]}{[illegible]}. \qquad (2)\quad 2\frac{[illegible]}{[illegible]} \times \frac{43089}{77500}.$$

4. Subtract 3·05 from 5·015 and divide 12·031 by 5300 and 10·24 by ·0128. Add together ·35 of 10*s*., ·54186 of 12*s*. 4*d*. and ·238 of 12*s*. 6*d*., and reduce the result to the decimal of £1.

5. A clock which was 1·4 minutes fast at a quarter to 11 P.M. on Dec. 2 was 8 minutes slow at 9 A.M. on Dec. 7. When was it exactly right?

6. Extract the square root of 546121, of $65\frac{[illegible]}{[illegible]}$ and of ·169 to three places of decimals.

7. After paying income-tax at the rate of 4*d*. in the pound, a man has £491. 13. 4 remaining. What is his income?

8. The sum which will pay *A*'s wages for $61\frac{1}{4}$ days, will pay *B*'s wages for $81\frac{2}{3}$ days. For how many days will it pay the wages of *A* and *B* together?

9. Find the present value of £578. 1. 4 due 3 years and 4 months hence at $4\frac{1}{2}$ per cent. per annum simple interest.

10. How many pounds of tobacco at 5*s*. 3*d*. per pound must a tobacconist mix with 4 lbs. at 6*s*. 6*d*. that he may sell the mixture at 7*s*. 10*d*. per pound and gain $33\frac{1}{3}$ per cent. upon his outlay?

11. The difference between the incomes derived from investing a certain sum in Six per Cent. Stock at 126 and in Nine per Cent. Stock at 210 is £22. 10*s*. What is the amount invested?

12. *A* can beat *B* by 5 yards in a 100 yards race and *B* can beat *C* by 10 yards in a 200 yards race. By how much can *A* beat *C* in a 400 yards race?

13. Two boats start to row a race at 3 o'clock. The race is over at $6\frac{7}{8}$ minutes past 3, the losing boat being 40 yards behind at the finish. At 4 minutes past 3 this boat was 700 yards from the winning-post. Find the speed of each boat in miles per hour.

XXVI. ST JOHN'S COLLEGE, CAMBRIDGE. *Dec.* 1864.

*** *Algebraical* symbols may not be used.

1. PROVE that the order of multiplication of two numbers is immaterial. Multiply 4327 by 814 and explain the process. Multiply 32856 by 121711 using 3 lines of multiplication only.

2. A person shooting at a target at a distance of 500 yards hears the bullet strike the target 4 seconds after he fired. A spectator equally distant from the target and the shooting-point hears the shot strike $2\frac{1}{2}$ seconds after he heard the report. Find the velocity of sound.

3. Define the G.C.M. of two numbers. Find that of 8029 and 23791. Find the G.C.M. and L.C.M. of 157 days 7 hrs. 4 min. 7 sec. and 243 days 2 hrs. 11 min. 49 secs.

4. Add together $\frac{1}{11}$, $\frac{6}{155}$, $\frac{4}{217}$, $\frac{2}{259}$, $\frac{8}{333}$, $\frac{7}{387}$,
and simplify

$$1\tfrac{5}{8} \text{ of } \frac{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}}{2\frac{1}{2}-3\frac{1}{3}+4\frac{1}{4}} \times \frac{\frac{3\frac{1}{2}}{3\frac{1}{4}}+\frac{\frac{3}{16}}{1\frac{3}{8}}}{\frac{3}{4\frac{1}{2}}+\frac{4\frac{1}{2}}{3}}.$$

5. Find the area of a room 17 ft. 4 in. long, and 14 ft. 3 in. broad, and the cost of carpet 1 ft. $11\frac{3}{4}$ in. broad, at 5*s*. $2\frac{1}{2}$*d*. per yard.

6. Find by Practice the cost of (i) 4321 articles at £7. 11. 8; (ii) 1978 at £3. 9. 2; and (iii) 214 acres 3 rds. 29 poles at £125. 7. 6 per acre.

7. Divide (i) 3·003 by 148·28; (ii) ·003003 by ·014828; and (iii) 300·3 by 1·4828.

Find the value of ·428571 of £3. 4. $3\frac{3}{4}$ + ·1875 of £5. 11. 8 + $\frac{2}{3}$ of ·81 of 4*s*. $9\frac{3}{4}$*d*. + ·001 of £1. 0. 10, and reduce it to the decimal of £5.

8. If 78 tons 10 cwts. 3 qrs. 10 lbs. 1 oz. cost £722. 4. $4\frac{1}{4}$, find the cost of 123 tons 8 cwts. 1 qr. 23 lbs. 13 oz.

9. Find the difference between the simple and compound interest on £2757. 3. $2\frac{3}{4}$ for 5 years, at 5 per cent.

10. Define discount. Find the discount on £201. 4. $7\frac{1}{2}$ for 25 days, at $3\frac{1}{4}$ per cent.

11. A person buys an article and sells it so as to gain 5 per cent. If he had bought it at 5 per cent. less, and sold it for 1*s*. less, he would have gained 10 per cent. Find the cost price.

12. A person has £5000 Three per Cent. Stock, which he sells and invests in Three-and-a-half per Cents. at $87\frac{1}{2}$. If the increase in his income be £5, what is the price of the Three per Cents.?

13. A person invests £14970 in the purchase of Three per Cents. at 90 and of Three-and-a-quarter per Cents. at 97. His total income being £500, how much of each Stock did he buy?

XXVII. ST JOHN'S COLLEGE. *Dec.* 1869.

1. EXPLAIN the principle of the decimal notation for integers. Write in figures one hundred and five millions one thousand five hundred and ninety eight: and divide that number by the number represented by MCDXCIX.

2. Find the number of hours from noon on Dec. 11th, 1769, to noon on Dec. 11th, 1869. What would be the amount of one farthing for each hour?

3. Find by Practice the cost

(α) of 23875 articles at £2. 18. 9 per 100,

(β) of 14 acres, 3 roods, $26\frac{1}{4}$ poles, at £52. 7. 6 per acre.

4. Find the Greatest Common Measure of 441441 and 844272 and the Least Common Multiple of 7, 11, 21, 63, 91, 99, 117, 143.

5. Simplify $\frac{1}{7}-\frac{1}{9}-\frac{1}{11}+\frac{1}{13}$; and multiply the result by the sum of $\frac{17}{64}$ and $\frac{9}{25}$.

6. State the rules for reducing terminating and circulating decimals to vulgar fractions.

Express as vulgar fractions $\cdot 037185$ and $\cdot 637185$; and reduce to a single decimal.

$$\frac{\cdot 04275}{3\cdot 05} \times \frac{4\cdot 216}{\cdot 342} \times \frac{2\cdot 7}{1\cdot 5318}.$$

7. Reduce 3 cwts. 3 qrs. 5 lbs. 4 oz. to the decimal of 1 ton; and find the value of that decimal of £4.

8. An article which cost 19 guineas per cwt. is retailed at 4*s*. 6*d*. per lb., there being a waste of 5 per cent. What is the rate of profit per cent.?

9. If the interest on £253. 2. 6 at 5 per cent. be equal to the discount on £257. 6. 10½ for the same time at the same rate, when is the latter sum due?

10. A person buys Six per Cent. Bonds, the interest on which is payable yearly and which are to be paid off at par, 3 years after the time of purchase: if money be worth 5 per cent., what price should he give for the Bonds?

11. The Three per Cents. are at 91½ and the Three-and-a-half per Cents. at 96¼. A person has a sum of money to invest which will give him £100 more of the former stock than of the latter. Find the difference of the income he could obtain by investing in the two stocks.

12. A contractor employs a fixed number of men to complete a work. He may employ either of two kinds of workmen: the first at 26*s*. 6*d*. per week each, the second at 18*s*. 6*d*. per week each; the work of one of the former being to that of one of the latter as 5 to 4. If he finishes it as quickly as possible, he spends £270 more than he would have done if he had finished it as cheaply as possible, but takes 4 weeks less time. What would it have cost if he had employed equal numbers of the two kinds of workmen?

XXVIII. St John's College. *Dec.* 1870.

1. Express in figures three hundred and sixty-two millions two thousand and seven, and in words 916892363207421, and divide the latter number by the former.

2. Multiply 1389 by 3175, and explain the process.

By what factor less than 1000 must 4389 be multiplied so that the last three figures of the product may be 438?

3. A gallon contains 277·274 cubic inches: a cubic foot of water weighs 1000 ounces: how many gallons will weigh a ton? and what is the weight of a pint?

4. Define a fraction, and shew that its value is not altered if its numerator and denominator be both multiplied by the same number.

Find the sum of

$$\left(\frac{5}{7}+\frac{5}{2}\right),\quad \left(\frac{5}{2}-\frac{5}{7}\right),\quad \frac{5}{7}\times\frac{5}{2},\quad \frac{5}{7}\div\frac{5}{2},\ \text{and}\ \frac{5}{2}\div\frac{5}{7}.$$

Reduce 16 days 12 hrs. 47 min. 25 secs. to the fraction of 23 days 3 hrs. 30 min. 23 secs.

5. Enunciate the rule for the division of decimals.

Divide (i) 56·259 by 1·316.
(ii) ·56259 by 13·16.
(iii) 5625·9 by ·001316.

Find the value of ·0001122334455667789 of 4 miles 4 furlongs.

6. Find the area of a court 59 ft. 4 in. long, and 48 ft. 9 in. wide, and also (by Practice) the cost of paving it at 4*s.* 6*d.* per sq. yard.

7. A sum of money was put out to compound interest: the first year's interest was £65. 2. 1, and the fourth year's interest was £73. 4. 8. What was the sum, and the rate per cent.?

8. If the discount on £261. 2*s.* at $2\frac{1}{2}$ per cent. be 17*s.*, when is the sum due?

9. A person by selling an article, which cost £14 per cwt., at 2*s.* $9\frac{5}{8}d.$ per lb. makes 5 per cent. more profit than he would do if he sold the whole for £55. 15. $3\frac{3}{4}$. What was the amount sold?

10. A person having to pay £402. 3. 9 two years hence invests a certain sum in the Three per Cent. Consols, and also an equal sum next year together with the interest already received. Supposing the price of Consols to remain throughout at 96, what must be the sum invested on each occasion so that there may be just sufficient to pay the debt at the proper time?

11. Three men are employed in a work, working respectively 8, 9, 10 hours per day, and receiving the same daily wage. After three days each works one hour a day more, and the work is finished in three days more. If the total sum paid for wages be £2. 7. $6\frac{1}{4}$, how much of it should each receive?

12. Ash saplings after five years' growth are worth 1*s.* 3*d.*, and increase in value 1*s.* 3*d.* each year afterwards. For their growth they require each twice as many square yards as the number of years they are intended to grow before cutting. A plantation is arranged so that each year the same number may be ready for cutting. Find the greatest annual income which can be obtained per acre, allowing 20 per cent. for expenses.

ANSWERS.

N.B. *The answer has not been given when it is to be written off directly from the question.*

Ex. 3.

1. 149050. 2. 340667. 3. 47423136. 4. 382169. 5. 959905480.
6. 488592013. 7. 31817108. 8. £75249177.

Ex. 4.

1. 84489. 2. 757569. 3. 533884537. 4. 14223165.
5. 22371. 6. 1273. 7. 6140.
8. 57; 426; 1; 457944; 140936.

Ex. 5.

1. 52070352; 45561558; 58579146; 71596734; 78105528.
2. 1014848586; 1973316695; 2706262896; 4059394344; 7442222964.
3. 241497865244; 296988105062; 5327809224181.
4. 2863151878 4; 2132481188 64; 6384027856 7472.
5. 2299320000; 51734700000; 24717690000; 2684456100000000.
6. 0. 7. 18489025. 8. 147603 yards. 9. 157739562; 303794712; 4288179204. 49110419796; 144872064531; 63723226584.
10. 1515868.

Ex. 6.

1. 99130823; 66087215...4; 113192369...1; 88116287...1; 72095144.
2. 25280669...14; 21753134...5; 2687887...91; 193421...811.
3. 42439...5498; 26171...7874; 5822...6639; 4167...2898.
4. 2733534...7; 1195921...15; 1063041...7; 797280...79.
5. 28716695...68; 19339815...68; 10028052...152.
6. 1308913...46; 98168...426; 6544...6826; 24542...426.
7. 12433128...54; 653266...334; 6566...27734; 42401...18834.
8. 9396...538. 9. 3507. 10. 6905. 11. 1887378. 12. 109; 278.
13. 67; 999. 14. 67. 15. 191919 miles...95 miles. 16. 13212.
17. 5193171.

Ex. 7.

1. 1055559. 2. 82152. 3. 15610; 7814. 4. 45653376; 8862.
5. 1639799; 1; 5185. 6. 1877; 1217. 7. 73; 1504. 8. 1008; 1380.
9. 8969578. 10. 5761; 76. 11. $(325-293)\times(306\div17)+(1000+99)$; 1675. 12. $34856-763\times41+1998\div(663-441)$; 3582.
13. 106; 1050. 14. 9801; 639. 15. 33211521848. 16. 10091401.
17. Each side is equal to (1) 1870; (2) 792; (3) 717409; (4) 1481040; (5) 1771561; (6) 290521; (7) 879499511; (8) 755301239.

Ex. 8.

1. 17; 27; 55. 2. 3; 5; 707. 3. 37; 999; 1. 4. 365; 571.
5. 127; 2476099. 6. 11; 21. 7. 59. 8. 69. 9. 3431.
10. 21. 11. 84. 12. No, G.C.M. is 27. Yes. 13. Yes; Yes.
14. ·63 gallons. 15. 40 grains. 16. 35. 17. 25. 18. No.

Ex. 9.

1. 1716; 734877; 1225449. 2. 9896425; 637225; 4040400.
3. 67868155; 203667; 20044430912. 4. 159137; 4019051.
5. 1628055; 7258671. 6. 27720; 334639305. 7. 720720.
8. 456326325. 9. 10080; 7560. 10. 11880; 352800.
11. 180180; 27720. 12. 25200; 9828. 13. 863940. 14. 98280.
15. 602910. 16. 2520 sec. = 42 m. 17. 8. 18. 76. 19. 981832.
20. 875. 21. 60 min.

Ex. 10.

1. 17 : pr.; 23 : 23. 2. pr.; 37 : 7.
3. 41; pr.; 53. 4. 4. 5. 5. 6. 5.
7. $2.3.7.11$; $2^2.5.23$; $2^6.5$; $2^2.3^2.11$.
8. $7.11.17$; $3^2.5.7$; $2^4.5^4$.
9. $3^2.7.29$; $3^3.5.11$; $2^4.3^4.5^2.7$.
10. $2^2.3.5.7.131$; $2^2.3^2.7.11$; $5^2.7.11^2.19$.
11. $3^6.5.7.11$; $7.13.19.31$; $7^2.31^2$.
12. $3.5^2.7.13^2$; $5.7^2.13.31$; $7^2.13^2.23$.
13. $11^2.13.17.19$; $7^2.17^2.293$.
14. $17^2.29.31$; $13^2.17^2.89$.
15. 3^2; $2^2.3^3.5.7.11.13=540540$.
16. 1; $2^2.3^3.5^2.7.11^2=2286900$.
17. $3.5.7=105$; $3^2.5^2.7^2=11025$.
18. $3^2.7.11=693$; $3^3.7^2.11^2=160083$.
19. 19; $13.17.19.21=88179$.

Ex. 11.

5. [illegible]
6. [illegible]

7. [illegible]; [illegible]; [illegible]; [illegible]; [illegible]; [illegible].
8. 12; 11; $11\frac{[illegible]}{[illegible]}$; 11; $109\frac{[illegible]}{[illegible]}$; $22\frac{[illegible]}{[illegible]}$; $16\frac{[illegible]}{[illegible]}$; $43\frac{[illegible]}{[illegible]}$; $135\frac{[illegible]}{[illegible]}$.
9. $176\frac{[illegible]}{[illegible]}$; $63\frac{[illegible]}{[illegible]}$; $76\frac{[illegible]}{[illegible]}$; $281\frac{[illegible]}{[illegible]}$; $83\frac{[illegible]}{[illegible]}$; $1012\frac{[illegible]}{[illegible]}$.
10. $2\frac{1}{7}$; $3\frac{4}{[illegible]}$; $3\frac{[illegible]}{[illegible]}$; $58\frac{74}{100}$; $28\frac{70}{[illegible]}$; $46\frac{[illegible]}{[illegible]}$; $884\frac{[illegible]}{[illegible]}$.
11. $142\frac{[illegible]}{[illegible]}$; $137\frac{[illegible]}{17}$; $40\frac{7}{[illegible]}$.

Ex. 12.

1. [illegible]; [illegible]; [illegible]. 2. [illegible]; [illegible]; [illegible]. 3. [illegible], [illegible], [illegible], [illegible].
4. [illegible]; [illegible]; [illegible]. 5. [illegible]; [illegible]; [illegible].
6. [illegible]; [illegible]; [illegible]; [illegible]. 7. $5\frac{2}{3}$; 12; $8\frac{[illegible]}{11}$; $37\frac{[illegible]}{[illegible]}$; 161; $70\frac{[illegible]}{[illegible]}$.
8. [illegible]; [illegible]; [illegible]; [illegible]; [illegible]; [illegible]. 9. [illegible]; [illegible]; $1\frac{[illegible]}{[illegible]}$; $3\frac{[illegible]}{[illegible]}$.
10. $1\frac{[illegible]}{[illegible]}$; [illegible]; $1\frac{[illegible]}{[illegible]}$. 11. 12; $2431\frac{[illegible]}{[illegible]}$. 12. $68\frac{[illegible]}{[illegible]}$.

Ex. 13.

1. [illegible], [illegible], [illegible]. 2. [illegible], $\frac{4}{7}$, $\frac{7}{11}$. 3. [illegible]; [illegible]; [illegible].
4. [illegible], [illegible]. 5. [illegible], [illegible]. 6. [illegible]; [illegible].
7. [illegible], [illegible], [illegible]. 8. [illegible]; [illegible]; [illegible]. 9. [illegible]; [illegible].
10. [illegible]; [illegible]. 11. [illegible]; [illegible]. 12. [illegible]; [illegible].
13. $7\frac{[illegible]}{[illegible]}$; $17\frac{[illegible]}{[illegible]}$; $55\frac{[illegible]}{[illegible]}$. 14. 63; $94\frac{1}{7}$; $87\frac{[illegible]}{[illegible]}$.

Ex. 14.

1. [illegible], [illegible], [illegible], [illegible]; [illegible], [illegible], [illegible], [illegible].
2. [illegible], [illegible], [illegible], [illegible], [illegible]; [illegible], [illegible], [illegible], [illegible].
3. [illegible], [illegible], [illegible], [illegible]; [illegible], [illegible], [illegible], [illegible].
4. [illegible], [illegible], [illegible], [illegible], [illegible]; [illegible], [illegible], [illegible], [illegible], [illegible].
5. [illegible], [illegible], [illegible], [illegible], [illegible], [illegible]; [illegible], [illegible], [illegible], [illegible], [illegible], [illegible]. 6. [illegible], [illegible], [illegible], [illegible], [illegible]; [illegible], [illegible], [illegible], [illegible], [illegible].
7. [illegible], [illegible], [illegible]; [illegible], [illegible], [illegible]; [illegible] of $9\frac{[illegible]}{[illegible]}$, $3\frac{[illegible]}{[illegible]}$, [illegible]. 8. $\frac{15+8}{19+8}$, [illegible].
9. [illegible], $\frac{8+9}{17+25}$, [illegible]; [illegible], $\frac{13+15}{14+16}$, [illegible]; [illegible], [illegible], $\frac{5+6+7+9}{6+7+8+10}$, [illegible], [illegible].
10. [illegible], [illegible]; [illegible], [illegible]; [illegible], [illegible]. 11. [illegible]; [illegible]. 12. [illegible].
13. [illegible], [illegible], [illegible], [illegible], [illegible].

Ex. 15.

1. 3; $16\frac{[illegible]}{[illegible]}$; $33\frac{[illegible]}{[illegible]}$. 2. $3\frac{[illegible]}{[illegible]}$; $3\frac{[illegible]}{[illegible]}$; $3\frac{[illegible]}{[illegible]}$.
3. 2; $2\frac{77}{[illegible]}$; $3\frac{[illegible]}{[illegible]}$. 4. 25; $10\frac{[illegible]}{[illegible]}$; $2\frac{[illegible]}{[illegible]}$. 5. 5000; $4\frac{[illegible]}{[illegible]}$.
6. $18\frac{[illegible]}{[illegible]}$; $73\frac{[illegible]}{[illegible]}$. 7. $15\frac{[illegible]}{[illegible]}$. 8. $49\frac{[illegible]}{[illegible]}$. 9. 1. 10. [illegible] + [illegible].

Ex. 16.

1. [illegible]; 3[illegible]; [illegible]; 2[illegible]. 2. 6[illegible]; 17[illegible]; [illegible]; [illegible]. 3. [illegible]; 3[illegible]; 1[illegible].
4. 16[illegible]; 5[illegible]; 1[illegible]; 13[illegible]; 1[illegible]; 2[illegible]. 5. 1; [illegible].
6. 0; 0; [illegible]. 7. 1[illegible]; [illegible]. 8. [illegible]; [illegible]; 1[illegible];
9. 10[illegible]; 1[illegible]. 10. [illegible]. 11. [illegible]. 12. 29[illegible].
13. 3[illegible]; 34. 14. 1[illegible]. 15. 12[illegible]. 16. [illegible].

Ex. 17.

1. [illegible], [illegible], [illegible], 17, 52, 61[illegible]. 2. 451[illegible], 1293[illegible], 90, 5[illegible].
3. 26, 1267[illegible], 8[illegible]. 4. 1[illegible], 1[illegible]. 5. 37[illegible], 8.
6. 2[illegible], 60[illegible], 3. 7. 1[illegible], 4981[illegible].
8. 117, 441[illegible], 1947[illegible], 37389[illegible], 168726[illegible]. 9. 66[illegible], 33[illegible], 14.
10. 22[illegible], 2[illegible], 14[illegible]. 11. 26[illegible], 18[illegible].
12. 372399[illegible], 9989992[illegible], 492[illegible]. 13. 2500.
14. [illegible]. 15. 109[illegible]. 16. 13[illegible].

Ex. 18.

1. 43[illegible], 194[illegible], 70[illegible], 182[illegible], 92[illegible]. 2. 156, 9[illegible], 1[illegible], 4[illegible], 5[illegible].
3. 1[illegible], [illegible], 1[illegible], [illegible]. 4. [illegible], [illegible], 4[illegible]. 5. 8, [illegible].
6. [illegible], [illegible]. 7. 1[illegible], 16. 8. 1[illegible].
9. 41772[illegible], [illegible], [illegible]. 10. 7[illegible], 9[illegible], 1[illegible].
11. [illegible], 1½. 12. [illegible], [illegible]. 13. [illegible]. 14. 1[illegible].
15. [illegible]. 16. 5[illegible]. 17. 22[illegible]. 18. 8[illegible].

Ex. 19.

1. [illegible]; 24. 2. [illegible]; 146[illegible]. 3. [illegible]; 60. 4. [illegible]; 18[illegible].
5. [illegible]; 236[illegible]. 6. [illegible]; 1801[illegible]. 7. [illegible]; 70[illegible].
8. [illegible]; 4458[illegible].

Ex. 20.

1. [illegible]; 3[illegible]; [illegible]; [illegible]; [illegible]; [illegible]; [illegible]; 1[illegible]. 2. 1; 1; 2[illegible]; [illegible].
3. 3; [illegible]; [illegible]; [illegible]. 4. 7; 5[illegible]; 1[illegible]; 4[illegible].
5. 3[illegible]; [illegible]; [illegible]. 6. 2[illegible]; [illegible]; [illegible]; [illegible]. 7. 1; 1.
8. 3[illegible]. 9. [illegible]; 7[illegible]. 10. [illegible], 1[illegible], [illegible]. 11. 1; 1.
12. [illegible]; [illegible]. 13. 1[illegible]; 2[illegible]; 4[illegible]. 14. 3[illegible], 4[illegible]; 14[illegible].
15. 6[illegible]; 2. 16. [illegible]. 17. [illegible]. 18. [illegible]. 19. [illegible]. 20. 38[illegible].

Ex. 21.

13. $\frac{36}{5}$; $\frac{11}{16}$; $\frac{333}{500}$; $\frac{72003}{3125}$. 14. $\frac{11}{16000}$; $\frac{15031}{3200}$; $\frac{3475}{16384}$.

15. $13\frac{27}{1600}$; $9\frac{711}{1600}$; $23\frac{11}{1600}$. 16. $89\frac{1024}{78125}$; $12\frac{288}{2048}$.

Ex. 22.

1. 6876·219354. 2. 417·11157. 3. 1582·357845.
4. 669·3001. 5. 28·88544; 6·024696.
6. 8·00001; 8·99949. 7. 405·970973. 8. 9932·1237939.
9. 30·836; 55·92641; ·00099. 10. 4·911002; ·088913.
11. 16·6799884. 12. 15·635563. 13. 6·171256.
14. ·034353. 15. ·2218487; ·0457575; ·999644; 2·345679; 1·101.
16. 3·1415927.

Ex. 23.

1. 982·99883; 230·625593; 3·399844.
2. ·8499745; ·00656448; ·82008. 3. ·0415584; ·0001; 49·070707.
4. ·001353; ·00003738028; ·7614948.
5. ·0487291247; 290; ·819. 6. ·0177775; ·001.
7. 12·66806; 23·68676; ·001. 8. ·0672; 9·56709.
9. 10; 10·01; 1545; 110000. 10. 129300; ·7567; 2260.
11. 1·37; ·0081; 3267·578125.
12. 104·69785..., 2·8546136..., ·2181659...
13. ·012; 790; 91·428571...
14. 24018000; 2590; 2895·764013.
15. 567·8; ·576; ·1385. 16. ·007853; ·0032; ·0046.
17. 21·32; 2500000; ·009990009990... 18. 300.
19. ·00213...; ·0065008... 20. 6·0360...; 91·336...
21. ·0121681...; 36542894·0328...
22. ·30685; 20500. 23. 147; 2296.
24. 15·845686; ·00011... 25. ·9; 85·1; 326·4.
26. 2812·5. 27. 961·6832321. 28. 6400.
29. 2299·95; 5909·00011875. 30. 11531·48614...

Ex. 24.

1. 31·06; 267·90. 2. 643·319; 2·246. 3. 27·00295; 15·45645.
4. 2180·5103. 5. 64·20153. 6. 5·74. 7. 30969. 8. ·95423.
9. ·392754. 10. 14·26. 11. ·1495. 12. 634·3. 13. 26·451577.
14. ·13059. 15. 30·799. 16. ·434294; ·318309.
17. 1·1037; 1596; 1·5446. 18. 1·15864; 1144·32158.
19. 1·2312; 290·4484. 20. 7238·88888; ·70318; 2148·58183.

Ex. 25.

1. ·875; ·376; ·8125; ·03375; 1·84375; ·3216; ·7578125.

2. ·32768; ·68359375; ·0135546875; ·222464; 1·0009765625; 1·6878417796875.

3. ·00081, ·0722; ·35912125; 37·6650390625; ·112.

4. 5·83925. 5. 2·183125. 6. 83·03355. 7. ·0001; ·008686.

8. ·7142...; 1·923... 9. 89·235294...; 1·88872323...

10. 0. 11. [illegible]. 12. 2, 4, 5, 8, 10, 16.

13. 1·6; 4·8; 3·614583; ·464; ·572.

14. ·9146; 1·90; 9·369; 1·1392045.

15. 18·1893; ·0126; 5·783; 1·2594.

16. 2·733108; 5·2871; 1·00198; 1·0850694.

17. ·72509; 3·3567; ·20432; ·98712.

18. 1·95121; ·73800; 1·29573170; ·4990774.

19. ·714285; 7·461538; 4·803571428; ·126984; 7·1893.

20. 13·942307609; ·944055; 2·457002; ·75240384615.

21. 3·4556097; ·907967032; ·204633; ·999000.

22. 1·761145; 3·780003; ·94117647058823352; ·894736842105263157.

23. [illegible]; 8[illegible]. 24. [illegible]; 3[illegible]; 5[illegible].

25. [illegible]. 26. [illegible]; 2[illegible]; [illegible]; 3[illegible].

27. [illegible]; 42[illegible]. 28. [illegible]; 8[illegible]; [illegible].

29. 37[illegible]; [illegible]. 30. 3[illegible]; 5[illegible].

31. 2[illegible]; 18[illegible]. 32. [illegible]; 13[illegible].

Ex. 26.

1. 408·6420311; 13·259435526. 2. 73·52494221; 100.

3. 44·5012779837. 4. 107·77164749359.

5. ·0066; 8·6198; 734·668367003.

6. 2·9957; 3·661923286; ·7131493506.

7. ·95436; ·9216; 74·3579629. 8. 3·3878; 4·1406; 13·927; 13·36.

9. 38·0375; ·510146306; 8·3876426.

10. 731·164399; 2715·1213693; 71·88031782 35.

11. ·432244318; 5·376548977; ·74878703.

12. ·490578703; 4·81816396103 89.

13. 1·185; ·63; 270[illegible] = 270·3300633967; 3·63.

14. 36·6; 1·36; 67·45; 580·90.

15. 12[illegible] = 12·847923681257014 5903; 46178·683621;

$$\frac{181313}{8\cdot4\cdot9\cdot9} = 69·951003086419 75.$$

16. 2·2; 1·42066; 3·27; 134·4; 8·18.

17. 6·945126; 2·5227; ·24; ·77.

18. ·4459; 2·9714285; 21; 8·18746; 6·7047619.

19. ·16; 1·6875. 20. ·8863; 4[illegible]; 37[illegible].

21. 4·216893; ·0245706. 22. 2530·6. 23. 3·083; [illegible].

24. ·0002938. 25. (1) is greatest and (2) least. 26. 1120·11200069.

27. 3887·755... 28. $\frac{355}{113}$, 3·1415926 and $3+\dfrac{1}{7+\frac{1}{16}}$.

29. ·36; ·85; 1·115; ·0094.

30. (1) ·249999... (2) ·166666... (3) ·00097059. (4) 1·718281...
(5) ·202732... (6) 3·141592...

Ex. 27.

1. 234; 897; 907; 3579; 9878; 4607.

2. 9·68; ·359; ·1679; ·0907; 60·49; ·7008.

3. 76963; 56804; 87056; 76008; 80047.

4. 15·367; 534·762; 20·0137; ·0708069.

5. 203·975; 90888; 600·098; 9688669.

6. 347·6905; 76050·4009; 887145333.

7. [illegible]; 268[illegible]; 5[illegible]; 23[illegible]; 55[illegible]; 4·3; 1·83; 68·83.

8. 4·92; 55·9; 27·5028; ·25; ·083.

9. 4·4721..., ·00031159; ·3162..., ·00001756; 1·7724..., ·00019089;
·1095..., ·00000975; 13·2382..., ·00062476; 3·5449..., ·00004399.

10. ·0824621...; ·7745966...; ·9607689...; 4·6612522...; 5·2164104...;
20·4939015...; ·0967000.

11. ·01705544487...; 3·12889756943...; 9·89444288483...;
·97014250014...; ·92195444573...; ·26134268690...;
·58881236400...; ·37176791431...

12. 8·68026497291413...; 1·73205080756887...; ·845154254728 51...
13. ·587869106393 28146...; ·00594306318324145...;
·265908011739155 21...
14. ·164575131106459059050...; 1·25299640861416677 88 49...;
7·100704190430692769065...

Ex. 28.

1. 97; 234; 6·16; ·192; ·0299. 2. ·289; 56·09; ·4339; 7084.
3. ·3842; 1·305; 5836; 20·37. 4. 8888; ·4607; 9·889; 800·8.
5. 45333; 1·9989; 563·89. 6. 459684; 886437163.
7. $\frac{11}{11}$; $\frac{24}{17}$; $1\frac{4}{9}$; $7\frac{7}{10}$; $3\frac{2}{3}$; $3\frac{4}{7}$; $15\cdot\dot{6}$.
8. 10·1120 72614...; 1·8081931033...; ·2152134170...; 1·7099759466...;
·85058074056...; 1·9813073175...
9. 1·5135076271833...; 1·395612425086...; ·7539474411293...;
·4135115219901 1...; ·4286472727229153...; ·687634552177 16...
10. ·9226318777 0011...; 2·6487514390 4066...; ·1259921049 8948...;
·2884499140 6148...
11. 1·2980246132766675 44089...; ·1442249570307408 3823...;
7·3554728085677827 4984...; ·2351334687720075 7489...;
4·3411712066252992 30788...
12. 1·5511335180712...; ·6446898173744...; 2·1286880406 3096...
13. 214·4014925321183...; 1·2840254166877...; 4·5694638635 18...;
·1693123346 56...; 2·8060662632 96...; 3·7789942409 59...
14. 799; 4·1844684350 6...; 2·0551537959234...; ·868301469035 55...
15. 1·6597013896947...; ·4937448592186...; ·9588131805648...
16. ·9595889443 9...; 2·4341470225193...; ·9474839202664...;
1·2863841294624...; 1·6335444707528...
17. $\frac{8}{15}$ = ·5$\dot{3}$; 5·$\dot{3}$; $\frac{163}{38}$ = 4·289473684 2...
18. Each expression = ·2679491924 3112... 19. 1·35.
20. 886437165·3932815252769293...

Ex. 29.

1. 3 : 5; 9 : 49; 7 : 8; 2 : 3; 209 : 510; 12 : 6 : 4 : 3; 16 : 18 : 20 : 21.
2. 2 : 5; 9 : 50; 100 : 483. 3. 7 : 9; 8 : 15; $4\frac{1}{3}$: $11\frac{2}{3}$.
4. No. 5. Yes. 6. 6. 7. $3\frac{41}{141}$.
8. $9\frac{3}{4}$; 70; ·0001; 770; ·0135... 9. 6; ·01; 2·6933...
10. 91 : 81. 11. 16 : 24 : 30 : 35. 12. 323 : 324.
13. 72. 14. $\frac{1}{2}$. 15. 9 : 8.

Ex. 30.

1. 82481*d.*; 122721*f.*; 46079*f.*; 422353 ($\frac{1}{2}d.$).

2. 66374375*s.*; 260273971. 3. 53600 yds.; 205862 in.

4. 1635033 in.; 1600555 in. 5. 83061$\frac{1}{4}$ sq. yds.; 374951$\frac{1}{4}$ sq. ft.

6. 54650895 sq. in.; 333274481 sq. in.

7. 29194441 cu. in.; 129139623 cu. in.

8. 3220671 oz.; 534793 dr. 9. 79480 grs.; 113633 grs.

10. 132401045 grs.; 459705 m. 11. 3046 gills; 3518 gall.

12. 2735$\frac{5}{8}$ in.; 667960276$\frac{1}{2}$ grs.

13. £1440. 6. 7; £2057. 12. 2$\frac{1}{2}$; £135. 8. 7$\frac{1}{4}$.

14. £1041. 13. 8$\frac{3}{4}$; 1 year (of 365 days) 47 wks. 10 h. 40 m.;
7 weeks 3 d. 21 h. 4 m. 17 s.

15. 5 weeks 8 h. 48 m. 29 s.; 42 weeks 8 h. 30 m. 45 s.;
4 miles 1 f. 36 p. 1 yd. 1 ft. 7 in.

16. 349 miles 7 f. 18 p. 1 ft.; 32 miles 4 f. 33 p. 1 yd. 1 ft. 6 in.;
49 miles 2 f. 31 p. 3 yd. 1 ft. 11 in.

17. 1 acre 1 r. 19 yds. 8 in.; 2 acres 13 yds. 7 ft. 121 in.;
15 acres 14 p. 14 yds. 7 ft. 72 in.

18. 185 acres 2 p. 26 yds. 4 ft. 72 in.; 1 acre 2 r. 15 p. 2 yds. 2 ft. 100 in.;
4 acres 13 p. 5 yds. 7 ft. 109 in.

19. 21 cu. yds. 4 ft. 189 in.; 45 cu. yds.; 39 tons 2 cwt. 2 qrs. 14 lb.

20. 10 tons 11 cwt. 26 lbs. 11 oz.; 2 tons 19 cwt. 7 lbs. 6 oz. 3 dr.;
1 ton 5 cwt. 2 qrs. 1 lb. 2 oz. 125 grs.

21. 73 lbs. 9 oz. 12 dwts. 7 grs.; or 885 Tr. oz. 295 grs. (Art. 209);
11 C. 3 O. 19 fl. oz. 23 m.; 234 gall. 3 qt. 1 pt.

22. 761 gall. 1 pt. 1 gill; 13 loads 3 qrs. 2 pk. 1 gall.;
38 loads 2 pk. 1 gall. 2 qt.

23. 625008 Tr. oz. 24. 789 miles 1 f. 33 p. 5 yd. 1 ft. 4 in.

25. 124897 ($\frac{1}{4}d.$); 2289 sixpences.

26. 106900 threepences; 18853 half-crowns and 18*d.* over.

27. 172 thalers and 2*s.* 9*d.* over; 2130 dollars and 1*s.* 8*d.* over;
3478 fr. and 1$\frac{1}{2}d.$ over. 28. 3097600 sq. yds.; 102400 p.

29. 80 yds. 1 qr. 3 nl. 1$\frac{1}{4}$ in.; 263 ells 2 qr. 1 nl. $\frac{3}{4}$ in.; 43 yds. 2 ft. $\frac{3}{4}$ in.

30. 394 lb. 11 oz. 11 dwt. 16 grs.; 703 lb. 5 oz. 18 dwt. 13 grs.

31. 27192*d.* 32. £2191. 10*s.* 33. 189000.

34. 2678400*s.*; 6958800*s.*; 15195600*s.*

Ex. 31.

1. £23089. 9. $8\frac{3}{4}$. 2. £10086. 4. $8\frac{1}{4}$. 3. £145568. 6. 7.
4. £4908. 1. $4\frac{3}{4}$. 5. £59599. 17. $9\frac{3}{4}$.
6. 55 lb. 413 grs. 7. 44 cwt. 1 qr. 13 oz.
8. 107 tons 8 cwt. 3 qr. 2 lb. 12 oz. 9. 51 lb. 11 oz. 4 dwt. 3 grs.
10. 8 c. 6 o. 7 fl. oz. 11. 187 gall. 2 qt. 1 gill.
12. 17 yds. 1 qr. 2 nl. $1\frac{1}{4}$ in. 13. 12 ells 2 qr. 3 nl.
14. 21 lds. 3 qr. 1 pk. 1 gall. 15. 65 days 23 h. 26 m. 32 s.
16. 65 mo. 1 da. 15 h. 8 m. 17. 40 years 262 da. 54 m. 17 s.
18. 2 fur. 11 p. 3 yds. 2 ft. 10 in. 19. 2 miles 1 f. 15 p. 5 yds.
20. 29 miles 4 f. 6 p. 2 yds. 1 ft. 6 in. 21. 2 roods 6 p. 15 yds. 8 ft. 141 in.
22. 2 ac. 1 r. 19 p. 15 yds. 1 ft. 135 in. 23. 169 cu. yds. 6 ft. 763 in.
24. 4 tons. 25. 35 lbs. 15 oz. $116\frac{1}{2}$ grs. 26. 79 miles 4 f. 15 p.
27. 124 ac. 7 p. 29 yds. 4 ft. 72 in. 28. £79. 11. $4\frac{1}{4}\frac{6}{25}$.
29. £13. 11. $2\frac{6}{7}$[illegible]. 30. £92. 7. $2\frac{1}{2}$[illegible].
31. 19 cwt. 1 qr. 17 lbs. $7\frac{7}{16}$ oz. 32. 2 fur. 8 po. 5 yds. 1[illegible] in.

Ex. 32.

1. £3452. 14. $9\frac{1}{2}$; £446. 14. $9\frac{1}{2}$; £19. 16. $1\frac{1}{2}$.
2. £478. 15. $7\frac{3}{4}$; £76. 16. $9\frac{3}{4}$; £1. 19. $9\frac{1}{2}$.
3. £26. 18. $8\frac{3}{4}$. 4. £361. 2. $2\frac{1}{4}$. 5. £37. 19. $11\frac{1}{4}$.
6. £6. 14. $7\frac{1}{2}$. 7. 6 lb. 11 oz. 12 dr. 8. 6 cwt. 3 qr. 25 lb. 13 oz.
9. 12 tons 15 cwt. 1 qr. 23 lb. 10 oz. 10. 4 lb. 8 oz. 16 dwt. 17 grs.
11. 3 c. 4 o. 14 fl. oz. 3 fl. dr. 35 m. 12. 30 gall. 2 qt. 1 pt. 3 gills.
13. 5 yds. 1 qr. 1 nl. $\frac{3}{4}$ in. 14. 2 ells 2 qr. 3 nl. $1\frac{3}{4}$ in.
15. 3 lds. 3 qr. 5 bsh. 2 pk. 1 gall. 16. 4 days 19 h. 53 m. 53 s.
17. 3 mo. 1 wk. 5 d. 18 h. 51 m. 18. 3 yrs. 220 d. 18 h. 51 m. 48 s.
19. 4 po. 2 yds. 2 ft. 4 in. 20. 2 fur. 28 p. 4 yds. 1 in.
21. 21 miles 4 f. 31 p. 3 yds. 2 ft. 6 in. 22. 11 per. 22 yds. 6 ft. 17 in.
23. 1 rood 32 p. 20 yds. 8 ft. 139 in. 24. 51 cu. yds. 11 ft. 964 in.
25. 4 po. 2 yds. 1 ft. 3 in.; 22 acres 2 r. 26 p. 25 yds. 2 ft. 36 in.
26. 6 cwt. 76 lbs. 11 oz. 13 dr. 27. 4 lbs. Av. 10 oz. $215\frac{1}{3}$ grs.
28. 1 lb. 6 oz. 6 dwts. 16 grs.; 13 oz. Av. $312\frac{1}{2}$ grs.
29. £13. 4. $8\frac{1}{4}$[illegible]; £8. 7. $6\frac{3}{4}$[illegible].
30. £1. 17. $8\frac{1}{2}$[illegible]; 3 cwts. 10[illegible] lbs.
31. 5 fur. 10 p. 2 yds. $9\frac{6}{17}$ in. 32. 10 oz. 16 dwts. 18[illegible] grs.
33. £317. 10. $3\frac{1}{2}$. 34. £118. 7. $6\frac{3}{4}$.

Ex. 33.

1. £159. 8. 10; £41. 15. 9; £874. 9. $9\frac{1}{2}$.
2. £341. 18. $1\frac{1}{2}$; £2622. 9. $11\frac{3}{4}$; £6046. 10. $1\frac{1}{4}$.
3. £4152. 1. $10\frac{1}{2}$; £62879. 3. 6; £43084. 6. 10.
4. £98049. 9. 9; £2170. 6. $2\frac{3}{4}$; £4130. 2. 0.
5. £2249. 12. $2\frac{1}{4}$; £2320. 18. $0\frac{3}{4}$; £821. 2. 5.
6. £268. 18. $11\frac{1}{2}$; £9614. 5. $11\frac{1}{4}$; £13144. 7. $10\frac{1}{4}$.
7. £4921. 19. $6\frac{3}{4}$; £68866. 4. $4\frac{3}{4}$.
8. £23723. 13. 8; £53904. 3. $1\frac{1}{2}$.
9. £44154. 18. 5; £120612. 15. $9\frac{1}{4}$.
10. £8729. 1. 6; £225382. 12. $3\frac{1}{2}$.
11. £4959308. 18. 0; £56051090. 3. 0.
12. 131 lb. 1 dwt. 16 grs.; 359 lb. 4 oz.
13. 1006 gall. 3 qt. 1 pt.; 6600 gall. 2 qt. 1 pt.
14. 59 qrs. 2 bush. 2 pks.; 717 qrs. 6 bush.
15. 158 weeks 6 d. 1 h. 8 m. 24 s.; 1394 weeks 3 d. 7 h. 20 m. 24 s.
16. 1150 miles 7 f. 6 ch.; 1241 ac. 1 r. 5 p.
17. 185 cu. yds. 23 ft. 216 in.; 1672 cu. yds. 19 ft. 216 in.
18. 172 lbs. 1 oz. 70 grs.; 1720 lbs. 11 oz. $262\frac{1}{2}$ grs.
19. 107 tons 10 cwt. 12 lbs.; 4485 tons 18 cwt. 12 lbs.
20. 5 miles 15 p. 2 yds. 2 ft. 6 in.; 60 miles 4 f. 26 p. 1 yd.
21. 1446 miles 1 f. 7 p. 3 yds. 10 in.; 2143 miles 3 f. 7 p. 3 yds. 2 ft. 4 in.
22. 4 ac. 5 p. 4 yds. 5 ft. 36 in.; 6 ac. 2 r. 35 p. 7 yds. 5 ft. 108 in.
23. 302 ac. 3 r. 39 p. 16 yds. 3 ft. 130 in.; 2840 ac. 2 r. 15 p. 21 yds. 7 ft. 21 in.
24. 26 yds. 3 nl. $\frac{1}{2}$ in.; 220 yds. 3 qr. 1 nl. 1 in.
25. £1[illegible]8. 9. $5\frac{1}{2}\frac{4}{5}$; £441. 10. $11\frac{1}{2}\frac{7}{8}$.
26. £317. 4. $0\frac{1}{4}\frac{8}{9}$; £386. 14. $11\frac{3}{4}\frac{8}{9}$.
27. £81. 6. $11\frac{1}{2}\frac{6}{7}$; £167. 1. $11\frac{1}{2}\frac{11}{13}$.
28. £435. 6. $8\frac{3}{4}\frac{4}{17}$; £395. 10. $0\frac{1}{4}\frac{13}{23}$.
29. 282 lbs. 9 oz. 4 dwt. $7\frac{11}{13}$ grs.
30. 71 tons 18 cwt. 16 lbs. $4\frac{4}{7}$ oz.
31. 47 miles 7 f. 8 p. 1 yd. $10\frac{4}{5}$ in.
32. £586. 10s.
33. £545. 11. $6\frac{1}{2}$.
34. £48. 3. $1\frac{1}{2}$.
35. £13896. 15. $10\frac{1}{2}$.
36. £16778. 13. $5\frac{1}{4}$.

Ex. 34.

1. 184; 47. 2. 290; 2697. 3. 278; 6545. 4. 2219.
5. 1142. 6. 187. 7. 845. 8. 17...$2\frac{3}{4}$ yds.
9. 96. 10. 262. 11. 61. 12. 297.
13. £87. 14. $7\frac{3}{4}$; £65. 15. $8\frac{3}{4}$, and 1*d.* rem.; £679. 18. $11\frac{3}{4}$, and $2\frac{1}{4}$*d.* rem.
14. £586. 16. $10\frac{3}{4}$; £12. 17. $8\frac{3}{4}$, and $2\frac{1}{4}$*d.* rem.; £23. 14. $6\frac{1}{4}$, and 4*d.* rem.
15. £39. 19. $11\frac{3}{4}$, and $7\frac{3}{4}$*d.* rem.; £35. 14. $11\frac{1}{2}$, and 2*d.* rem.; £8. 15. $2\frac{3}{4}$.
16. £8. 15. $2\frac{3}{4}$; £96. 11. $9\frac{1}{4}$, and $2\frac{1}{4}$*d.* rem.; £32. 9. $8\frac{3}{4}$, and 5*d.* rem.
17. £10. 17. $8\frac{1}{2}$, and 8*s.* rem.; £2358. 15. $5\frac{3}{4}$, and 6*s.* 10*d.* rem.;
£676. 19. $0\frac{1}{4}$, and $1\frac{1}{4}$*d.* rem.
18. £5. 11. $5\frac{3}{4}$; £38. 3. $6\frac{3}{4}$, and $3\frac{3}{4}$*d.* rem.; £37. 10. 1, and 3*s.* 1*d.* rem.
19. £8. 15. $9\frac{1}{4}$, and 10*s.* $4\frac{1}{4}$*d.* rem.; £16. 18. $10\frac{3}{4}$, and 13*s.* 8*d.* rem.
20. £3. 8. $4\frac{1}{2}\frac{4}{11}$; £6. 12. $9\frac{3}{4}\frac{3}{13}$. 21. £1. 18. $8\frac{3}{4}\frac{4}{11}$; £3. 14. $6\frac{3}{4}\frac{8}{31}$.
22. £2. 13. $9\frac{1}{2}\frac{28}{73}$; £18. 16. $6\frac{3}{4}\frac{44}{63}$.
23. 19*s.* $2\frac{1}{4}\frac{8}{17}$; £23. 11. $6\frac{3}{4}\frac{21}{325}$.
24. £14. 15. $7\frac{3}{4}\frac{11}{141}$; 14*s.* $8\frac{1}{2}\frac{188}{235}$.
25. 97 wks. 4 d. 9 h. 44 m. 10 s., and 6 s. rem.;
15 wks. 4 d. 19 h. 59 m. 36 s.
26. 61 miles 6 f. 111 yds. 1 ft. 3 in., and 9 in. rem.;
5 miles 5 f. 72 yds. 1 ft. $7\frac{3}{4}$ in.
27. 186 cu. yds. 8 ft. 899 in., and 20 in. rem.; 50 cu. yds. 11 ft. $345\frac{13}{16}$ in.
28. 54 tons 3 cwt. 2 qrs. 24 lb. 10 oz., and 4 oz. rem.;
1 ton 4 cwt. 2 qrs. 7 lbs. $9\frac{3}{5}\frac{1}{7}$ oz.
29. 330 lbs. 4 oz. 16 dwts. 7 grs., and 6 grs. rem.;
40 lbs. 10 oz. 9 dwts. $20\frac{3}{7}$ grs.
30. 44 miles 2 f. 9 ch. 18 yds., and 9 yds. rem.; 4 m. 5 f. 3 ch. $14\frac{8}{11}$ yds.
31. 37 tons 3 cwt. 3 qrs. 20 lbs. 2 oz. 15 dr., and 3 dr. rem.;
4 tons 5 cwt. 27 lb. 1 oz. $10\frac{3}{4}$ dr.
32. 732 gall. 3 q. 1 p. 3 g., and 1 pt. 3 gills rem.; 78 gall. 1 pt. $1\frac{5}{7}$ gills.
33. 1477 lds. 7 b. 3 pk., and 3 gall. rem.; 110 lds. 1 qr. 1 bsh. 2 p. $0\frac{3}{7}$ g.
34. 46 miles 4 f. 20 p. 4 yds. 1 ft. 4 in., and 5 in. rem.;
29 miles 5 f. 2 p. 1 yd. 2 ft. $6\frac{4}{11}$ in.
35. 4 miles 7 f. 36 p. 7 in., and 1 ft. 4 in. rem.;
34 po. 4 yd. 1 ft. $11\frac{8}{9}\frac{4}{7}$ in.
36. 94 ac. 3 r. 38 p. 17 yds. 7 ft. 25 in.; 3 ac. 1 r. 36 p. 1 yd. 3 ft. $72\frac{8}{11}$ in.
37. 69 yds. 1 qr. 2 nl. 1 in., and $6\frac{1}{4}$ in. rem.; 44 yds. 2 nl. $2\frac{1}{4}$ in.
38. 11 lbs. 12 oz. $9\frac{3}{7}$ dr.; 15 gall. 1 qr. $1\frac{6}{11}$ g.
39. 7*s.* $5\frac{3}{4}$*d.*; 1*s.* $10\frac{1}{4}\frac{3}{4}$. 40. 34 yds. 3 qrs. 4 in.
41. £3. 17. $10\frac{1}{4}$. 42. 2 ac. 3 r. 18 p. 43. 2*s.* $10\frac{3}{4}$.
44. £4. 17. 10; £34. 6. $9\frac{1}{4}$.

Ex. 35.

1. 49. 2. 91 miles 3 f. 15 p. 5 yds. 1 ft. 3 in. 3. £24. 18s.
4. £73. 11. 10½; &c. 5. £1. 6. 0½.
6. 7 tons 17 cwt. 2 qrs. 2 lbs. 400 grs. 7. 1¾*d.*
8. £43. 2. 7¾, and 1¼*d.* over. 9. 115. 10. 2s. 10*d.*
11. 1059, and 14s. 10½*d.* over. 12. 15 miles 3 f. 16 p. 1 yd. 3 in.
13. 36 miles 2 f. 18 p. 1 yd. 14. 437½; 12 oz. 350 grs.
15. 11 days 17 h. 36 m. 16. Gain £1. 17. 6.
17. £86. 13. 11½. 18. £1. 15s.
19. 2s. 10½*d.* 20. £14. 7. 6; £5. 15s.
21. £7. 12. 9; £6. 7. 3½. 22. 1 lb. of sugar; 1 oz. of gold; 90 lbs. 11 oz. 3 dwt. 17 grs.; 428 tons 4 cwt. 13 lbs. 11 oz.
23. 3 miles 2 f. 40 yds.; 11 hours. 24. £1. 11. 7½.
25. £253. 16. 3. 26. *A*, £1044. 17. 2½; &c. 27. £425.
28. 4272. 29. £500. 30. £648.
31. £993. 6s. 32. 29 miles 1 f. 11 p. 2 yds. 2 ft. 4 in.; 9 ac. 1 r. 37 p. 18 yds. 3 ft. 47 in.
33. *A*, £6. 14. 6; &c. 34. £17619. 2. 1½.
35. £1. 18. 6. 36. 408 lbs. 4 oz. 37. 14s. 7¾*d.*; 9*d.*
38. £100. 3. 1½. 39. 3550. 40. 13.
41. £1. 16. 11¼, and 1½*d.* over. 42. £1. 19. 6½; &c. 43. 6 cwt. 1 qr.
44. 8 tons 18 cwt. 3 qrs. 45. 2 gall. 46. 2 tons 4 cwt. 2 qrs. 16 lbs.
47. 19. 48. 1 mile 7 f. 22 p. 5 yds.
49. 54s. 6¾*d.* 50. £494812. 10s.
51. 12359. 52. £72. 14. 0¾. 53. 4350.
54. 7 tons 18 cwt. 1 qr. 16 lbs. 12 oz. 350 grs. 55. *A*, £27. 17. 9; &c.
56. £875. 57. 1930; 1152 cu. in. 58. Receives £13. 13. 9.
59. 585. 60. 58 miles.
61. £306. 4. 1. 62. *A*, by £10. 13. 4½.
63. 46 ac. 3 r. 27 p. 13 yds. 8 ft. 65 in. 64. £2. 17. 8½; £3. 16. 5½.
65. 564394385 grs. 66. £15. 15. 8. 67. 4½*d.*
68. 20. 69. 45. 70. 1636.
71. £12. 15. 1¼. 72. 5 yds. 2 ft. 5 in.
73. 2 cwt. 1 qr. 17 lbs. 74. 2400; £5. 75. 6s. 4½*d.*
76. 3s. 7½*d.* 77. £4. 7. 6.
78. 5397 remdr.; 24 fur. 27 p. 1 yd. 1 ft. 3 in. 79. £2. 19. 6.
80. 20 min. 81. £3679. 11. 10½. 82. 48243, and 5*d.* over.
83. 26 days 20 m.; 40000. 84. 14. 85. £1430.
86. 5*d.* 87. £3. 17. 10½. 88. 70 men.
89. 300 grs.; 3s. 5¼*d.* 90. 57 min. 45 s.

Ex. 36.

1. $\frac{7}{8}s.$; £$\frac{3}{37}$. 2. $\frac{1}{78}s.$; £$\frac{1}{585}$. 3. $19\frac{2}{14}s.$; £$\frac{[illegible]}{[illegible]}$.
4. $\frac{1}{350}$ lb. Tr.; $\frac{1}{10080}$ wk. 5. $\frac{8}{315}$ cwt.; $5\frac{1}{3}$ hrs. 6. $\frac{1}{567}$ cu. yd.
7. $\frac{1}{180}$ fur.; $\frac{1}{5632}$ ac. 8. $\frac{18}{875}$ yr. 9. $23\frac{1}{2}l.$; $18\frac{1}{3}s.$; $194\frac{2}{11}d.$
10. $\frac{2}{11}d.$; $\frac{2}{15}l.$; $\frac{2}{4}$. 11. $\frac{[illegible]}{[illegible]}$ lb.; $33\frac{1}{3}$ qrs.; $332\frac{1}{2}$ grs.
12. $107\frac{1}{7}$ dwts.; $\frac{1}{24}$ pt. 13. $147\frac{1}{33}$ po.; $6091\frac{2}{3}$ yds.
14. $2660\frac{2}{3}$ yds.; $887\frac{[illegible]}{[illegible]}$ p. 15. $\frac{7}{60}$ lb. Tr.; $\frac{7}{9}$ ft. 16. $\frac{1}{5}$ ell; $\frac{1}{3}$ ft.
17. $\frac{[illegible]}{[illegible]}$ lb. Tr.; $\frac{[illegible]}{[illegible]}$ cwt. 18. £$\frac{[illegible]}{[illegible]}$; £$\frac{[illegible]}{[illegible]}$; £$\frac{[illegible]}{[illegible]}$.
19. £$4\frac{[illegible]}{[illegible]}$; £$3\frac{[illegible]}{[illegible]}$; £$5\frac{[illegible]}{[illegible]}$. 20. $\frac{[illegible]}{[illegible]}$ lb. Tr.; $\frac{[illegible]}{1400}$ lb. Av.
21. $15\frac{[illegible]}{[illegible]}$ cwts.; $\frac{[illegible]}{[illegible]}$ ton. 22. $\frac{[illegible]}{[illegible]}$ qr. 23. $\frac{[illegible]}{[illegible]}$ cwt.
24. $\frac{[illegible]}{[illegible]}$ mile. 25. $8\frac{[illegible]}{[illegible]}$ per.; $\frac{107}{1920}$ ac. 26. $\frac{[illegible]}{[illegible]}$ yr.
27. $9\frac{3}{4}d.$; $4\frac{1}{2}d.$ $\frac{3}{5}$; 10s. $7\frac{1}{2}$; 18s. $10\frac{1}{4}$; 13s. $10\frac{[illegible]}{[illegible]}$; £8. 16. $4\frac{[illegible]}{[illegible]}$;
12s. $8\frac{1}{3}\frac{10}{11}$; £5. 12. $11\frac{[illegible]}{[illegible]}$.
28. 7 oz. 4 dwts.; 9 oz. $2\frac{2}{7}$ dr.; 3 dwts. 23 grs.; 2 lb. 5 oz. 13 dwt. 8 grs.;
2 qrs. 26 lbs. 4 oz.
29. 15 cwts. 13 lbs. 12 oz.; 2 roods 21 p. 24 yds. 6 ft. 108 in.;
5 fur. 32 p. 1 yd. 1 ft. 6 in.; 4 fur. 22 p. 2 ft.
30. 16 hrs. 19 m. 12 s.; 37 per. 24 yds. 6 ft. 108 in.; 3 qrs.;
3 ells 1 qr. 2 nl. $\frac{3}{4}$ in.
31. 109 days 13 h. 20 m.; 3 cwt. 1 qr. 6 lbs.; 3 days 11 h. 13 m.
32. 1 gall.; 20 sq. yds. 5 ft. 90 in. 33. £8. 5. 6; £4. 0. $8\frac{1}{4}$.
34. 2 qrs. 17 lbs. 1 oz. $3\frac{1}{3}$ dr.; 12 cwt. 2 qr. 14 lbs. 10 oz. $10\frac{2}{3}$ dr.
35. 2 miles 6 f. 22 p. 3 in.; 100 ac. 3 r. 17 p.
36. 4 days 23 h. 31 m. $31\frac{3}{5}$ s.

Ex. 37.

1. £1. 12. $10\frac{1}{2}\frac{3}{7}$; £4. 15. $11\frac{[illegible]}{[illegible]}$.
2. £402. 17. $5\frac{1}{4}\frac{9}{11}$; £76632. 18. $9\frac{1}{4}\frac{1}{3}$.
3. 35 lbs. 11 oz. 9 dwts. $8\frac{4}{11}$ grs.; 6 cwts. 3 qrs. 13 lbs. $0\frac{[illegible]}{[illegible]}$ oz.
4. 10 days 10 hrs. 36 m. $5\frac{1}{16}$ s.; 119 ac. 1 r. 1 p. $21\frac{[illegible]}{[illegible]}$ sq. yds.
5. £42. 7. $3\frac{[illegible]}{[illegible]}$; £5. 2. $8\frac{1}{2}\frac{[illegible]}{[illegible]}$. 6. £3. 12. $4\frac{1}{2}\frac{[illegible]}{[illegible]}$; £5. 4. $1\frac{[illegible]}{[illegible]}$.
7. £167. 11. $4\frac{[illegible]}{[illegible]}$; £102. 19. $9\frac{1}{2}\frac{[illegible]}{[illegible]}$.
8. 5 lbs. 9 oz. 5 dwts. $13\frac{1}{7}$ grs.; 1 day 19 h. 29 m. $46\frac{2}{3}$ s.
9. 3 cwt. 3 qrs. 18 lbs. $12\frac{1}{3}$ oz.; 2 miles 8 p. $1\frac{[illegible]}{[illegible]}$ yds.

10. £3. 15. $10\frac{3}{4}\frac{9}{11}$; £3. 8. $7\frac{1}{4}\frac{19}{135}$.

11. £33. 15. $8\frac{3}{4}\frac{5}{9}$; £3760. 12. $8\frac{3}{4}\frac{114}{125}$.

12. 176 lbs. 13 oz. $2\frac{4}{7}$ drs.; 275 gall. 1 qt. $2\frac{299}{473}$ gills.

13. £14. 3. $5\frac{1}{4}\frac{48}{83}$. 14. £36. 4. $11\frac{1}{2}\frac{1277}{1885}$. 15. 30 yds. 1 ft. $0\frac{8}{15}$ in.

16. £2. 14s. 17. $\frac{9}{25}d.$ 18. 12s. $10\frac{1}{2}d.$ 19. 4d.

20. 2 cwts. 26 lbs. 21. 4s.; 1s. $1\frac{1}{2}d.$; 12s.

22. £4; 2s. 3d.; 6s. $4\frac{3}{4}\frac{44}{45}$. 23. £12. 3s.; £34. 6. $4\frac{3}{4}\frac{1}{5}$.

24. 4 ft. $6\frac{18}{23}$ in.; 3 sq. ft. 96 in.; 3 yds. 2 nl. $1\frac{1}{2}$ in.

25. 9 cwt. 2 qrs.; 102 miles 4 f. 11 p. $3\frac{7}{30}$ yds.

26. 2 roods $18\frac{2}{7}$ p.; 26 cwts. $24\frac{1}{4}$ lbs.

27. 2 cu. ft. 604 in.; 32 days 9 h. 49 m.

28. 17 lbs. 4 oz. 29. £5. 12. 6. 30. 10s. $11\frac{1}{4}\frac{17}{18}$.

31. 13s. 32. 9s. $1\frac{1}{2}d.$ 33. £6. 15. $10\frac{3}{4}\frac{1}{7}$. 34. £50.

35. £3. 5. $10\frac{1}{2}\frac{1}{7}$. 36. 16s. $7\frac{1}{2}d.$ 37. £5. 12. $8\frac{1}{4}$.

38. £6. 5. $5\frac{1}{2}\frac{7}{11}$. 39. £18. 10. $5\frac{3}{4}\frac{6}{13}$.

40. 7 years (of $365\frac{1}{4}$ days) 169 d. 34 m.

41. 1 lb. 7 oz.; 2 days 4 h. 47 m. 16 s. 42. 2800 : 2793 : 3040.

43. 7040 : 7056 : 7007.

44. 8820 : 8415 : 8806.

Ex. 38.

1. $\frac{17}{81}$; $\frac{8}{21}$. 2. $\frac{89}{108}$. 3. $\frac{173}{384}$. 4. $\frac{7}{25}$. 5. $\frac{9}{11}$.

6. $\frac{118}{17797}$. 7. $\frac{20899}{74888}$. 8. $\frac{8}{21}$. 9. $\frac{17}{48}$. 10. $\frac{93}{135}$.

11. $\frac{185}{384}$. 12. $\frac{8}{45}$. 13. $\frac{179}{8785}$. 14. $\frac{9}{10}$. 15. $\frac{1}{2}$.

16. $\frac{4}{7}$. 17. $\frac{13}{20}$. 18. $\frac{128}{243}$. 19. $\frac{28}{1280}$. 20. $\frac{481}{600}$.

21. $\frac{1}{8}$. 22. $\frac{731}{1000}$. 23. $\frac{80}{101}$. 24. $\frac{13}{35}$. 25. $\frac{35}{48}$; $\frac{13}{15}$.

26. $\frac{22}{81}$. 27. $\frac{8}{17}$. 28. $\frac{307}{4050}$. 29. $\frac{168}{1351}$. 30. $\frac{65}{378}$.

31. 7 cwt. 11 lbs. 32. $\frac{36}{385}$. 33. $\frac{1}{320}$. 34. $\frac{1}{240}$.

35. $\frac{187}{780}$. 36. $\frac{488}{747}$. 37. $\frac{13}{84}$. 38. $\frac{8}{15}$.

39. £7. 7. $1\frac{1}{4}$.

Ex. 39.

1. $\frac{26}{9}$. 2. £90. 3. $5\frac{5}{8}$ days. 4. 2 h. 10 m. $54\frac{6}{13}$ s.

5. 132. 6. £2. 13. $7\frac{1}{2}$. 7. £598. 10s.; $\frac{1}{37}$. 8. $\frac{11}{64}$ mile.

9. $5\frac{1}{4}$ days. 10. 3 m. $27\frac{3}{11}$ s.; 1080 yds. 11. $\frac{224}{375}$; $\frac{151}{496}$.

12. £19. 4s. 13. 3 h. $38\frac{4}{7}$ m. 14. £3. 13. $8\frac{1}{2}$. 15. 172.
16. [illegible]s. 17. £504. 18. 20[illegible] in. 19. $1\frac{1}{2}$ hrs. 20. $2\frac{2}{3}$ ft.
21. $\frac{3}{8}$; $4\frac{1}{2}$. 22. £44. 8. $10\frac{2}{3}$[illegible]. 23. 15 days. 24. 35[illegible].
25. $24\frac{8}{11}$. 26. 12. 27. £1270. 1. $9\frac{2}{3}$[illegible]. 28. £22. 8s.
29. 70 days. 30. $\frac{1}{17}$. 31. [illegible]. 32. 4s. $4\frac{1}{2}d$.
33. $3\frac{3}{4}$ hrs. 34. 6 o'cl. 35. Yes; $\frac{1}{28}$.
36. £50. 12. $4\frac{1}{2}$. 37. £8. 6s. 38. 6.
39. $10\frac{2}{3}$ days. 40. 5[illegible] hrs. 41. £4. 14. $3\frac{1}{4}$[illegible]; 6s. 8d.
42. £44. 17. $8\frac{1}{2}$[illegible]. 43. 54 days. 44. 1 h. $51\frac{6}{11}$ m. 45. $\frac{1}{3}$ hr.
46. 429. 47. 6d.; 2s. 6d. 48. 8 h. 55 m. 49. 9 hrs.
50. 5 miles. 51. [illegible]. 52. £1. 2. 6. 53. 4 days.
54. 3 h. 45 m. 55. $7\frac{3}{8}$[illegible] miles; 2 hrs. 55 m.
56. At 3 h. 35 m.; $34\frac{1}{6}$ miles. 57. 111,835[illegible] metres. 58. 16720 tons.
59. $\frac{1}{15}$. 60. 1 mile 980 yds.; $13\frac{8}{9}$ miles. 61. [illegible].
62. 1s. 63. $21\frac{3}{7}$ days; 50 days. 64. 6 h. $16\frac{4}{11}$ m.; 6 h. $49\frac{1}{11}$ m.
65. 684. 66. 89. 67. 37,610,528. 68. 29 yds.
69. 3 d. 12 h. $46\frac{2}{16}$[illegible] m. 70. $\frac{2}{9}$. 71. 3 days 10 h. 25 m. 3 s.
72. 5s. 6d. 73. 12 days. 74. 26[illegible]. 75. $\frac{5}{11}$ mile; 12 hrs.
76. 175. 77. 9s. 4d. 78. 4. 79. At $34\frac{2}{7}$ m. past 10. 80. 3000.
81. $\frac{3}{8}$; £17. 4. 2. 82. £80. 83. $45\frac{1}{2}$ da.; $10\frac{1}{3}$[illegible] da.
84. 11 h. $38\frac{2}{11}$ m. 85. 5 hrs. 86. 160 ; 24.
87. 254,500,885[illegible] sq. miles. 88. $11\frac{9}{41}$ in. 89. 10 h. 55 m.
90. $7\frac{74}{111}$ hrs. 91. 9[illegible] miles, $9\frac{1}{3}$ miles.

Ex. 40.

1. ·57d.; ·15l.; ·859l. 2. 1408·3328 oz.; 3570 grs.
3. 4135·2 yds.; 17847·016 sq. yds.
4. ·38 pt.; 6840 min.
5. £·1675; £·0265625; £·00042375. 6. ·021309375 ton; ·0390625 lb.
7. ·19965875 mile; ·0004396 day.
8. ·086945 ac.; ·000558035714286 cwt.
9. ·5952380 g.; 1·190476 half-g. 10. ·89725 ch.; ·34608 ell.
11. ·9114583 oz. Tr.; 7·158857142 oz. Av.
12. 326·095 sq. yds.; ·000625 sq. fur.

13. ·0191... 14. ·4375*s.*; ·0625*s.*; 8·9375*s.*; 34·875*s.*
15. £·509375; £853125. £·0072916; £·0078125.
16. £3·946875; £2·790625; £5·385416.
17. ·628125; ·00628125; 628·125. 18. ·9027; ·9027.
19. ·42106481; ·09975. 20. ·77083; ·136904761.
21. ·01663390625; ·1614735449. 22. ·33878968253; ·2667135416.
23. ·864; ·7251984126; ·7230109126984. 24. ·882899305; ·7265.
25. 4·34375; ·89533203125. 26. ·00625; ·4125; ·00390625.
27. 8·495; 849·5; ·59. 28. ·455921875; 1·9375.
29. ·7882571428. 30. ·62357954; 13·125; 13·201904296875.
31. ·1289043367...; 29·530588194; ·7498046875.
32. ·1469; ·9471590; ·509837962. 33. ·2905; ·538461; ·78409.
34. ·256581439; ·74572. 35. ·900449.
36. $\frac{3}{4}$*d.*; 8*s.* 9*d.*; 6*s.* 8$\frac{1}{4}$*d.* 37. 11$\frac{3}{4}$*d.*; 15*s.* 3$\frac{1}{4}$*d.*; $\frac{1}{2}$*d.*
38. 3*s.* 5$\frac{1}{2}$*d.*; £5. 15. 11$\frac{3}{4}$; £1. 1. 6$\frac{1}{4}\frac{1}{8}$.
39. 2 qrs. 3 lbs. 10·4448 drs.; 1 rood 14 p.; 3 qrs. 3 bush. 3 pk.
40. 1 ell 1 qr. 1 nl. 1·35 in.; 3 fur. 32 p. 2 yds. 2 ft. 3 in.;
2 days 12 h. 55 m. 21 s.
41. 17 cwt. 1 qr. 20 lbs. 8 oz. 8$\frac{8}{15}$ drs.; 12 lbs.;
3 roods 5 p. 13 yds. 6 ft. 108 in.
42. 27 lbs. 10 oz. 6$\frac{2}{3}$ drs.; 2 lbs 3 oz. 6 dwt. 5 grs.; 2$\frac{1}{4}$ in.
43. 5*s.*; 3 roods 26 p. 26 yds. 1 ft. 18 in. 44. 3 cwt. 2 qrs. 26·05 lbs.

Ex. 41.

1. £4. 13. 9; £11. 14. 6$\frac{1}{2}$·88; £31. 13. 4.
2. 4*s.* 10$\frac{3}{4}$·68; 7*s.* 7$\frac{1}{4}$*d.*; 12*s.* 10$\frac{3}{4}$·2.
3. £4. 0. 0$\frac{1}{4}$·8; 11*s.* 5$\frac{3}{4}$*d.*; £1. 14. 4$\frac{3}{4}$·8296.
4. 1$\frac{1}{2}$*d.*; 10*s.* 1$\frac{1}{2}$*d.*; £6. 15*s.*
5. £1. 9*s.* 7$\frac{1}{2}$*d.*; 0*s.* 1$\frac{3}{4}$·8594; 0*s.* 3$\frac{0}{4}$·24.
6. £1640; £117. 10. 2$\frac{1}{2}$·725.
7. £25. 10. 10$\frac{3}{4}$·008; £5161. 5. 8$\frac{1}{4}$·78.
8. £23. 10. 10; £1. 15. 5. 9. £19. 2. 9$\frac{3}{4}$; £3. 15. 1$\frac{1}{4}$·8.
10. 12 lbs. 2 oz. 1·152 drs.; 177 lbs. 2 oz. 1 dwt. 13·61 grs.
11. 2 tons 17 cwt. 1 qr. 27 lbs. 7 oz. 4 drs.

12. 1 po. 1 yd. 2 ft. 6 in.; 25 days 21 h. 3·0028... m.
13. £18. 19. 11; 1s. 8$\frac{3}{4}$·808.
14. 2 hrs. 5 m. 4 s.; 1 cwt. 2 qrs. 6 lbs. 6 oz.
15. 4 ac. 12 p.; 52 miles 2 f. 250. 3 yds.
16. 1 oz. 17 dwt. 22 grs. 17. 6s. 8$\frac{2}{7}$. 18. £11. 7. 5$\frac{3}{4}$·691.
19. £9. 4. 8$\frac{1}{4}$. 20. 5s. 1$\frac{1}{4}$d. 21. 10s. 2$\frac{1}{4}$·93309.
22. 19s. 1$\frac{1}{2}$d. 23. £4. 4. 4$\frac{3}{4}$. 24. 8s. 2d. 25. £2. 7. 1.
26. 1 ton 17 cwt. 2 qrs. 4 lbs. 27. 1 sq. yd. 6 ft. 32·4 in.; £2. 10. 9.
28. $\frac{3}{4}$d. ·84. 29. £1. 1s. 30. £15. 11. 2$\frac{1}{2}$·875.

Ex. 42.

1. ·1; ·0625; $\cdot 4\dot{7}2\dot{9}$. 2. $\cdot 0158\dot{7}\dot{3}$; $\cdot 48\dot{7}8$.
3. ·875; ·3675; 14·49; $\cdot 0\dot{7}1428\dot{5}$. 4. $\cdot 82\dot{9}\dot{3}$. 5. 59·52.
6. $\cdot 10885416\dot{6}$. 7. ·95. 8. $\cdot 0103\dot{6}$. 9. $\cdot 3\dot{6}$. 10. ·0625.
11. ·02952380. 12. $\cdot 630\dot{5}$. 13. $\cdot 1302\dot{7}$. 14. ·0203125.
15. ·171296. 16. 3·28. 17. ·0072562358...
18. 10; 2·5. $1\cdot\dot{6}$; $2\cdot 08\dot{3}$. 19. ·236075. 20. ·45.
21. $\cdot 38\dot{2}3\dot{7}$. 22. $\cdot 3\dot{6}$. 23. ·003. 24. ·0801.

Ex. 43.

1. 45170 *m.*; £876. 3 *f.* 4 *c.* 5 *m.* 2. 25025 *m.*; 38·09 *f.*
3. 8435 *m.*; 2625 *m.* 4. £965. 8 *f.* 6 *c.* 6 *m.* 5. £194. 4 *f.* 6 *c.* 8 *m.*
6. £34. 3 *f.* 2 *c.* 5 *m.* 7. £474. 2 *f.* 3 *c.* 7 *m.* 8. £1. 8 *f.* 9 *c.* 7 *m.*
9. £3051. 7 *f.* 2 *c.* 1 *m.*; £28303. 5 *f.* 7 *c.* 6 *m.*
10. £875; £1619141. 8 *f.* 8 *c.* 11. 7 *f.* 8 *c.* 2·2359... *m.*
12. £2. 9 *f.* 9 *c.* 9 *m.* 13. 789.
14. ·732; ·01464. 15. £26. 2 *f.* 2 *c.* 9 *m.*
16. 2 *f.* 1 *c.* 2$\frac{1}{2}$ *m.*; 4 *f.* 7 *c.* 5 *m.*; £3. 6 *f.* 8 *c.* 7$\frac{1}{2}$ *m.*; 1 *c.* 8$\frac{3}{4}$ *m.*; £1. 7 *f.* 9 *c.* 3$\frac{3}{4}$ *m.*
17. 1 *f.* 5 *c.* 3$\frac{1}{8}$ *m.*; 7 *f.* 8 *c.* 4$\frac{3}{8}$ *m.*; £8. 9 *f.* 4 *c.* 6$\frac{1}{8}$ *m.*; 1 *c.* 5$\frac{5}{8}$ *m.*; £7. 9 *f.* 7 *c.* 8$\frac{1}{8}$ *m.*
18. 3 *f.* 1 *c.* 6$\frac{2}{3}$ *m.*; 1 *f.* 5·208$\dot{3}$ *m.*; £1. 4 *f.* 6 *c.* 9·791$\dot{6}$ *m.*; 2 *c.* 2·91$\dot{6}$ *m.*; £2. 4 *f.* 2 *c.* ·8$\dot{3}$ *m.*
19. 6 *f.* 5 *c.* 1·041$\dot{6}$ *m.*; 8 *f.* 8 *c.* 2·291$\dot{6}$ *m.*; £3. 2·08$\dot{3}$ *m.*; 4 *c.* 1$\frac{2}{3}$ *m.*; £2. 6 *f.* 4 *c.* 8·958$\dot{3}$ *m.*
20. 7 *f.* 8 *c.* 3$\frac{3}{4}$ *m.*; 6 *f.* 4 *c.* 8·958$\dot{3}$ *m.*; £1. 3 *c.* 9·58$\dot{3}$ *m.*; 4 *c.* 4·791$\dot{6}$ *m.*; £5. 8 *f.* 9 *c.* 7·91$\dot{6}$ *m.*

21. £5. 12. 6; 9*s*. 9*d*.; 2*s*. 1½*d*.; £2. 9. 2¼.
22. £3. 12. 5¾; 19*s*. 11¼; 17*s*. 0¾; 15*s*. 8*d*.
23. 2*s*. 3¼; £4. 16. 5½; £1. 15. 11¼·352; £19. 17. 6¾·96.
24. £92. 15. 5¾; £5. 18. 10¾; 18*s*. 1½·0864; £2. 9. 4¼·232.
25. 6*f*. 3*c*. 3*m*.; 7*f*. 6*c*. 7*m*.; £2. 6*f*. 4*c*.; 4*c*. 3*m*.; £3. 9*f*. 2*c*. 8*m*.
26. 8*f*. 6*c*. 9*m*.; 7*f*. 1*c*. 2*m*. (or 3*m*.); £3. 9*f*. 4*c*. 6*m*.; 1*c*. 5*m*.; £7. 4*f*. 6*c*. 8*m*.
27. 7*f*. 5*c*. 2*m*.; 8*f*. 8*c*. 9*m*.; £1. 6*f*. 8*c*. 3*m*.; 3*c*. 7*m*. (or 8 m.); £8. 9*f*. 9*c*. 9*m*.
28. 8*s*. 4½; £3. 13. 0¼; 16*s*. 7¾; £5. 13. 6¾.
29. £2. 6. 10¾; £4. 5. 2½; 16*s*. 5½; £1. 1. 5½.
30. £45. 15. 0¾; 15*s*. 9¼; £25. 13. 9½; £87. 19. 11¼.

Ex. 44.

1. £3005. 11. 6¾.
2. £2529. 19. 4½.
3. £15362. 6. 6¾.
4. £2139. 6. 1.
5. £120. 1. 10½.
6. £122. 18. 3.
7. £572. 1. 9.
8. £62. 0. 11½.
9. £310. 2*s*.
10. £388. 5. 5½.
11. £59. 13. 2¾.
12. £658. 12. 0¼.
13. £1203. 7. 5¼.
14. £695. 0. 8½·9...
15. £133. 8. 6.
16. £19. 6. 5½.
17. £130. 6. 9.
18. £1213. 15. 11½.
19. 10*s*. 9*d*.
20. 33 miles 51 ch. 7 yds.
21. 583 tons 8 cwt. 14·889... lbs.
22. £142. 7. 0¾.
23. £5. 17. 0½.
24. 2*s*. 10½*d*.; £48. 10. 8.
25. 15*s*.; £386. 14. 4½; £305. 5*s*.; £220.
26. £7. 9. 4½.
27. £3. 13. 9½.
28. £1. 7. 3¼; £261. 2. 8¾..., £84. 2. 5½..., £166. 7. 4½...
29. 26·5435416.
30. 128·5016...
31. 62 lbs. 5·136... oz.; 7·82936... oz.
32. 388; 11·32 grs.
33. 365·242218.
34. 14 ft. 9½ in.; 1043.
35. 561; ·0023 in.
36. 3 cwt. 21 lbs. 12 oz.

Ex. 45.

1. 345 kilom. 6 hectom. &c.
2. 207 kilol. 8 hectol. &c.
3. 66 sq. kilom. 78 sq. hectom. 95 sq. decam. 60 s. m.
4. 357 cu. m. 82 cu. decim. 670 cu. centim.
5. 345706·809 m.; 345·706809 kilom.
6. 3·5678095 tonn.; 35678095 decigr.
7. 708·0905 hect.; 7080905 centiares; 7·080905 sq. kilom.
8. 457·024060 cu. m.
9. 67·805008 cu. m. of water.

10. 65 hect. 25 ares 46 centiares. 11. 18 gr. 613 milligr.
12. ·55·75588. 13. 334 cu. m. 855 cu. decim. 14. 422379 fr. $14\frac{3}{8}$ c.
15. 13 kilog. 598 gr. 16. 508 fr. $91\frac{3}{8}$ c. 17. 7 kilogr. 721·746682 gr.
18. ·914386 m.; 2·53995 centim. 19. 4 kilom. 502 m.
20. 1088 m. 4 dec. 2 centim. 21. ·994123... m.
22. From Art. 226, 1 kilom. = ·621382 mile :—12733·56... kilom.
23. 59 miles 68 ch. 8 yds. 24. 7 miles 48 ch. 3 yds.
25. 15784 feet. 26. 29·9218... in. 27. 24855·2... miles.
28. 40·4671 ares; 2·5899 sq. kilom. 29. 130497 sq. kilom.
30. 156 hect. 97 ares 20 centiares. 31. 204749 sq. miles.
32. 17 ac. 3 r. 26·83... p. 33. 1956 cu. m. 335 cu. decim.
34. 1139 cu. yds. 5 ft. 770 in. 35. 47 gall. 3 qts.
36. 2·6096... 37. 373·24158 gr.; ·45359 kilog.; ·050802 millier.
38. 5·06345 tonn.; 5063·45 kilog. 39. 91·6941. 40. 21 fr. 80 c.
41. 9*s*. $4\frac{3}{4}$*d*. ·8... 42. 358 fr. 45·939... c. 43. £1. 6. $6\frac{3}{4}$ ·7...
44. 1·03706... kilog. 45. 10694·617... gr.; 802·724... yds.
46. 25·3995... centim.; 60·9373... kilog.; 4457·63... m.
47. £1,713,296. 16. $0\frac{1}{2}$. 48. 49·44... lbs.
49. 8·49 gr. 50. 5*s*. 4*d*.

Ex. 46.

1. £847. 16. 8; £953. 16. 3; £980. 6. $1\frac{3}{4}$.
2. £1428. 3. 4; £1785. 4. 2; £1095. 9. $4\frac{1}{2}$.
3. £866. 2. 0; £893. 3. $3\frac{3}{4}$; £1055. 11. $2\frac{1}{4}$.
4. £5. 14. $0\frac{3}{4}$; £95. 16. 3; £444. 16. $10\frac{1}{2}$.
5. £13658. 12. 5; £18467. 1. 7.
6. £964. 4. 7; £1219. 5. 8; £15657. 16. 9.
7. £610. 3. $1\frac{1}{2}$; £189. 15. $6\frac{3}{4}$. 8. £493. 17. 1; £204. 19. 9.
9. £18321. 15. 6; £20833. 12. $7\frac{1}{2}$.
10. £5107. 2. $11\frac{1}{4}$; £27821. 15. $11\frac{1}{4}$.
11. £1358. 12. $3\frac{3}{4}$; £21014. 7. $3\frac{3}{4}$. 12. £10869. 16. $4\frac{1}{4}$; £35978. 18. $1\frac{1}{2}$.
13. £2821. 8. $4\frac{1}{2}$; £12216. 8. $0\frac{1}{2}$. 14. £31559. 14. $4\frac{3}{4}$; £650621. 0. $1\frac{1}{4}$.
15. £1408. 3. 4; £1925. 17. 6; £2692. 1. 8.
16. £42. 7. $10\frac{1}{2}$; £966. 19. $7\frac{1}{2}$; £4829. 17. $2\frac{1}{4}$.
17. £4724. 6. $1\frac{1}{2}$; £13988. 4. $7\frac{1}{2}$; £20504. 0. $1\frac{1}{2}$.
18. £17289. 0. $6\frac{1}{4}\frac{1}{2}$; £596. 0. $5\frac{1}{2}\frac{1}{2}$.
19. £91949. 1. $10\frac{1}{2}\frac{3}{4}$; £10278. 2. $10\frac{3}{4}\frac{3}{4}$.
20. £626. 18. $2\frac{3}{4}\frac{1}{2}$. 21. £1364. 19. $7\frac{1}{2}\frac{7}{16}$; £3956. 14. $5\frac{1}{4}\frac{88}{100}$.
22. £59. 7. $4\frac{1}{2}\frac{3}{4}$; £570. 16. $2\frac{1}{4}\frac{1}{4}$. 23. £510. 14. $4\frac{1}{2}$; £31945. 4. $0\frac{3}{4}\frac{4}{17}$.
24. £491. 7. $11\frac{1}{2}\frac{1}{4}$; £90987. 17. $7\frac{1}{2}$. 25. £10. 15. $0\frac{3}{4}$; £169. 8. $11\frac{1}{3}$.

26. £48. 3. $1\frac{1}{2}$. 27. £593. 8. $1\frac{3}{4}\frac{1}{5}$. 28. £269. 7. $5\frac{1}{4}\frac{1}{3}$.
29. £14877. 0. $10\frac{3}{4}\frac{1}{3}$. 30. £2688. 4. $0\frac{3}{4}$.
31. £10303. 17. $4\frac{2}{4}\frac{1}{3}$. 32. £44726. 7. $4\frac{2}{4}\frac{1}{4}$.
33. 53703 lbs. 18 dwts. 6 grs.; 19 cwts. 3 qrs. 25 lbs. $13\frac{7}{15}$ oz.
34. 576 qrs. 7 bu. $0\frac{7}{12}$ pk. 35. 14 tons 8 cwt. 1 qr. $14\frac{8}{10}$ lbs.
36. £181. 1. $1\frac{1}{4}\frac{1}{5}$. 37. £3667. 5. $11\frac{3}{4}\frac{1}{3}$. 38. £450. 10. $1\frac{2}{4}\frac{1}{4}$.
39. £37. 15. $2\frac{1}{4}$. 40. £121. 2. $5\frac{1}{4}$. 41. £2. 15. $11\frac{1}{4}$; £28. 3. $6\frac{1}{4}$.
42. £8095. 4. $2\frac{1}{4}$. 43. £23264. 13. $3\frac{1}{2}$. 44. £281. 18. $2\frac{3}{4}\frac{1}{4}$.
45. £14230. 5. 0. 46. £26. 12. 0. 47. £370703. 2. 6.
48. £2057. 14. $2\frac{1}{4}$. 49. £595. 6. $11\frac{3}{4}\frac{3}{20}$. 50. £15. 18. $8\frac{3}{4}$.
51. £6. 8. $7\frac{3}{4}$. 52. £55. 6. $10\frac{1}{2}$. 53. 2377 ac. 1 r. $15\frac{5}{9}$ p.
54. 59 cu. ft. $1202\frac{144}{360}$ in. 55. 143 qrs. 7 bu. $5\frac{27}{81}$ pk.
56. 530 lbs. $14\frac{3}{4}\frac{1}{4}$ oz. 57. 182 miles 3 f. $143\frac{126}{226}$ yds.

Ex. 47.

1. £118. 13. 6. 2. 2 tons 3 cwt. 3 qrs. 3. 361 days.
4. £1718. 9. $10\frac{3}{4}\frac{1}{5}$. 5. 365. 6. £10. 2. $8\frac{1}{2}$. 7. £136. 16. 3.
8. 54756. 9. £106. 11. 6. 10. £57. 6. $8\frac{1}{4}$. 11. £20. 2. 9.
12. 43 ells 3 qr. 1 nl. 13. 5 tons 1 cwt. 2 qrs. 14. £5. 7. $9\frac{1}{4}\frac{21}{47}$.
15. $74\frac{11}{13}$ days. 16. 65. 17. 5389. 18. £4. 4. 0.
19. 1 lb. $15\frac{1}{2}$ oz. 20. £257. 10*s.* 21. £5. 22. $29\frac{3}{7}$ lbs.
23. £45. 24. £23. 13. $11\frac{1}{2}$; 16 yds. 25. 39 hours.
26. $4682\frac{18}{173}$. 27. £1. 4. 6. 28. 113 miles 592 yds.
29. £1400; $3\frac{8}{11}$*d.* 30. 10 hrs. 40 m. $36\frac{7}{13}$ s.
31. £61. 12*s.* 32. £211. 19. 3. 33. £18. 3. $3\frac{3}{4}\frac{1}{5}$.
34. 260 lbs. 8 oz. 17 dwts. 35. £1. 11. $10\frac{1}{2}$.
36. £3. 7. 10. 37. 1 hr. 10 m. $13\frac{3}{8}\frac{3}{6}$ s. 38. 2400.
39. 9*s.* 10*d.* 40. 3 p.m. Dec. 3. 41. 2*s.* $8\frac{3}{4}$*d.*; £61. 18. $0\frac{1}{4}\frac{3}{5}$.
42. 78 yds 2 ft. $1\frac{1}{7}$ in. 43. 288. 44. 2 cwt. 1 qr. $20\frac{8}{21}$ lbs.
45. £22. 8. $0\frac{1}{2}\frac{7}{13}$. 46. $78\frac{3}{4}$. 47. £1. 15. 9. 48. $4166\frac{2}{3}$.
49. 9 min. 50. £1963541. 13. 4. 51. 28. 52. $25\frac{3}{5}$.
53. 2 cwt. 3 qrs. 20 lbs. $5\frac{5}{7}$ oz. 54. 111 yds. 8 in. 55. 27 in.
56. £10. 5. $1\frac{3}{4}\frac{4}{13}$. 57. 6 min. $52\frac{1}{2}$ s. 58. £1507. 10*s.*
59. 8 p.m. Thursday. 60. $\frac{8}{11}$ mile. 61. 17 qrs. $3\frac{1}{4}$ pks. (in 365 days).
62. 15. 63. $13\frac{1}{3}$ m...; $13\frac{3}{4}$ m. 64. £6012. 6*s.*; 15*s.* $3\frac{3}{4}$*d.*
65. 104 days. 66. 10 days; $12\frac{6}{7}$ days. 67. 275625.
68. $1\frac{1}{2}$ miles. 69. 3 h. 25 m. p.m. 70. 2 hrs. 32 m. $19\frac{1}{2}$ s.

Ex. 48.

1. £28. 5. 9; 97 miles; 63 tons. 2. 1878.
3. 5 days 6 h. 10 m. 40 s. 4. $2\frac{1}{4}$ years. 5. 5 months.
6. 20; 15*s.* 9*d.* 7. 3 lbs.; 1*s.* 6*d.* 8. 10; 7. 9. 12; 2250.
10. 300. 11. 156. 12. 8 cwt. 1 qr. 20[illegible] lbs. 13. 27.
14. 1 ton 3 qrs. 16 lbs. 15. 9; 1000 ft.
16. 48 days; £56. 8. 4; £40. 9. 9[illegible].
17. 20. 18. 98. 19. 9. 20. 7 ft. 4[illegible] in. 21. 45.
22. 17 cwt. 2 qrs. $9\frac{3}{5}$ lbs. 23. 1170.
24. 13[illegible]. 25. 19·36. 26. 26 lbs. 0[illegible] oz.; 9*s.* 0[illegible].
27. 12[illegible]. 28. £297. 6. 9[illegible]; £1. 13. 6[illegible]...; 7*s.* 4[illegible]...
29. 45. 30. 1050. 31. 14. 32. 11. 33. 24 times.
34. 18. 35. 5 : 2; 34. 36. 15. 37. 47 hrs. 20 m. $54\frac{6}{11}$ s.
38. $27\frac{1}{13}$ ft.; 1[illegible] s. 39. 2*d.* 40. 165. 41. £9. 16. $10\frac{1}{2}$.
42. 6 tons. 43. 129 acres 2 r. 16 p.

Ex. 49.

1. 2601. 2. 494[illegible]. 3. 200. 4. 392[illegible].
5. 1 metre = 3·192...ft. 6. 44 : 65. 7. $31\frac{6}{11}$ oz. 8. 113·0016...
9. 24*f.* 44[illegible]*c.* 10. 860. 11. 201[illegible].
12. 4 lbs. 8 oz. 12 dwts. 19·7...grs. 13. 3457[illegible].
14. £2. 1. 7[illegible]. 15. 2*s.* $11\frac{1}{2}$·53691... 16. 1*s.* $8\frac{1}{4}$·2646...
17. 2*s.* 0[illegible]... 18. £878. 15. 8[illegible]... 19. 114 miles 6 f. 13 p. 3[illegible] yds.
20. 1·7242...*d.* 21. £3. 0. $8\frac{1}{2}$... 22. 16·0069...*fl.*
23. 21 *fr.* 80 *c.* 24. 6*s.* 6[illegible]·9...
25. £116. 17. $5\frac{3}{4}$·6...

Ex. 50.

1. 2197, 2197, 8788, 15379; 2197, 4394, 8788, 13182.
2. 1925, 3640, 4212, 1950, 2496. 3. £22. 8*s.*, £40. 16. 8, £66. 5. 4.
4. 2 tons 2 cwt. 1 qr. 18 lbs., 1 cwt. 3 qrs. 4 lbs., 1 qr. 22 lbs.
5. 30 cwt. 1 qr. 16[illegible] lbs., 3 cwt. 3 qrs. 5[illegible] lbs., 5 cwt. 3 qrs. 5[illegible] lbs.
6. 5 cwt. 3 qrs. 4·128 lbs., 5 cwts. 2·445 lbs., 3 qrs. 5·427 lbs.
7. 28 kg. 502·5 gr., 14 kg. 470·5 gr., 701·6 gr., 175·4 gr.
8. £121. 5. 6, £179. 11*s.*, £292. 12*s.*
9. £183. 15*s.*, £122. 10*s.*, £91. 17. 6, £73. 10*s.*
10. 5*s.*, 6*s.* 8*d.*, 8*s.* 4*d.*; 5*s.* 4*d.*, 6*s.* 8*d.*, 8*s.*
11. £39. 7. $7\frac{1}{2}$, £65. 12. $8\frac{1}{2}$, £105. 0. 4, £118. 2. $10\frac{1}{2}$.
12. 16[illegible] gall., 25[illegible] gall. 13. £3500, £4900, £7000, £8400.

14. £5. 16. $5\frac{1}{4}$, £7. 13. $6\frac{2}{3}$. 15. £375, £270, £105.
16. £65. 12s., £28. 16s.
17. £179. 1. 4; £257. 8. 2. 18. 100, 175, 250.
19. £8. 11. 6, £13. 9. 6. 20. £11. 5s. 21. £4. 4s., £1. 16s.
22. 2 cwt. 1 qr. 21 lbs., 20 cwt. 3 qrs. 16 lbs. 23. $\frac{414}{1575}$, $\frac{38}{145}$, $\frac{89}{358}$.
24. 4 gall. 1 qt. $1\frac{4}{13}$ pt., 6 g. 3 q. $0\frac{12}{13}$ p., 8 g. $1\frac{5}{13}$ p., 13 g. 1 q. $1\frac{9}{13}$ p.
25. 3s. 8d., 1s. $7\frac{1}{4}$d., $5\frac{1}{4}$d.
26. £1. 3. 4, 18s. 8d., 16s. 4d. 27. 309440, 313308, 377130.
28. £59. 3s., £27. 19s., £6. 10s. 29. 55, 50, 32, 17, 46.

Ex. 51.

1. 2s. $4\frac{8}{9}\frac{1}{3}$; 2s. $2\frac{2}{3}\frac{1}{3}$. 2. 18 carats.
3. 2 : 7. 4. 467, 467, 836 of alloy. 5. 3 : 13.
6. 4, 5, 3, 2; or 5, 4, 2, 3; or 9, 9, 5, 5. 7. 3, 3, 3, 2.
8. 7, 2, 2. 9. 98; 416, 224.
10. $22\frac{1}{2}$ lb., 9 lb., 18 lb., $31\frac{1}{2}$ lb.; or 9 lb., $22\frac{1}{2}$ lb., $31\frac{1}{2}$ lb., 18 lb.
11. 16 lbs., 64 lbs. 12. 3, 3, 5. 13. 21 lbs. and 12 lbs.
14. 1 ton 7 cwt. 2 qrs. $26\frac{6}{7}$ lbs. 15. 115 oz., 85 oz.
16. 10 : 7; 5s. $1\frac{1}{2}$d.

Ex. 52.

1. £72. 9. 3; 18s. $1\frac{1}{4}\frac{1}{3}$. 2. £31. 1. $8\frac{1}{4}$; £29. 10. $7\frac{9}{4}\frac{1}{18}$.
3. 43797. 4. 75844. 5. 34877. 6. 58000.
7. £337. 19. $4\frac{2}{3}\frac{4}{5}$. 8. £200. 9. $12\frac{1}{2}$; $28\frac{4}{7}$; $49\frac{1}{3}$; $26\frac{17}{18}$; $18\frac{13}{19}$.
10. $2\frac{1}{3}$; $19\frac{3}{4}$; $11\frac{387}{583}$ = 11·6638...; $5\frac{66}{87}$. 11. 1s.; 2s. 6d.; 8s. 4d.
12. 71·358..., 10·556, 16·980..., ·453..., ·651...
13. 34·947..., 49·3938..., 26·643..., 155·316... 14. 12·018..., 6·276...
15. 52·312..., 91·158... 16. 5·349...
17. 75·084..., 15·226..., 9·689... 18. 9·3167... 19. £44150.
20. 61808. 21. $4\frac{1}{4}$. 22. £208. 23. £376. 24. £360.
25. £363. 12. $8\frac{1}{3}\frac{10}{11}$. 26. £8100; £168. 15s. 27. £3200.
28. £14400. 29. £1148. 30. $7\frac{1}{2}$ per cent.; £115. 15. 5.
31. 72 gallons. 32. 176 tons 18 cwt. 2 qrs. $15\frac{9}{11}$ lbs.

Ex. 53.

1. £6. 11. $6\frac{1}{2}$ 2. £1. 18. $7\frac{3}{4}\frac{1}{10}$. 3. £105. 1. $2\frac{3}{4}$.
4. £350. 14. $10\frac{1}{2}\frac{3}{16}$. 5. £6. 4. $3\frac{1}{2}\frac{3}{8}$. 6. £51. 1. $0\frac{3}{4}\frac{1}{8}$.
7. £129. 5. $6\frac{1}{2}\frac{3}{8}$. 8. £2314. 18. $8\frac{1}{4}\frac{1}{16}$; £76666. 12. $3\frac{1}{4}\frac{1}{5}$.
9. £3. 17. $4\frac{1}{4}\frac{1}{2}$. 10. £297. 5. $0\frac{1}{2}$. 11. $4\frac{1}{2}$ p.c.

12. £125. 13. £1576. 10. $10\frac{2}{3}\frac{7}{11}$. 14. £486. 3. $10\frac{1}{2}$[illegible].
15. Prem. alone £172. 14. $9\frac{3}{4}$[illegible]; whole expense £198. 6. $7\frac{1}{2}$[illegible].

Ex. 54.

1. 28. 2. £27. 10s. 3. 7s. 4. $7\frac{1}{2}$. 5. $18\frac{3}{4}$. 6. 4s. $1\frac{1}{2}d$.
7. 18s. 4d. 8. £27. 9. 12 p.c. gain. 10. 1s. $2\frac{1}{4}\frac{1}{2}$.
11. $23\frac{3}{4}$ p.c. gained. 12. $42\frac{6}{7}$. 13. $10\frac{1}{2}$. 14. $18\frac{2}{11}$.
15. £1. 16. 6. 16. $96\frac{4}{11}$ p.c. 17. £1. 8. $7\frac{5}{8}$; $8\frac{1}{3}$ p.c.
18. 3s. $3\frac{3}{4}d$. 19. 2s. $11\frac{1}{3}\frac{1}{3}$. 20. £232. 0. $7\frac{1}{2}\frac{7}{8}$. 21. £14. 6. $0\frac{2}{3}\frac{1}{2}$.
22. £6. 15. $1\frac{1}{2}\frac{1}{3}$; 0. 23. 10. 24. 392. 25. 6. 26. 11s. 8d.
27. $37\frac{1}{2}$[illegible]. 28. 25; 20. 29. 7. 30. 16. 31. $10\frac{10}{21}$ p.c.
32. 10 p.c. 33. £35. 34. 17s. $7\frac{1}{2}d$. 35. 2s. $4\frac{1}{2}d$.
36. 1s. 3d. 37. 12 lbs. 38. 3 : 13. 39. 11 : 2. 40. 55 : 53.

Ex. 55.

N.B. *The Answers in Ex. 55—61 are usually given exactly: but in most cases it ought to be considered sufficient, to obtain them to the nearest farthing.*

1. £40. 17. 6. 2. £30. 19. 4. 3. £475. 16. $4\frac{1}{2}$.
4. £102. 9. $9\frac{1}{4}\frac{1}{2}$. 5. £16. 19. $7\frac{3}{4}\frac{9}{10}$. 6. £23. 7. $1\frac{1}{2}$.
7. £2. 3. $3\frac{1}{4}$. 8. £132. 9. $2\frac{1}{4}\frac{9}{10}$. 9. £1. 19. $7\frac{1}{4}\frac{1}{10}$.
10. £256. 14. $7\frac{1}{4}$[illegible]. 11. £75. 10. $3\frac{3}{4}\frac{3}{5}$. 12. £5. 10. $9\frac{3}{4}$[illegible].
13. £47. 8. $4\frac{1}{2}$[illegible]. 14. £1356. 10. $6\frac{3}{4}$[illegible]. 15. £2. 4. $0\frac{1}{2}$[illegible].
16. £73. 6. $8\frac{1}{4}$[illegible]. 17. £36. 16. $7\frac{1}{2}$[illegible]. 18. £4. 12. $6\frac{3}{4}$[illegible].
19. £65. 1. $6\frac{3}{4}$. 20. £81. 10. $3\frac{3}{4}$[illegible]. 21. £73. 15. $1\frac{1}{4}$[illegible].
22. £979. 5. $6\frac{1}{4}$[illegible]. 23. £19. 6. $3\frac{1}{4}$[illegible]. 24. £9. 11. $8\frac{1}{4}$[illegible].
25. £3. 8. $2\frac{1}{4}\frac{1}{3}$. 26. £32. 15. $11\frac{3}{4}$[illegible]. 27. £15. 13. $2\frac{3}{4}$[illegible].
28. £8. 18. $6\frac{1}{4}\frac{4}{5}$. 29. £30. 9. $2\frac{0}{4}$... 30. £3. 18. $3\frac{3}{4}$·8...
31. £7. 10. $4\frac{1}{4}$·8. 32. £81. 3. $6\frac{1}{4}$... 33. £7. 15. $3\frac{1}{4}$·59...
34. £3. 15. $3\frac{3}{4}$... 35. £2. 19. $8\frac{1}{4}$... 36. £114. 10. $6\frac{3}{4}$...

Ex. 56.

1. £649. 4. 2. 2. £221. 3. $0\frac{3}{4}$[illegible]. 3. £591. 10. $8\frac{3}{4}$[illegible].
4. £468. 16s. 5. £392. 17. $9\frac{1}{4}\frac{1}{2}$. 6. £490. 6. 8.
7. £1316. 6. $3\frac{3}{4}$[illegible]. 8. £678. 3. $1\frac{3}{4}$[illegible]. 9. £848. 18. $7\frac{3}{4}$[illegible].
10. £559. 8. 10. 11. £339. 13. $8\frac{3}{4}$[illegible]. 12. £56. 19. $9\frac{3}{4}$...

13. £72. 7. $3\frac{1}{8}$... 14. £5117. 5. $4\frac{1}{2}$... 15. 4. 16. $3\frac{1}{4}$.
17. $3\frac{1}{5}$. 18. $2\frac{1}{3}$. 19. $5\frac{1}{2}$. 20. $3\frac{1}{3}$. 21. $3\frac{3}{4}$. 2[illegible] 6.
23. $5\frac{1}{4}$. 24. $4\frac{1}{2}$. 25. $4\frac{3}{4}$. 26. $2\frac{3}{4}$. 27. 3 yrs. 28. $4\frac{1}{2}$ yrs.
29. $3\frac{1}{2}$ yrs. 30. 3 yrs. 7 mo. 31. $4\frac{1}{4}$ yrs. 32. 5 yrs. 7 mo. 20 da.
33. 6 yrs. 8 mo. 34. 16 yrs. 35. 97 days. 36. 125 days.
37. 87 days. 38. Jan. 6th, 1871. 39. 25 yrs.
40. £291. 17. $5\frac{3}{4}$ [illegible]. 41. $2\frac{1}{2}$ p.c. 42. £38. 0. 5; £10950.
43. £217. 19. $4\frac{1}{2}$; $4\frac{1}{2}$. 44. £10. 15. $3\frac{0}{4}$·72; $2\frac{9}{32}$ or £2. 5. $7\frac{1}{2}$.
45. £31[illegible]. 10. 8; $2\frac{1}{2}$ years.

Ex. 57.

1. £850. 2. £530. 3. £1003. 16. 8. 4. £120.
5. £650. 18. $10\frac{1}{4}\frac{4}{23}$. 6. £1149. 17. $8\frac{1}{4}\frac{19}{25}$. 7. £24. 17. $11\frac{0}{4}\frac{100}{251}$.
8. £212. 4. $9\frac{1}{4}\frac{213}{463}$. 9. £45. 11. $0\frac{1}{2}\frac{30}{329}$. 10. £591. 9. $11\frac{1}{3}\frac{947}{1171}$.
11. £1123. 4*s*. 12. £1044. 11. $10\frac{3}{4}\frac{3}{7}$. 13. £120. 0. 3.
14. £152. 17. 4. 15. £5607. 16. £70. 17. 6.
17. £8. 9. $11\frac{3}{4}\frac{49}{185}$. 18. £11. 11. $10\frac{1}{2}\frac{6}{13}$. 19. £49. 11. $1\frac{0}{4}\frac{773}{853}$.
20. £46. 4. $0\frac{1}{2}\frac{214}{1409}$. 21. £48. 5. 10. 22. £4. 2. $7\frac{1}{3}\frac{5533}{7381}$.
23. £276. 15. $4\frac{3}{4}\frac{1100}{3789}$. 24. £6. 2. $7\frac{0}{4}\frac{3148}{9803}$. 25. £62. 5. $5\frac{3}{4}\frac{1152}{18507}$.
26. 19*s*. $9\frac{1}{2}\frac{80}{181}$. 27. 4 p.c. 28. 8 months. 29. £813. 9*s*.
30. 5 p.c. 31. *B*'s by £33. 6. 8. 32. 6*s*. $0\frac{1}{4}\frac{4}{5}$. 33. 6 p.c.
34. £106. 6. $0\frac{1}{2}\frac{14}{23}$. 35. $3\frac{1}{2}$ yrs. 36. £71. 11. 9.
37. £5. 2. 1. 38. 18*s*. 5*d*. 39. £16. 19. $3\frac{1}{4}\frac{4}{7}$. 40. 0.
41. $\frac{3}{505}$. 42. 80 : 83; £32. 43. £45. 16. 8; £13. 1. $10\frac{3}{4}\frac{3}{7}$.
44. £32. 5. $0\frac{1}{2}\frac{29}{37}$. 45. £973. 2. 6. 46. $6\frac{1}{4}$ p.c.; £574. 13*s*.
47. £22. 3. 4; £208. 16. 8. 48. $18\frac{14}{27}$. 49. £607. 16. $0\frac{0}{4}\frac{1844}{2037}$.
50. £1018. 7. $4\frac{1}{2}\frac{9482}{30627}$. 51. £701. 8. $8\frac{1}{4}$...
5[illegible]. £211. 10. $2\frac{0}{4}$...; 4.66...mo.

Ex. 58.

1. £3[illegible]. 12. $2\frac{1}{2}\frac{93}{73}$. 2. £18. 11. $2\frac{1}{4}\frac{3}{25}$.
3. £547. 2. $2\frac{0}{4}\frac{70}{73}$; 7*s*. $9\frac{3}{4}\frac{3}{73}$. 4. £1. 16. $11\frac{1}{2}\frac{372}{1825}$; $3\frac{1}{2}$*d*. $\frac{272}{1825}$.
5. £158. 11. $1\frac{0}{4}\frac{22}{73}$; £155. 7. 3.
6. £5123. 17. $9\frac{0}{4}\frac{318}{365}$; £5127. 9. $1\frac{1}{4}\frac{8}{71}$.
7. £1. 4. $5\frac{1}{4}\frac{141}{160}$; £1. 3. $11\frac{3}{4}\frac{435}{1021}$.
8. £651. 14. $10\frac{0}{4}\frac{842}{1825}$; $10\frac{1}{4}$*d*. $\frac{12021}{13775}$.
9. £1664. 8. $8\frac{3}{4}\frac{1747}{1885}$; £1664. 14. $10\frac{0}{4}\frac{7976}{73987}$.
10. £5. 14. $0\frac{1}{2}\frac{8018}{8115}$; 2*s*. $0\frac{1}{2}\frac{8018}{8115}$.

Ex. 59.

1. £139. 4. 7¾. 2. £9091. 15. 10½. 3. £60. 4. 3½.
4. £806. 2. 3¾. 5. £703. 9. 6¾. 6. £3107. 13. 1¼...
7. £40. 12. 1¾. 8. £9. 7. 3. 9. £17. 2. 3½.
10. £10. 15. 11. £345. 8. 6¼. 12. £40. 8. 3.
13. £29. 14. 10. 14. £96. 3. 8½. 15. £128. 11. 4¼.
16. £169. 12. 11¼. 17. £652. 16. 11¼. 18. £125. 1. 4¼.
19. £123. 8. 6½. 20. £448. 10. 7¼. 21. £14. 14. 6½...
22. £47. 15. 2½... 23. £2. 12. 0¾... 24. £18. 14. 8¾...
25. £1408. 1. 3. 26. £1554. 0. 10¾'9... 27. 874278.
28. 23851351. 29. £3. 3. 9½... 30. £12542. 17. 0¾.
31. £333. 6. 8. 32. £336. 10. 7¾. 33. A's by £336. 13. 5¼.
34. £561. 11. 1. 35. £1058. 5. 10. 36. £2104. 5. 7.
37. £129. 1. 3¼. 38. £196. 13. 4¼. 39. £124. 10. 11¾.
40. £1003. 7. 5.

Ex, 60.

1. £1420. 1. 9¾ 178/183. 2. £4978. 16. 10¼ 5/6. 3. £2676. 10. 8.
4. £2966. 1. 1¾ 6/25. 5. £867. 3. 7½ 27/37. 6. £507. 10. 6¼ 25/31.
7. £9458. 4. 7. 8. £7786. 14. 5¼. 9. £805. 3. 10¾ 23/48.
10. £806. 11. 9¼ 3/35. 11. £609. 7. 4. 12. £5418. 3. 9.
13. £2980. 17. 3¼ 11/22. 14. £21137. 10s. 15. £3863. 3. 3¼ 49/75.
16. £5019. 15. 6½ 314/361. 17. £3. 5. 3½ 14/19. 18. £4. 14. 6¾ 1/141.
19. £4. 13. 3¾ 203/413. 20. £5. 1. 4¾ 42/71. 21. 80.
22. £58. 16. 5½ 10/17. 23. £74. 1. 5¾ 1/3. 24. 160.
25. £149. 5. 9¼ 13/26. 26. £648. 14. 9¼ 8/85. 27. £84. 7. 2¾ 8/13.
28. £903. 17. 8¾ 143/287. 29. £365. 30. £109. 7. 7¾ 87/176!
31. £175. 18. 6. 32. 3730f. 3'2...c. 33. £4407. 4. 2.
34. £5805. 35. £3269. 36. £8194. 18. 5¼.
37. £13689. 7. 7¾ ½. 38. £23. 17. 9. 39. £50. 16.'6.
40. £45. 10. 8½ ⅞. 41. £155. 14. 6¾. 42. £2269. 18. 3¼ 7/19.
43. £2000. 44. 112 7/32 or £112. 4. 4½. 45. 92 1/16 or £92. 0. 9¾ 11/13.
46. 3½ p. c. 47. 4½ p. c. 48. 37 : 36. 49. 7909 : 8469.
50. Bank Stock by 8s. 4½... 51. £4318. 9. 1¾ 4/11. 52. £5391. 7. 10½.
53. £5300. 54. £5313. 4. 1¾ 11/13. 55. £908. 1. 0¼ 12/17.
56. £2729. 0. 1¾ 48/137; £5. 10. 3½ 24/137. 57. £9. 5. 11½ ⅓.
58. £5. 19. 5¾ 181/381. 59. £40. 1. 0¼ 7/17. 60. £5631. 5s.; £90. 6. 3 gain.

61. £3. 5. 6$\frac{2}{3}$... 62. £244. 6. 8. 63. £7320. 64. 91$\frac{1}{4}$.
65. 117$\frac{1}{4}$. 66. £9060. 67. £2124. 15. 9. 68. £13854. 17. 6.
69. £18. 15*s.* more. 70. £19. 16. 8 less.
71. £52. 10*s.*; £58. 6. 8 more. 72. In *A*; £31. 10*s.*
73. 3 p. c.; £582. 10*s.* 74. £32. 5*s.* more. 75. £1824.
76. 4 p. c.; £1143. 2. 10$\frac{1}{2}$. 77. 237500 f. 78. 15688 f.
79. £511. 80. 3$\frac{4}{77}$ or £3. 1. 0$\frac{3}{4}\frac{47}{77}$ p. c.; 2$\frac{74}{77}$ or £2. 19. 2$\frac{1}{2}\frac{40}{77}$ p. c.
81. $\frac{3}{100}$, $\frac{231}{8000}$; £4000. 82. £7. 10*s.* 83. £257. 5. 5.
84. £242914. 19. 7$\frac{9}{14}\frac{140}{247}$.

Ex. 61.

1. 17511 fr. 4$\frac{1}{2}$ c. 2. 11474 \$ 38$\frac{1}{2}$ c. 3. 11300 m. 12 pf.
4. 16328 fl. 33 kr. 5. 18772 r. 10 a. 9$\frac{3}{35}$ p. 6. 8997 fl. 98$\frac{1}{18}$ c.
7. £3225. 8. 2$\frac{1}{4}\frac{53}{57}$. 8. 30369 m. 18·75...pf. 9. £24. 17. 8$\frac{1}{2}\frac{10}{21}$.
10. 9*s.* 4$\frac{1}{2}$*d.*; 11 fr. 71$\frac{7}{8}$ c. 11. 7.
12. £1791. 13. 4; 45418 f. 75 c.; 23381 fl. 25 kr.
13. 124·2675...fr. or 124$\frac{1}{4}$ fr. nearly. 14. 52$\frac{1}{4}$...*d.*
15. 25 fr. 73...c. 16. 11 fl. 45 c. 17. 12 fl. 3 st.
18. £5. 9. 0$\frac{1}{4}$. 19. 20 marcs 42$\frac{1}{2}$ pf.
20. 1\$ = 4·22429...*s.* = 4*s.* 2$\frac{1}{2}$*d.* 21. 25·001...fr. or 25 fr. very nearly.
22. 5*s.* 0·8435...*d.* or 5*s.* 0$\frac{3}{8}$*d.* nearly. 23. 288 oz. 15 dwts. 5$\frac{1}{4}$ grs.
24. 12 fl. 51 c. 25. 25 fr. 61 c. very nearly. 26. Scarcely any.
27. About 4 per mille. 28. 25 fr. 32 c. 29. 2·2179... or nearly 2$\frac{1}{4}$ p. c.
30. Gain 14·673... p. c. 31. 94 fl. 68$\frac{3}{4}$...kr. 32. £21. 16. 4$\frac{3}{4}$.

Ex. 62.

1. 11 sq. ft. 68 in. 2. 35 sq. yds. 7 ft. 32 in. 3. 9 sq. ft. 121$\frac{1}{2}$ in.
4. 96 sq. ft. 93 in. 5. 41 sq. yds. 8 ft. 125$\frac{5}{8}$ in.
6. 6 sq. yds. 7 ft. 52 in. 7. 12 ac. 2 r. 6·016 p.
8. 1 ac. 1 r. 11 p. 6$\frac{5}{12}$ yds.; 977$\frac{1}{2}$ sq. yds. 9. 2·85 ac.
10. 110 sq. yds. 1 ft. 10 in. 11. 1·19 sq. yds. 12. 21 sq. ft. 50 in.
13. 764$\frac{27}{35}$ sq. yds. 14. 203 sq. yds. 2 ft. 15. 2 ft. 6$\frac{3}{4}$ in.
16. 13 ft. 1$\frac{5}{7}$ in. 17. 18 ft. 9 in. 18. 2 ft. 8$\frac{5}{8}$ in.
19. 70 yds. 20. 2$\frac{7}{16}$ in. 21. 2 ft. 5 in. 22. 142 sq. yds. 90 in.
23. 1913 sq. yds. 3 ft. 24. 72$\frac{3}{37}$. 25. 759. 26. 16$\frac{1}{3}$ yds.

27. 131 ft. 28. 800 yds. 29. 4 ac. 1 r. 35 p. 30. 24 cu. ft. 8 in.
31. 4 cu. yds. 24 ft. 1504 in. 32. 3 cu. yds. 26 ft. 297 in.
33. 91 cu. ft. 216 in.; 121$\frac{1}{3}$ sq. ft. 34. 17572[illegible] cu. ft. 35. 648.
36. 109 cu. ft. 408 in. 37. 3 ft. 1 in.: 9 sq. ft. 73 in. 38. 3 ft. 6 in.
39. 3 ft. 8 in.; 8 tons 3 cwts. 3 qrs. 1·023 lbs. 40. ·0000459...inch.
41. (1) 7 yds. 2 ft. 11$\frac{3}{5}$ in. (2) 7 sq. yds. 8 ft. 116$\frac{2}{3}$ in.
(3) 3 cu. yds. 2 ft. 934[illegible] in. (4) 47 yds. 2 ft. 6[illegible] in.
(5) 21 sq. yds. 8 ft. 1[illegible] in. (6) 65 cu. yds. 8 ft. 1283[illegible] in.
(7) 93 sq. yds. 7 ft. 66[illegible] in. (8) 86 cu. yds. 19 ft. 800[illegible] in.
42. 19 sq. yds. 128 in. 43. 33 sq. yds. 3 ft. 28$\frac{1}{2}$ in.
44. 33 sq. yds. 6 ft. 13 in. 45. 62 sq. yds. 3 ft. 38 in.
46. 154 sq. yds. 6 ft. 90$\frac{3}{4}$ in. 47. 3577 sq. yds. 7 ft. 81$\frac{3}{4}$ in.
48. 2203 sq. yds. 2 ft. 98[illegible] in. 49. 70 sq. yds. 5 ft. 125[illegible] in.
50. 168 sq. yds. 2 ft. 5[illegible] in. 51. 147 sq. yds. 6 ft. 35[illegible] in.
52. 120 sq. yds. 8 ft. 88[illegible] in.
53. 5 sq. yds. 8 ft. 82[illegible] in.; 33 sq. yds. 5 ft. 10[illegible] in.;
1696 sq. yds. 6 ft. 38[illegible] in.
54. 113 cu. yds. 9 ft. 620[illegible] in. 55. 132 cu. yds. 4 ft. 1497[illegible] in.
56. 56 cu. yds. 22 ft. 1196[illegible] in.; 203 cu. yds. 19 ft. 1673[illegible] in.;
44 cu. yds. 16 ft. 298[illegible] in.
57. £4. 4. 5[illegible]. 58. £1. 10. 0. 59. £13. 11. 4[illegible].
60. 14 yds. 1 ft. 61. £8. 18. 9. 62. £17. 12. 10[illegible].
63. 115 yds.; £2. 3. 1$\frac{1}{2}$. 64. 113 yds. 65. 34[illegible] sq. ft.
66. £1071. 17. 6. 67. 1 sq. ft. 116[illegible] in.
68. 11 cwts. 1 qr. 21 lbs. 9 oz. 9[illegible] drs. 69. 347[illegible].
70. £3. 1. 8[illegible]. 71. 2178 tons. 72. 23 lbs. 13$\frac{1}{2}$ oz.
73. £4. 1. 10[illegible]. 74. £4166. 13. 4. 75. 136$\frac{2}{3}$ days. 76. 1$\frac{1}{3}$ yds.
77. 7·15 in. 78. 6 ft.; 2 ft. 8 in.; 2 ft. 79. 83148[illegible].
80. 37 sq. yds. 5 ft. 81. 10 ft. 82. 32 ft. 83. 2 ft. 3 in.
84. £156. 85. 968 years; 180 yds. 2 ft. 4[illegible] in.

EXAMINATION PAPERS.

I. 1. £8346. 2. 6$\frac{1}{4}$*d.* 3. 1421114$\frac{1}{2}$. 4. 54. 5. £10. 18. 10.
6. $\frac{1}{120}$. 7. ·15625. 8. 6216. 9. £42. 18. 4. 10. 1820.

II. 1. 52084. 2. 2*s.* 8$\frac{1}{2}$*d.* 3. 7 tons 12 cwts. 3 qrs. 3 lbs. 4 oz.
4. 3 ft. 9 in. 5. [illegible]; 594. 6. ·00018. 7. [illegible] = ·0069.
8. 1$\frac{1}{2}$*d.* 9. £37. 10*s.* 10. 673.

III. 1. 321 qrs. 5 b. 3 pk. 1 qt. 1 pt. 2. 17. 3. £511. 3. $8\frac{3}{4}$.
4. £9584. 6. 6. 5. [illegible]. 6. $\frac{94}{111}$. 7. 2. 8. 14.
9. 634·898367. 10. 26·31772.
11. ·2163294. 12. 6·056. 13. ·01875.
14. 2122. 15. [illegible]; £1. 12. 0·750027*d*.
16. 5 p. c.; £136762. 10*s*.

IV. 1. 90,000. 2. £9. 0. $11\frac{3}{4}$; 81 miles 6 f. 39 p.
3. £650. 16. 8; £8. 3. $6\frac{1}{4}$. 4. $\frac{43}{88}$; $\frac{45}{88}$; 1. 5. £1.
6. 30; ·03; 3. 7. $\frac{1}{64}$; ·00$\frac{5}{8}$. 8. 29897; 1·21550625.
9. £4. 6*s*.; £4. 12. $8\frac{0}{4}\frac{43}{1000}$. 10. 25 p. c.
11. £54. 3. $2\frac{1}{4}$. 12. 14.

V. 1. 296; $\frac{1}{4}$*d*. 2. 4 cu. yds. 20 ft. 635·2 in. 3. £312. 13. $8\frac{3}{4}$.
4. 18. 5. ·0095; 1·05; 47500. 6. ·00375; ·1785714$\dot{2}$; $\frac{17}{72}$.
7. 3 tons 7 cwts. 3 qrs. 22 lbs. 8. 4701; $14\frac{1}{2}$. 9. 5.
10. £4802. 15. $3\frac{3}{4}$. 11. 62; $83\frac{1}{3}$ cu. ft. 12. $91\frac{1}{5}$.

VI. 1. 1 fur. 5 po. 2 yds. 2 ft. $8\frac{2}{82}$ in. 2. $95\frac{211}{350}$; $\frac{18}{35}$; $\frac{1}{47}$; $\frac{21}{155}$.
3. 45816·11479. 4. 10; ·1; 1·047619. 5. 11*s*. $1\frac{1}{2}$*d*.; ·11125.
6. 5·9726...; ·6. 7. £4166. 13. 4. 8. £5; £787. 8. $0\frac{1}{4}\frac{65}{127}$.
9. 300. 10. First kind; $1\frac{131}{300}$ or 1·35501... p. c.

VII. 2. 761...16. 3. £781328. 4. $4\frac{1}{2}$; 16 tons 17 cwts. 2 qrs.
4. [illegible]; $3\frac{29}{30}$; $1\frac{13}{30}$. 5. £4011. 6. $6\frac{3}{4}$.
6. ·82008; ·002; 200; 2; 20000.
7. ·4047...; 2 fur. 14 po. $2\frac{1}{3}$ yds.
8. 2 tons 1 cwt. 2 qrs. $14\frac{4}{3}\frac{3}{7}\frac{2}{5}$ lbs. 9. 2 men. 10. $15\frac{5}{8}$.
11. £14520. 12. 3·60555...; 7 sq. yds. 7 ft. 6 in.

VIII. 1. £17. 5. $11\frac{3}{4}\frac{1}{2}$. 2. $1\frac{1}{18}$. 3. 7·3772916.
4. 3·0688259...; [illegible]. 5. 23·171102... 6. £155. 15. $0\frac{0}{4}\frac{12}{35}$.
7. 1 hr. $57\frac{1}{7}$ m. 8. £35. 11. $6\frac{1}{4}\frac{21}{130}$. 9. 1 cwt. $21\frac{1}{2}$ lbs.
10. $118\frac{4}{7}$ p. c. 11. £8. 12. $3\frac{3}{4}\frac{177}{353}$.

IX. 2. 105 in. 3. £1820. 4. 164 yds. 2 ft. $11\frac{1}{5}$ in. 5. ·0071875.
6. 170 cu. ft. $3'\ 9''\ 2'''\ 9^{iv}\ 4^{v}$ = 170 cu. ft. $542\frac{3}{8}$ in.
7. Loses £4. 8. 1·0490734... 9. £506.
10. At 11 h. $10\frac{10}{17}$ m. A.M. on 10 April, 1871; 11 h. $26\frac{8}{17}$ m.; 10 h. $56\frac{4}{17}$ m.

X. 1. £1905. 15*s*.; £360. 2. English navvies; £4000.
3. 15; £384; 3.361 = $\frac{121}{36}$, &c. 4. £1050.

XI. 1. 6*s*. 5$\frac{1}{2}$*d*. 2. $9\frac{9}{16}$ yds. 3. £17. 3. 4. 4. 4*s*. 8*d*.
5. ·00118359375. 6. £34. 8. 11[illegible]·544.

XII. 1. 71874. 2. £32. 5. 0$\frac{1}{2}$. 3. 3 lbs. 7 oz. 15 dwts.
4. 192; No. 5. £1. 0. 10$\frac{1}{4}$. 6. 156.
7. 1199·365234375 sq. yds. 8. 2$\frac{1}{2}$ p. c.
9. £19. 4. 11; £1. 13. 3.

XIII. 1. ·015625; ·00704; ·45056; [illegible]. 2. 914 yds. 1 ft. 7 in.
3. 7 ft. 8 in. 4. 3 : 7. 5. 7$\frac{1}{8}$ in. 6. 6$\frac{1}{4}$ p. c.; £574. 13*s*.

XIV. 1. [illegible]; 7; ·00029296875. 2. ·00015625; 1 cwt. 1 qr. 20 lbs.
3. £105. 8. 4; £104. 7. 3. 4. £500; £541. 13. 4.
5. 100372; ·17157...; Art. 164.

XV. 1. $\frac{3}{8}$; ·0000152. 2. $2^2.7.13$; $2^3.3^2.5.7$; $3^2.5.11^2$.
3. £3. 6. 4[illegible]; ·298828125. 4. [illegible]; 109$\frac{1}{4}$ ft.
5. 14479673·94...

XVI. 1. 2415. 2. $3\frac{1}{2}$; $3\frac{9}{17}$ = 3·5294... 3. [illegible]; [illegible].
4. ·3183098...; ·808. 5. 18·0000015...; 7·745966...
6. 47 hrs. 3 m. 20 s.

XVII. 1. £1530. 2. £18. 15*s*. 3. £297. 13. 5$\frac{1}{4}$ [illegible].
4. ·57004987...; 570049·87...; ·47691.
5. [illegible]; [illegible]. 6. £30. 14. 8$\frac{1}{4}$ [illegible]. 7. 31920; 8.
8. 165. 9. £21. 9. 3$\frac{1}{2}$... 10. 2 miles.

XVIII. 1. £65. 2. 1. 2. £6. 3. 6. 3. [illegible]. 4. [illegible]; [illegible].
5. £4. 13. 4; 3*s*. 7$\frac{1}{2}$*d*. 6. ·00027825; 6·94. 7. £119. 4. 8$\frac{1}{2}$ [illegible].
8. 36 : 110. 9. 3*s*. 7$\frac{1}{4}$ $\frac{1}{3}$. 10. 37$\frac{1}{2}$ p. c.
11. £64. 6. 10$\frac{1}{4}$ [illegible].

XIX. 1. Art. 31, 2; 152. 2. Art. 67; 19. 3. 442...11; Art. 52.
4. 73·52494221; 3·628693554. 5. [illegible]. 6. [illegible]; £17. 4. 2.
8. 12*s*. 4[illegible]. 9. £4307. 11. 10$\frac{1}{2}$: £75. 10. 4$\frac{1}{2}$.
10. 1 qr. 13[illegible] lbs. 11. 9*s*. 7[illegible]*d*. [illegible].
12. 2[illegible] or 2·50015... p. c. 13. 8 p. c. 14. 3$\frac{1}{2}$ hrs.
15. 5*s*. 10*d*.

XX. 1. Art. 15. 2. Art. 67; 2003. 3. 3.
4. 982·99883; 6·01; 60100; ·0601. 5. 270·33006339967.
6. $\frac{128881}{301183}$; $\frac{5}{7}$. 7. £64. 8. 70 yds.
9. £1377. 12. 6; £36. 3. $1\frac{1}{2}\frac{5}{14}$. 10. £275.
11. $4\frac{1}{2}$ p. c. 12. 20 yards from it.

XXI. 1. 436985; &c. 2. 1864000; 1848000. 3. 2 cwts. 26 lbs.
4. £34. 6. $4\frac{3}{4}\frac{1}{5}$. 5. £67. 12. $0\frac{1}{2}\frac{7}{18}$. 6. $11\frac{1}{12}$; 20.
7. 60320000; 20500. 8. 3366·204361; 137·65.
9. £18. 19. 11; 1s. $8\frac{3}{4}\frac{1}{5}$. 10. £5600. 11. 7 ft. 12. 23 p.c.

XXII. 1. 270728,547466. 2. $\frac{25}{351}$; $\frac{1}{4}$. 3. 117; 23400.
4. 7371 : 2089. 5. 35 and 25. 6. 3; $\frac{61}{79}$.
7. 121·11458...; 14·857142; Art. 157.
8. 6·5504777; 18s. $1\frac{1}{4}$ ·4048.
9. £100. 0. $0\frac{3}{4}\frac{3}{500}$; £28. 12. $3\frac{3}{4}$... 10. £675; £705. 16. $7\frac{1}{4}\frac{2}{5}$.
11. $13\frac{473}{876}$ or 13·765... p. c. 12. 3 : 2.

XXIII. 1. 56725. 2. $1\frac{1}{2}$; $1\frac{1}{4}$; $\frac{3}{4}$; 2.
3. 652·20834; ·83; 8·3; 830000. 4. ·4. 5. $\frac{3}{7}$; $\frac{1}{5}$.
6. £13. 10s. 7. 1 ft. 7 in.
8. £6775. 2. $2\frac{1}{2}$; £13. 17. $4\frac{3}{4}\frac{1}{2}$. 9. £611. 3. 4.
10. 113; 339; 678; 791. 11. £22. 0. 3. 12. 0.

XXIV. 1. 101,001,001. 2. 66; 18.
3. ·0000029; 80,000,000; 4·05536; 5·07962; $\frac{10}{19}$; $\frac{133}{270}$.
4. 2 yds. 1 ft. $2\frac{5}{8}$ in.; ·0046037946428571. 5. 322 yds. 2 ft.
6. 30 of each. 7. 5678; $4\frac{1}{7}$; ·34785...
8. £26. 13. 6; £1. 2. 6. 9. 4 yds. 10. £702. 13. 4.
11. 90. 12. £75. 13. $34\frac{1}{8}$ miles.

XXV. 1. 74090,296787,694642. 2. 3596...52; Art. 52.
3. $\frac{78}{77}$; $1\frac{1}{2}$. 4. 1·965; ·00227; 800; 13s. $2\frac{1}{2}\frac{1}{5}$; ·66125.
5. 3 o'cl. p.m. Dec. 3. 6. 739; $8\frac{1}{3}$; ·411096...
7. £500. 8. 35 days. 9. £502. 13. 4. 10. 4.
11. £4725. 12. 39 yds. 13. 9 m. 1035 yds.; 9 m. 3 fur.

XXVI. 1. 3522178, Art. 41; 3998936616. 2. 1000 ft.
3. 37; 14 days 7 h. 11 m. 17 s.; 2674 days 0 h. 9 m. 59 s.
4. $\frac{448}{1885}$; $\frac{1}{4}$. 5. 27 sq. yds. 4 ft.; £10. 16. 8.
6. £32767. 11. 8; £6840. 11. 8; £26947. 0. $1\frac{1}{4}\frac{1}{4}$.

7. ·0202522255...; ·2025...; 202·52225...£2. 11. $1\frac{3}{4}$; ·3114583.
8. £1134. 18. $3\frac{1}{4}$. 9. £72. 9. 4. 10. 9s. $7\frac{1}{2}d$. 11. £10.
12. $77\frac{1}{2}$. 13. £10166. 13. $4\frac{2}{3}$ and £6000 Stock.

XXVII. 1. 70047...1145. 2. 876576 hrs.; £913. 2s.
3. £701. 6. $6\frac{2}{3}$; £781. 5. $2\frac{1}{2}$. 4. 1287; 9009.
5. $\frac{190}{3009}$; $\frac{1}{24}$. 6. $\frac{361}{8750}$; $\frac{335}{9008}$; ·3115.
7. ·18984375; 15s. $2\frac{3}{4}d$. 8. 20 p. c. 9. 4 months.
10. 102·723...or £102. 14. 6. 11. £6. 3s. 12. £2000.

XXVIII. 1. 2532837...285803562. 2. 4410075; 942.
3. 223·358...; 20·0574... oz. 4. $10\frac{4}{7}$; $\frac{4}{7}$.
5. 42·75; ·04275. 4275000; 2 ft. 8 in.
6. 321 sq. yds. $3\frac{1}{2}$ ft.; £72. 6. 3. 7. £1627. 12. 1; 4 p. c.
8. $47\frac{713}{1042}$ or 47·68... days. 9. 3 cwts. 2 qrs. 23 lbs.
10. £192. 11. 13s. $11\frac{3}{4}d$.; 15s. 10d.; 13s. 9d. 12. £7. 11. 3.

CAMBRIDGE: PRINTED BY JOHN CLAY, M.A. AT THE UNIVERSITY PRESS.